Reduced Admittance Matrix Method for Asymmetrical Load-Flow in Sequence Domain

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Abstract – **In this paper a new linear method for asymmetrical load-flow solution is presented. With 6×6 matrix model of the power system elements, the node-admittance matrix is formed by "overlapping" procedure. Applying the new scaling concept and enhanced bus classification, also synthesizing the low voltage nodes of step-up transformers in the high voltage nodes, the node-admittance matrix dimensions are reduced.**

Keywords – **Node-admittance matrix, Sequence domain, Linear method, Node reduction, Asymmetrical load-flow.**

I. Introduction

Always, the three-phase electrical power systems states are asymmetrical, which deviate more or less from the symmetrical states. The reasons for asymmetrical states are presence of long unbalanced (untransposed) lines and asymmetrical or single-phase loads (as induction furnaces and traction motors etc.). These states cause: negative-sequence currents at generator terminals rise heating in their rotors; malfunctions of protective relays; zero-sequence currents increase greatly the effect of inductive coupling between parallel transmission lines; higher power system loss etc. Therefore, for more precise analysis of three-phase power system states, the asymmetrical load-flow (ALF) analysis are required. Also, ALF calculations are required to study the effects of various phase arrangements of transmission lines, single pole switching, etc.

Because of mutual inductive and capacitive couplings between phases, 6x6 node-admittance matrices in phase domain, which describe the generators, transformers and all lines, are not sparse. But, 6×6 node-admittance matrices which describe the balanced power system elements (practically all generators, transformers and transposed lines) in sequence domain are sparse [1] (all mutual couplings between phases are eliminated). In this domain, only 6×6 node-admittance matrices, which describe untransposed lines, are not sparse. Usually, the solution of ALF problem is performed using methods in phase domain (Newton-Raphson and fast decoupled procedures) [2]. Taking into account that 6×6 node-admittance matrices which represent power system elements are sparse, it is obvious that memory for problem storage and CPU time for problem solution in the phase domain will be much greater against solutions in the sequence domain.

First solutions of the ALF problem in sequence domain are proposed in [3]. In these methods the transformer model in sequence domain is obtained by transformation of it's model in the phase domain. Because of this procedure, the advancements of direct modeling in sequence domain and application of 6×6 sparse matrices are lost. Applying the generator, transformer and transposed line models presented in [1], a new efficient linear method for ALF analysis in sequence domain is established. This method is based on the power system nodal voltage equations.

II. Power System Buses Reduction

Let us consider a three-phase (unbalanced) power system in (asymmetrical) steady state. The system consists of n three-phase buses, i.e. $3n$ phase nodes and the zero potential node R. These buses consist of: N_G generator internal buses (fictitious buses behind the generator synchronous impedances); N_G generator external buses (are buses connecting generators and their step-up transformers); N_G buses of the high voltage sides of step-up transformers; N_L load buses; N_{QV} buses in which synchronous and static compensators, capacitor and reactor units are connected; N_O transfer buses (all buses that do not belong to any of previous five groups).

The most widely used linear power system model is that of the nodal voltage equation. In the phase domain (*abc*) and sequence domain (*dio*), this model says:

$$\underline{\mathbf{Y}}^{abc}_{3n\times 3n}\underline{\mathbf{U}}^{abc}_{3n\times 1} = \underline{\mathbf{I}}^{abc}_{3n\times 1}\ , \tag{1}$$

$$\underline{\mathbf{Y}}^{dio}_{3n\times 3n}\underline{\mathbf{U}}^{dio}_{3n\times 1} = \underline{\mathbf{I}}^{dio}_{3n\times 1}\ . \tag{2}$$

The node-admittance matrix in phase domain $\mathbf{Y}^{abc}_{3n\times 3n}$ has more nonzero elements then the matrix $\mathbf{Y}^{dio}_{3n\times 3n}$ in the sequence domain which is sparse matrix.

The node-admittance matrix would be symmetrical if ideal transformers with complex turns ratios did not appear in the power systems equivalent circuits. It is always the case when the power system is treated in the phase domain, or when it is treated in the sequence domain, but transformed by the New Scaling Concept [4].

