Optimisation of Parameters of Output Filters for Direct Digital Synthesizers

Ognian I. Petkov[1]

Abstract – **Investigation of the possibility for optimum choice of phase and amplitude characteristics of the output filters of direct digital synthesizers for better noise and spurious performance.**

Keywords – **Direct Digital Synthesizers, noise, spurious and alias frequencies, output filters, amplitude and group delay time characteristics**

I. Introducion

Direct Digital Frequency Synthesis (DDFS) becomes a very popular technique for generating frequencies whenever very precise frequency resolution and fast frequency switching is needed. The most common DDFS architecture includes a periodically overflowed phase accumulator, for generating and storing phase information and uses a ROM based look up table to compute the sine function. The digitally generated sine from the ROM is next passed to a digital to analog converter (DAC) where it is converted to a pulsed analog form and finally this form is filtered by a low pass filter. The frequency of the generated sine wave is controlled by the Frequency Control Word (FCW). The output frequency could be defined from Eq. (1):

$$f_{\text{out}} = \text{FCW}\,\frac{f_{\text{clk}}}{2^j} \tag{1}$$

where f_{out} is the synthesized frequency, f_{clk} is the clock frequency and "j" is the word length of the phase accumulator. The spectral density of the generated signal could be expressed with the help of the formulae, given in Eq. (2):

$$S(f) = e^{-j\pi f/f_{\text{clk}}}\,\frac{\sin(\pi f/f_{\text{clk}})}{\pi f/f_{\text{clk}}} \times \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} c_m \sigma(f - n f_{\text{clk}} - m f_{\text{out}}) \tag{2}$$

where n and m are the numbers of harmonics of the clock and output frequencies.

As can be seen from Fig. 1 the DDFS causes all frequency components residing in the generated signal to be aliased about every harmonic of the clock frequency f_{clk}. If the output is a perfect sine wave, then the spectrum contains the frequencies $n f_{\text{clk}} \pm f_{\text{out}}$. The component, corresponding to n=0 is the desired sine wave and the others are the images or alias frequencies.

[1]Ognian I. Petkov is with the Faculty of Electronics, Technical University – Varna, Studentska 1, 9010 Varna, Bulgaria, E-mail : oipbg@yahoo.com

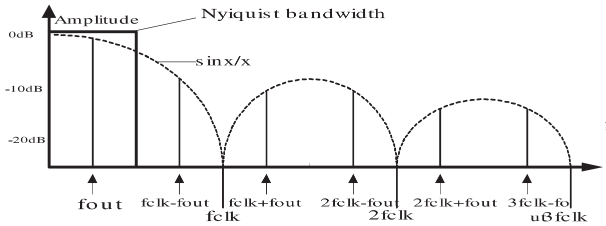

Fig. 1.

For many applications, the DDS solutions have distinct advantages over the equivalent frequency synthesizers, employing PLL circuitry. This advantages could be used more completely if we know the real nature of the synthesized output sine wave in DDS. One important subject is the generation of spurious spectrum lines and the noise at the output of the DDS and the appropriate measures for diminishing their influence on the purity of the output spectrum.

In this paper the sources of the noise and spurious frequencies will be analyzed and some solution for the output filter design will be proposed.

II. Noise and Spurious Sources in DDS

As we have seen before the most dangerous and difficult to handle are the spurious frequencies, arising from the digital nature of the generated signal. But the output spectrum contains additional spurious frequencies and wideband noise, which had to be taken in consideration. The most important of them are:

1. Spurious lines and noise due to the truncation of the phase accumulator bits addressing the sine ROM.
2. Noise due to the distortion from the compression of the sine ROM.
3. Errors, due to the limited precision of the samples, stored in the ROM.
4. Noise from the nonlinearities in the digital to analog conversion.

We need to know the carrier to noise (C/N) ratio for every one of these noise and spurious sources. From [2] it is calculated that the worst case C/N caused from the truncation of the least significant bits in the phase accumulator is from Eq. (3):

$$C_t/N \approx 6.02 - 3.92\ \text{dBc} \tag{3}$$

where k is the number of most significant bits of the phase accumulator, used for programming the ROM table.

