Methods for 2D Transformations in Modeling of Real Medical Signals

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Abstract – **The types of transformations and their combinations in modeling of real medical signals are given in this article.**

Keywords – **Medical images , 2D medical signals , Education concerning treatment of medical signals , Distance education.**

I. Introduction

The creation of real 2D and 3D medical images is a combination of two fundamental processes – modelling (defining geometry of medical objects) and representing (displaying) these mathematical models. In order to define geometry of medical signals and objects some polygons, curves and surfaces have been used. The process of modelling is connected with thousands of transformations of coordinates of the points, which define relevant images or signals.

II. Types of Transformations

In modelling, a lot of applications have to alter or manipulate medical signals and images by changing their size, position or orientation. Therefore, the basic types transformations are translation, scaling, rotation and shearing.

When it is necessary to translate medical signals, set of points must be shifted in new position. If x and y are coordinates of a point $P(x, y)$, the coordinates of the new point $P'(x', y')$ after the translation are given by:

$$\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \ , \end{aligned} \tag{1}$$

where t_x and t_y have constant value. This is shown in Fig. 1:

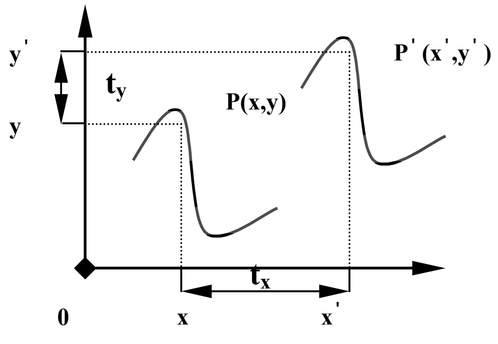

Fig. 1. Translation of signals

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The scaling is used to change the size of signals and images. The scaling about origin (0, 0) is multiplication of the coordinates x and y of points respectively with scale factors S_x and S_y.

The coordinates of the new point after that transformation are:

$$\begin{aligned} x' &= x.S_x \\ y' &= y.S_y \end{aligned} \tag{2}$$

This is shown in Fig. 2:

The possible cases of scaling are two:

1) $S_x = S_y$ – case of symmetric scaling transformation – the size of signals or images changes equally in x and y axes. If $|S_x| > 1$ and $|S_y| > 1$, the size of signals increases. If $|S_x| < 1$ and $|S_y| < 1$, the size reduces.

2) $S_x \neq S_y$ - case of asymmetric scaling transformation. If $S_x < 0$, the medical images reflect in the y axis. If $S_y < 0$ – the signals reflect in the x axis.

The rotation has been used to orientate medical signals and images in 2D plane, as they have been turning on angle α. The coordinates of the point $P(x, y)$, which belongs to the medical signal, are:

$$\begin{aligned} x &= R.\cos\beta \\ y &= R.\sin\beta \ , \end{aligned} \tag{3}$$

where R is the length of the line, connecting point $P(x, y)$ and point (0, 0). After the rotation the new coordinates of point $P'(x', y')$ are:

$$\begin{aligned} x' &= R.\cos(\alpha+\beta) = x.\cos\alpha - y.\sin\alpha \\ y' &= R.\sin(\alpha+\beta) = x.\sin\alpha + y.\cos\alpha \ , \end{aligned} \tag{4}$$

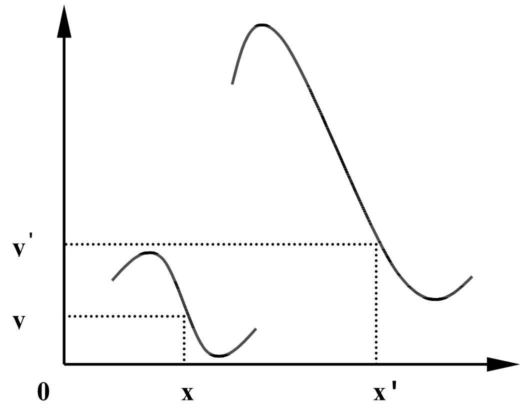

Fig. 2. Scaling

where β is the angle between x axis and the line R.

The rotation is displayed in Fig. 3:

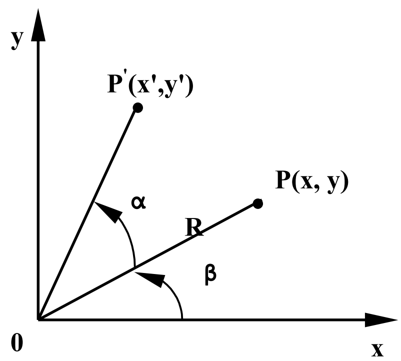

Fig. 3. Rotation

The shear transformation is a type of transformation that has the effect of distorting the signals and images in medicine. The coordinates of the new points, after the shearing, are:

$$x' = x + y.a$$
$$y' = y + x.b\ . \qquad (5)$$

If $a \neq 0$ then shear transformation in x axis appears and if $b \neq 0$ then shear of medical signals in y axis appears.