Applying this concept for normalization, the phase shifts introduced by the ideal transformers with complex turns ratios in the sequence circuits, are eliminated. Now, each generator and it's step-up transformer can be presented in the sequence domain, separate from the transmission network, as it is shown on the Fig. 1a).

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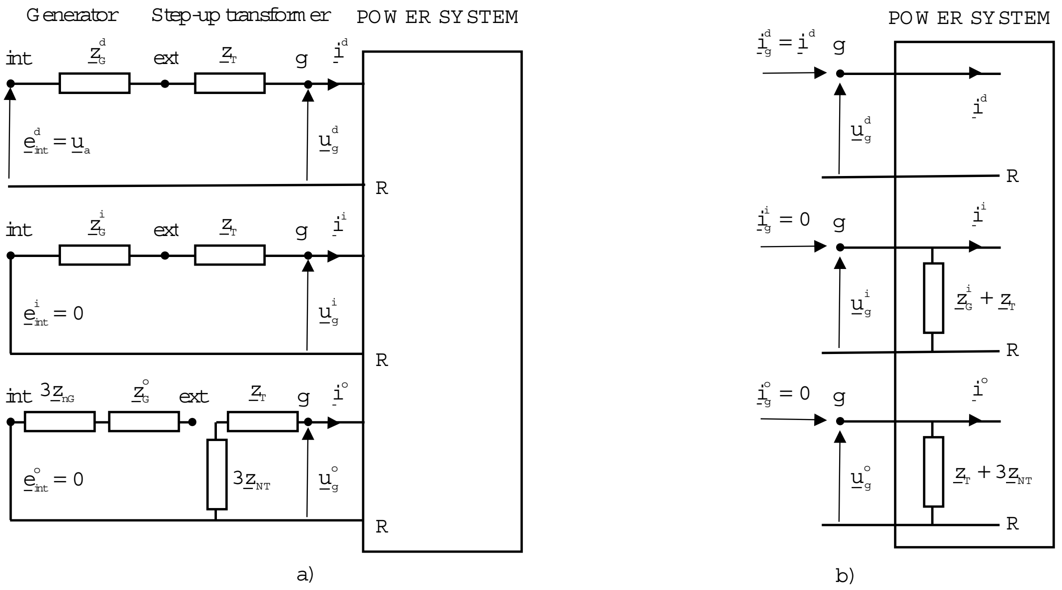

Fig. 1. Scaled sequence circuits of synchronous generator and step-up transformer a) and b) synthesized in the high voltage bus g of the step-up transformer.

The superscript of positive-sequence parameters is d, for negative-sequence is i and for zero-sequence is o; the internal and external generator buses are signed by *int* and *ext*, respectively.

The scaled sequence impedances of the synchronous generator are denoted by $\underline{z}_G^d$, $\underline{z}_G^i$ and $\underline{z}_G^o$; $\underline{z}_{nG}$ represents the generator grounding impedance; the phase a generator open-circuit voltage is denoted by $\underline{u}_a$ and it is equal to the positive-sequence internal bus voltage $\underline{e}_{int}^d$. The transformer positive-sequence and grounding impedances are denoted as $\underline{z}_T$ and $\underline{z}_{NT}$, respectively.

Because the generator internal bus voltage, as well as the voltage drops on the generator and transformer impedances are not of interest simultaneously with values of other power system quantities, the voltage control and the active power control are associated with the high voltage transformer bus g (that is the usual practice). Thus, the circuits presented in Fig. 1a are simplified as those presented in Fig. 1b, where the internal and external generator buses are synthesized in the high voltage bus of the step-up transformer. In this case, parameters of the positive-sequence are omitted in the Fig. 1b, but parameters of the negative and zero-sequence circuits are suppressed in the transmission network. The high voltage bus denoted by g may be of $P_\Sigma V$, θV or $P_\Sigma Q_\Sigma$ type [5].

It is obvious that synthesizing procedure enables power system buses reduction for $2N_G$ buses. Now, the power system can be treated as a system with $r = n - 2N_G$ buses or $3r$ nodes.

