Concatenation and Combination of Transformations of 2D Medical Signals

Penka Tz.Isaeva[1]

Abstract – **In this paper the concatenation and the combination between the basic types of transformations of 2D medical signals are given.**

Keywords – **Medical signals, Medical images, 2D signals, Education on treatment medical signals, Distance education.**

I. Introduction

The basic types of transformations of 2D medical signals can be combined at more complex operations. The composite transformations must be expressed by more complex matrices. For the general case a matrix representation of transformations is:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} . \qquad (1)$$

In the above matrix equation x and y are the coordinates of a point of the 2D medical signal before the transformation, x' and y' are coordinates of the point after the transformation and a, b, c, d, e, f are the constants for all types of transformations.

It is much easier if the square matrix of the constants is extended at (3x3) matrix and column vectors representing points have an extra entry.

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad (2)$$

If the bottom row of the matrix is [0, 0, 1] w' will be 1 and can be ignored.

II. Concatenation and Combination of 2D Transformations

Consider rotation of an image or a signal about its center $(x_c,\ y_c)$. This operation includes a number of basic transformations – translation of the signal by $(-x_c, -y_c)$ so that the center point coincides with the origin, rotation about the origin, back translation of the signal by (x_c, y_c) to its first position. Matrix representation of this combination of transformations is:

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad (3)$$

[1]Penka Tz. Isaeva is with the Faculty of Communications and Communications Technologies, Technical University, Kliment Ohridski 8, 1000 Sofia Bulgaria, E-mail: pepi_is@abv.bg

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad (4)$$

$$\begin{bmatrix} x_3 \\ y_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & (x_c - \cos\alpha.x_c + \sin\alpha.y_c) \\ \sin\alpha & \cos\alpha & (y_c - \sin\alpha.x_c - \cos\alpha.y_c) \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad (5)$$

Eq. (3) shows translation of the medical signal by $(-x_c, y_c)$, Eq. (4) – rotation by angle α about the origin point $(0, 0)$ and Eq. (5) – translation by (x_c, y_c).

The final effect of the transformations is to arrange point (x, y) onto the point (x_3, y_3). In the particular example, this is represented by matrix multiplication of three basic transformation matrices. Square matrices can multiply together to produce another square matrix on the same dimension. In this way a composite transformations can be represented by a single transformation matrix, which is obtained by multiplying together of the basic matrices. Each point of the medical signals to be transformed is multiplied by this matrix, which performs all the component transformations in one step.

III. Order in Combination of the Transformations

More than one transformation can be combined by multiplication together of the correspond matrices. Matrix multiplication is not commutative operation $\mathbf{M}_1.\mathbf{M}_2 \neq \mathbf{M}_2.\mathbf{M}_1$. Therefore the order of combination of the transformations is important. For example, the possible combinations of the basic transformations translation and rotation are two. In the first combination the medical signal is rotating and then is translating. In the second combination the signal is translating and then is rotating. The effect in the both cases is clearly not the same.

If the transformation matrix $\mathbf{M}_1$ arrange point $\mathbf{p}$ onto point $\mathbf{p}'$:

$$\mathbf{p}' = \mathbf{M}_1.\mathbf{p} \qquad (6)$$

A second transformation matrix $\mathbf{M}_2$ can to be combined with $\mathbf{M}_1$, so that $\mathbf{M}_1$ is applied first followed by $\mathbf{M}_2$. In this case $\mathbf{M}_2$ is postconcatenated with $\mathbf{M}_1$ so that:

$$\mathbf{p}' = \mathbf{M}_2.\mathbf{M}_1.\mathbf{p} \tag{7}$$

In the case when $\mathbf{M}_2$ is applied first then $\mathbf{M}_1$, $\mathbf{M}_2$ is preconcatenated with $\mathbf{M}_1$:

$$\mathbf{p}' = \mathbf{M}_1.\mathbf{M}_2.\mathbf{p} \tag{8}$$

The order in which transformations is applied can be seen by reading outwards from the vector being transformed.

IV. Homogeneous Coordinates

The square (3x3) matrices can be obtained by adding an additional row. In the same time an additional coordinate (w) must be added to the vector of the point. In this way a point from 2D space is presented in 3D homogeneous coordinates. This technique of representing a point in a space whose dimension is one greater than dimension of the point is called homogeneous representation.

The transformed point (x', y', w') on the medical signal is expressed by Eq. (2). On converting a 2D point (x, y) in homogeneous coordinates, the w – coordinate is fixed to 1, giving the corresponding homogeneous coordinate point $(x, y, 1)$. Each of the transformation matrices has bottom row [0, 0, 1]. Therefore w' always is 1 and transformed 2D point is (x', y').

If the matrix's elements of the bottom row (g, h, i) have values resulting in $w' \neq 1$, the effect of this transformation matrix is to transform point $(x, y, 1)$ in the $w = 1$ plane onto point (x', y', w') in the $w' \neq 1$ plane. The plane $w = 1$ is real-word coordinate space and the transformed point must be mapped back onto this plane. Therefore the point (x', y', w') must be projected onto the plane $w = 1$. This operation is known as homogeneous division.

The mathematical effect from the projection is in dividing the x- and y- components of the point by the w- component:

$$\begin{aligned} x' &= x'/w' \\ y' &= y'/w' \ , \end{aligned} \tag{9}$$

Therefore real – world point of the 2D medical signal is (x'', y''), where x'' and y'' are the coordinates of the projected point.

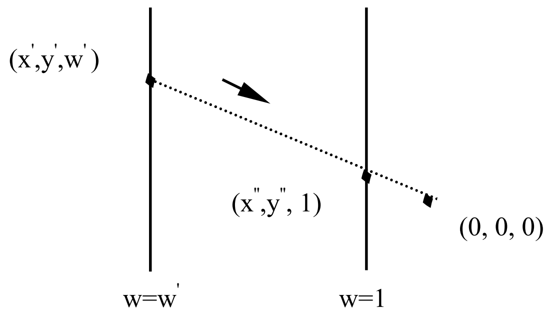

Fig. 1. Homogeneous division

V. Conclusion

The basic types of transformations of the medical signals can be expressed by matrix with dimension (3x3), which is multiplied with the vector of a point on the signal to obtain coordinates of transformed point. Thus different types of transformations of the medical signals can be combined by multiplication together of the corresponding matrices. This mean that the 2D point must be represented as 3D homogeneous point $(x, y, 1)$ to be transformed. After this transformation is obtaining point (x', y', w'). The real 2D coordinates are obtaining by division of the x and y coordinates by the w coordinate.

All these results can be applied as in the education on the medical signals and images so as theoretical basis in developing and researching of the particular problems and in creation of computer programs in this field of study.

References

[1] Foley J. D., van Dam A., Feiner S.K. and Hughes J. F., Computer Graphics: Principles and Practice (2 nd Ed.), Addison Wesley, 1990.

[2] W. Boehm et al., Computer Aided Geometric Design: A Survey of Curve and Surface Methods, 1984.

[3] Gerald Farin, Curves and Surfaces for Computed Aided Design, Academic Press, 2 nd edition, 1990.