III. Matrices of the 2D Transformations

In the common case, the coordinates of the new point after transformation are:

$$x' = a.x + b.y + c$$
$$y' = d.x + e.y + f\ , \qquad (6)$$

a, b, c, d, e, f are constants, x' and y' are linear function of x and y. The matrix forms of the above mentioned expressions are :

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ d & e \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ f \end{bmatrix} \qquad (7)$$

or

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} . \qquad (8)$$

The matrices of the constants a, b, c, d, e, f (for each transformation type) are:

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \text{ – for translation ,}$$
$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \end{bmatrix} \text{ – for scaling ,}$$
$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \end{bmatrix} \text{ – for rotation ,} \qquad (9)$$
$$\begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \end{bmatrix} \text{ – for shearing .}$$

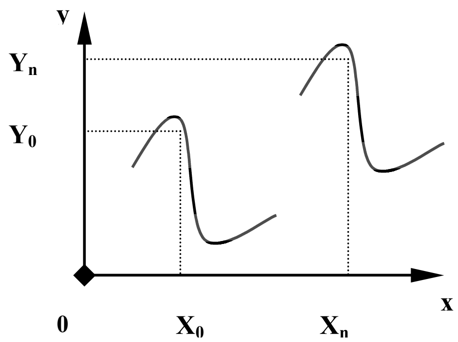

Fig. 4. Object transformation – shift object by (dx, dy)

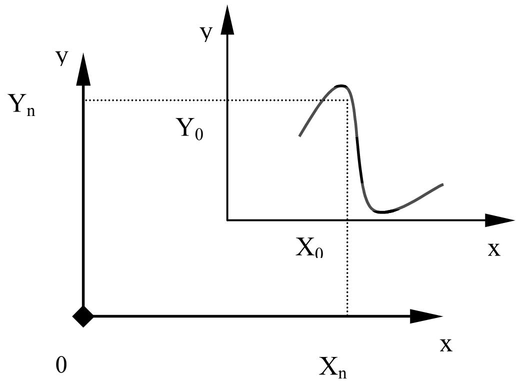

Fig. 5. Axis transformation – shift axes by $(-dx, -dy)$

IV. Object and Axis Transformation of the Medical Signals and Images

The transformations, which have been discussed in the above sections, are object transformations. The object transformation of medical signals is a method, where the signal is transforming, while the axes x and y staying fixed. Another method for transformation of medical signals and images is axis transformations, where the signal has remained fixed, while the axes have been changing. In the first case the set of points, that forms the signal, is shifted by (dx, dy). The transformed points are plotted relative to the same set of axes. The second case shows that the axes are shifted by $(-dx, -dy)$. The points on the medical signal are fixed in the space, but they are shifted by the new axes. These facts can be seen in the next figures – Fig. 4 and Fig. 5. After object transformation coordinates of the new point (x_n, y_n) are:

$$x_n = x_0 + dx$$
$$y_n = y_0 + dy\ . \qquad (10)$$

The new coordinates of the same point don't change after axis transformation by $(-dx, -dy)$. Therefore axis transformation is equivalent to an equal and opposite object transformation. This conclusion is valid for all types of transformation. Hence an object scaling transformation with coefficients S_x and S_y is equivalent respectively to an axis scaling transformation with coefficients $(1/S_x)$ and $(1/S_y)$. An object rotation with angle (α) is equivalent to an axis rotation with angle $(-\alpha)$.

The matrices for object and axis transformations of the medical signals are:

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \text{ – translate object by } (t_x, t_y) \text{ ;}$$
$$\begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \end{bmatrix} \text{ – translate } (t_x, t_y) \text{ ;}$$
$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \end{bmatrix} \text{ – scale object by } (S_x, S_y) \text{ ;}$$
$$\begin{bmatrix} 1/S_x & 0 & 0 \\ 0 & 1/S_y & 0 \end{bmatrix} \text{ – scale } (S_x, S_y) \text{ ;} \quad (11)$$
$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \end{bmatrix} \text{ – rotate object by } \alpha \text{ ;}$$
$$\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \end{bmatrix} \text{ – rotate axes by } -\alpha \text{ .}$$

As a common rule, the inverse of an object transformation is the corresponding axis transformation.

V. Conclusion

The main methods for transformation of points of 2D medical signals and images are translation, scale, rotation and shear. The medical transformations are divided into two groups – object transformations, where the points of signals are transformed, and axes transformations, where the coordinate axes are transformed and the signal points re-expressed relatively to the new axes.

Methods for 2D transformations can be used as in real applications, for observation, research, computation of particular tasks, also for education concerning of treatment medical signals and images.

References

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