III. Reduced Node-Admittance Matrix

The node-admittance matrix is formed for a sequence circuits without ideal transformers with complex turn ratios and for a power system model with reduced number of nodes $3r$. Thus, it will be symmetrical and can be derived very simply by inspection of the power system structure. Applying the power system elements models given by 6x6 matrices [1] and "overlapping" procedure [6], the reduced node-admittance matrix is forming step by step.

Let us consider "overlapping" of two three-phase power system elements. The element E is connected between buses p and j and it's 6×6 node-admittance matrix is signed as $\mathbf{Y}_E$. The element F is connected between buses j and k and it's 6×6 node-admittance matrix is signed as $\mathbf{Y}_F$. The corresponding matrices

$$\mathbf{Y}_E = \begin{bmatrix} \mathbf{Y}_E^{(pp)} & \mathbf{Y}_E^{(pj)} \\ \mathbf{Y}_E^{(jp)} & \mathbf{Y}_E^{(jj)} \end{bmatrix} \text{ and } \mathbf{Y}_F = \begin{bmatrix} \mathbf{Y}_F^{(jj)} & \mathbf{Y}_F^{(jk)} \\ \mathbf{Y}_F^{(kj)} & \mathbf{Y}_F^{(kk)} \end{bmatrix}$$

are consist of four 3×3 sub-matrices. The sub-matrices $\mathbf{Y}_E^{(pp)}$, $\mathbf{Y}_E^{(pj)}$, $\mathbf{Y}_E^{(jp)}$, $\mathbf{Y}_E^{(jj)}$ and $\mathbf{Y}_F^{(jj)}$, $\mathbf{Y}_F^{(jk)}$, $\mathbf{Y}_F^{(kj)}$, $\mathbf{Y}_F^{(kk)}$ are diagonal if the elements E and F are balanced. But if these elements are untransposed lines, these matrices are with non-zero elements.

The connection way of these two elements in the power system is shown on the Fig. 2.

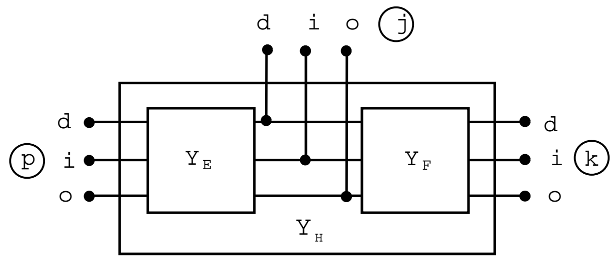

Fig. 2. Elements E and F connection in the power system.

The resultant node-admittance matrix for this part of the power system, consist of these two elements, is calculated as:

$$\mathbf{Y}_H = \begin{bmatrix} \mathbf{Y}_E^{(pp)} & \mathbf{Y}_E^{(pj)} & \mathbf{O} \\ \mathbf{Y}_E^{(jp)} & \mathbf{Y}_E^{(jj)} + \mathbf{Y}_F^{(jj)} & \mathbf{Y}_F^{(jk)} \\ \mathbf{O} & \mathbf{Y}_F^{(kj)} & \mathbf{Y}_F^{(kk)} \end{bmatrix}. \quad (3)$$

Thus, continuing by this procedure, adding element by element, the reduced node-admittance matrix $\mathbf{Y}^{dio}_{3r\times 3r}$ for the power system of $3r$ nodes, can be formed very easy.

IV. Linear Model in Sequence Domain

The nodal voltage equation for the power system represented with reduced number of buses shown on Fig. 1b, is:

$$\underline{\mathbf{Y}}^{dio}_{3r\times 3r}\underline{\mathbf{U}}^{dio}_{3r\times 1} = \underline{\mathbf{I}}^{dio}_{3r\times 1} \; . \tag{4}$$

The dimensions of the linear power system model given by Eq. (1) are $3n \times 3n$. In contrast to this model, the dimensions of the proposed model (Eq. (4)) are reduced to $3r \times 3r$. The proposed model is advanced with respect to the model given by Eq. (2), [3], as follows:

1. The node-admittance matrix is performed for a circuit without ideal transformers with complex turns ratios (thus, it is symmetrical and it can be derived very simply by inspection);
2. The node-admittance matrix dimensions are reduced for all generator internal and external buses;
3. There is no need to introduce small fictitious zero-sequence admittances at Δ-sides of step-up transformers to avoid the singularity of corresponding node-admittance matrices.

If the three-phase bus, type θV is a last numbered bus r, then Eq. (4) represented with sub-matrices, gets the form:

$$\begin{bmatrix} \mathbf{Y}^{dio}_{11} \mathbf{Y}^{dio}_{12} \cdots \mathbf{Y}^{dio}_{1s} \cdots \mathbf{Y}^{dio}_{1t} \cdots \mathbf{Y}^{dio}_{1r} \\ \mathbf{Y}^{dio}_{21} \mathbf{Y}^{dio}_{22} \cdots \mathbf{Y}^{dio}_{2s} \cdots \mathbf{Y}^{dio}_{2t} \cdots \mathbf{Y}^{dio}_{2r} \\ \cdots \\ \mathbf{Y}^{dio}_{s1} \mathbf{Y}^{dio}_{s2} \cdots \mathbf{Y}^{dio}_{ss} \cdots \mathbf{Y}^{dio}_{st} \cdots \mathbf{Y}^{dio}_{sr} \\ \cdots \\ \mathbf{Y}^{dio}_{t1} \mathbf{Y}^{dio}_{t2} \quad \mathbf{Y}^{dio}_{ts} \quad \mathbf{Y}^{dio}_{tt} \quad \mathbf{Y}^{dio}_{tr} \\ \cdots \\ \mathbf{Y}^{dio}_{r1} \mathbf{Y}^{dio}_{r2} \quad \mathbf{Y}^{dio}_{rs} \quad \mathbf{Y}^{dio}_{rt} \quad \mathbf{Y}^{dio}_{rr} \end{bmatrix} \begin{bmatrix} \mathbf{U}^{dio}_{1} \\ \mathbf{U}^{dio}_{2} \\ \cdots \\ \mathbf{U}^{dio}_{s} \\ \cdots \\ \mathbf{U}^{dio}_{t} \\ \cdots \\ \mathbf{U}^{dio}_{r} \end{bmatrix} = \begin{bmatrix} \mathbf{I}^{dio}_{1} \\ \mathbf{I}^{dio}_{2} \\ \cdots \\ \mathbf{I}^{dio}_{s} \\ \cdots \\ \mathbf{I}^{dio}_{t} \\ \cdots \\ \mathbf{I}^{dio}_{r} \end{bmatrix} \tag{5}$$

In common terms, the sub-matrix with dimensions $3 \times 3$is:

$$\mathbf{Y}^{dio}_{st} = \begin{bmatrix} \underline{Y}^{dd}_{st} & \underline{Y}^{di}_{st} & \underline{Y}^{do}_{st} \\ \underline{Y}^{id}_{st} & \underline{Y}^{ii}_{st} & \underline{Y}^{io}_{st} \\ \underline{Y}^{od}_{st} & \underline{Y}^{oi}_{st} & \underline{Y}^{oo}_{st} \end{bmatrix}, \; s,t = 1,2,..,r$$

.

If the three-phase element, connected in the power system between buses s and t is balanced, this sub-matrix is diagonal. But, if the element is unbalanced this sub-matrix is with all non-zero elements. The same conclusion can be established for $s = t$. Namely, if any three-phase element connected into bus s (or t) is unbalanced, the sub-matrix $\mathbf{Y}^{dio}_{ss}$ (or $\mathbf{Y}^{dio}_{tt}$) is with all non-zero elements. The sequence circuits voltages of bus s and injected currents in bus s (s=1,2,...,t,...,r) are given in matrix form, respectively $\mathbf{U}^{dio}_{s}$ and $\mathbf{I}^{dio}_{s}$.

Depending of the power system buses type, the following values are specified:

a) for the slack bus, type θV, the angle and magnitude of a voltage in positive-sequence circuit:

$$\underline{U}^{d}_{r} = \underline{U}^{d}_{r,sp} = U^{d}_{r,sp}\angle\theta^{d}_{r,sp}; \tag{6}$$

b) for the buses type $P_\Sigma V$, values of three-phase injected active powers (P^Σ_g) and magnitude of the positive-sequence voltage:

$$P^\Sigma_g = P^a_g + P^b_g + P^c_g = P^\Sigma_{g,sp} \text{ and } U^d_g = U^d_{g,sp}, \; g \in \{P_\Sigma V\};$$

c) for the bus type PQ, values of three pairs of injected active and reactive powers:

$$P^a_p = P^a_{p,sp}; \; P^b_p = P^b_{p,sp}; \; P^c_p = P^c_{p,sp},$$
$$Q^a_p = Q^a_{p,sp}; \; Q^b_p = Q^b_{p,sp}; \; Q^c_p = Q^c_{p,sp}, \; p \in \{PQ\}.$$

Because for the slack bus, the complex voltage of the positive-sequence circuit is known (specified), the corresponding equation should be excluded from the linear equations system given by matrix Eq. (5). This exclusion is performed by specific transformation of Eq. (5) presented in [7], without losing the dimensions of the equations system. After the transformation procedure, a new corrected system of linear equations (Eq. (7), with the same unknown values as a system given by Eqs. (4) or (5), is established:

$$\mathbf{Y}^{(dio)k}_{3r\times 3r}\mathbf{U}^{dio}_{3r\times 1} = \mathbf{I}^{(dio)k}_{3r\times 1}. \tag{7}$$

Although, the modules of the complex voltages in the nodes of the positive-sequence circuit for the $P_\Sigma V$ type buses are specified, in this method they are treated as unknown values.

V. Linear Equations System Solution

The main purpose of the asymmetrical load-flow solution is to obtain the complex voltages in all power system buses. With known complex voltages, the injected active and reactive powers into all buses, as well as the powers in all system elements, can be easy calculated.

The solution of the linear equations system, given in matrix form by Eq. (7) is iterative. For the initial iteration (h=0) the initial values are needed. The symmetrical phase voltage "flat profile" is preferred for the initial values: 1 p.u. for the PQ type and specified magnitude values for the $P_\Sigma V$ type buses. Assuming $\cos\varphi_g$ factor (usually nominal) for the block (generator and its step-up transformer), the initial values of the injected reactive powers into $P_\Sigma V$ type buses are:

$$Q^\Sigma_g = P^\Sigma_{g,sp} tg\varphi_g, \;\; g \in \{P_\Sigma V\} \; . \tag{8}$$

The positive-sequence injected currents initial values, in this type of buses, can be calculated from the equation, which expresses the total injected power in the same bus:

$$\underline{U}^d_g \underline{I}^{d*}_g - \underline{U}^i_g \underline{U}^{i*}_g \underline{Y}^{i*}_g - \underline{U}^o_g \underline{U}^{o*}_g \underline{Y}^{o*}_g = (P^\Sigma_{g,sp} + jQ^\Sigma_g/3, \in \{P_\Sigma V\}.$$

The negative- and zero-sequence injected currents into $P_\Sigma V$ and θV type of buses, in every iteration are:

$$\underline{I}^i_g = 0; \; \underline{I}^o_g = 0, \quad g \in \{P_\Sigma V\} \cup \{\theta V\}.$$

Zero valued currents are obtained by suppression of the negative- and zero-sequence admittances (equivalent of generator and it's step-up transformer) into reduced power system node-admittance matrix $\mathbf{Y}^{dio}_{3r\times 3r}$ (section II).

Taking into account all mentioned above, the complex voltages in the sequence circuits nodes, can be calculated in a few steps, for each iteration h.

Step 1. The positive-sequence injected current in buses type $P_{\Sigma}V$ (with subscript g) are calculated by following procedure:

a) Complex voltages in iteration h, are corrected with specified magnitude and calculated argument, as:

$$\underline{U}_g^{(d)k}(h) = \underline{U}_g^d(h)\, U_{g,sp}^d \Big/ U_g^d(h),$$

b) With the values from iteration h, the injected reactive powers are calculated as:

$$Q_g^{\Sigma}(h) \cong 3 I_m \left[\underline{U}_g^{(d)k}(h)\, I_g^{d*}(h) - \underline{U}_g^i(h)\, \underline{U}_g^{i*}(h)\, Y_g^{i*} - \underline{U}_g^o(h)\, \underline{U}_g^{o*}(h)\, Y_g^{o*}\right] .$$

c) The generator internal bus complex voltage $E_{int}^d(h)$ is calculated from the equation for total injected power in the bus g:

$$P_{g,sp}^{\Sigma} + jQ_g^{\Sigma}(h) \cong 3\left[\underline{E}_{int}^d(h) - \underline{U}_g^{(d)k}(h)\right]^* \cdot \underline{Y}_g^{d*} \underline{U}_g^{(d)k}(h) - 3\underline{U}_g^i(h)\underline{U}_g^{i*}(h)\underline{Y}_g^{i*} - 3\underline{U}_g^o(h)\underline{U}_g^{o*}(h)\underline{Y}_g^{o*} .$$

d) The new "more correct" value of the positive-sequence injected current is obtained as:

$$\underline{I}_g^d(h+1) \cong \left[\underline{E}_{int}^d(h) - \underline{U}_g^d(h)\right]\ \underline{Y}_g^d .$$

Step 2. In this step the positive-, negative- and zero-sequence injected currents in the buses type PQ are calculated.

a) By the complex sequence voltages $\underline{U}_p^d(h)$, $\underline{U}_p^i(h)$, $\underline{U}_p^o(h)$, the phase complex voltages $\underline{U}_p^a(h)$, $\underline{U}_p^b(h)$ and $\underline{U}_p^c(h)$ are calculated.

b) The phase injected currents are calculated by the specified powers, with equations:

$$\begin{aligned}
\underline{I}_p^a(h+1) &\cong -\left(P_p^a + jQ_p^a\right)^* \Big/ U_p^{a*}(h),\\
\underline{I}_p^b(h+1) &\cong -\left(P_p^b + jQ_p^b\right)^* \Big/ U_p^{b*}(h),\\
\underline{I}_p^c(h+1) &\cong -\left(P_p^c + jQ_p^c\right)^* \Big/ U_p^{c*}(h).
\end{aligned} \tag{9}$$

c) If the phase to sequence domain transformation is used, from the phase injected currents (Eqs. (9)), the sequence injected currents $\underline{I}_p^d(h{+}1)$, $\underline{I}_p^i(h{+}1)$ and $\underline{I}_p^o(h{+}1)$ are obtained very easy.

Step 3. The sequence injected currents calculated in previous steps are applied in the right side in Eq. (5). After appropriate transformation [7], the new modified system (Eq. (7)) with same sequence unknown complex voltages is obtained. A very suitable method for this linear equations system solution is Gauss's method of coefficient elimination.

Step 4. If the convergence criteria is achieved, the iteration procedure stops. But, if it is not achieved, a new iteration going on from the step 1.

VI. Method Verification

The ALF solution for several power systems (including the entire power system of the Republic of Macedonia) in several asymmetrical states are performed with above presented method. The results validity are compared with rezults obtained by the well known Newton-Raphson and fast decoupled procedures in phase domain. The results are identical among all methods. Althought the proposed method is linear, the eficiency in occupied CP memory and CP calculation time is on it's side, against the methods in phase domain [2].

VII. Conclusion

In this paper a new linear method for asymmetrical load-flow solution in sequence domain is presented. Several recently published procedures as: power system elements modeling in sequence domain and the negative- and zero-sequence equivalent admittances of the generator and it's step-up transformer suppression into the power system node-admittance matrix, are applied. A new reduced form of power system node-admittance matrix is obtained by "overlapping" procedure. Nodal voltage equations for the power system with reduced number of buses represents a linear model. A procedure for the node injected sequence circuits currents calculation, needed for equations system solution is explained.

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