The finite character of the quantization in the sine ROM values also leads to the output DDFS spectrum impairments.

The carrier to noise ratio for this case could be calculated from Eq. (4):

$$C_q/N \approx 6.02m + 1.76 \text{ dBc} \tag{4}$$

In high speed and high resolution (> 10 bits, > 50 MHz) DDFS most of the spurs are generated from the analog errors in the DAC. In the advanced high speed DDFS the DAC is the most critical component. The anomalies in the output spectrum, caused by the DAC do not follow the $\sin(x)/x$ roll off response.

The carrier to noise ratio on the output of the DAC could be defined from the well known formulae in Eq. (5):

$$C_q/N \approx 6.02n + 1.8 \text{ dBc} \tag{5}$$

where n is the number of bits in the DAC. In almost every state of the art DDS the number of bits after truncation in the phase accumulator is big enough in comparison with the bits in the DAC, so here one can assume that the main source of wideband noise at the output of the DDS is the quantizing noise of the DAC, which can be well predicted. For an example if we take the AD9854, which has 48 bits phase accumulator, 17 bits after truncation and 12 bits DAC. The C/N from the truncation can be calculated and is more than 102 dB and the C/N from the DAC in this case is 72 dB.

The nonlinear process in the DAC is generating harmonically related spurs in the output spectrum. The amplitude of the spurs is difficult to predict, but the location of the spurs is harmonically related to the output frequency of the synthesizer. A detailed analysis could be done for every particular DAC . At this stage of the development of the direct digital synthesis it could be seen that only the lack of high frequency and linear DACs is that stops their more intense application in the frequency bands for portable and computer wireless communications. The comparatively high level of wideband noise and the presence of alias and image frequencies prevents their applications in precise measurement equipment.

If we summarize the analysis we can make the conclusion that the main sources of spurious frequencies and noise at the DDS output are the digital nature of the signal at the output and the quantizing and nonlinearities noise from the DAC.

III. Output Filter Consideration

For every particular application of the DDS these spurious frequencies have to be minimized to a levels that are compatible with the spectrums of the other frequency sources. In almost every DDS this is accomplished from the lowpass filter after the DAC. The frequency response of an ideal lowpass filter would be 1 over the Nyquist band ($0 \le f \le F_s/2$) and 0 elsewhere. Such a filter is not physically realizable and this results in the sacrifice of some portion of the available output bandwidth in order to match the non-ideal response of the antialias filter.

The parameters of the output filters are very important for the overall performance of the DDS and the requirements they have to met define the purity of the synthesized signal.

The proper choice of the filter is connected with the reaching of some compromise between the contradictionary requirements for the great steepness of the transitional part of the frequency response of the filter and the flatness of the group delay time characteristic, which is needed for digital communication systems.

For a proper choice of a filter we need to define the different applications of the DDS and we can divide the applications in three types:

Direct digital synthesizers used individually or as a part of hybrid synthesizer where the whole possible frequency band has to be utilized.

DDS, used for a reference oscillators for hybrid DDS-PLL combination, where the only a part of the possible band of DDS is used and no need for wide band digital modulation.

DDS for a relatively narrow frequency band in mixer type hybrid synthesizers where wide band digital modulations have to be implemented.

Every one of this types has a need particularly designed low pass filter. The most serious demands are for the first application where the aim for achieving a wider bandwidth leads to reaches to the Nyquist limit of $f_{\text{clk}}/2$ in which case the first image $f_{\text{clk}} - f_{\text{out}}$ is getting closer to f_{out} and it becomes impossible to separate these two frequencies with the known filter approximations.

Eq. (6) could be used for exact calculation of the amplitude difference in dB for f_{out} and ($f_{\text{clk}} - f_{\text{out}}$). The calculations show that there difference when $f_{\text{out}} = 0.4 f_{\text{clk}}$ reaches approximately 4 dB.

$$\Delta A = 10 \lg \left(\frac{\sin\left(\frac{2\pi f_{\text{out}}}{f_{\text{clk}}}\right)}{\frac{2\pi f_{\text{out}}}{f_{\text{clk}}}} \right) - 10 \lg \left(\frac{\sin\left(\frac{2\pi (f_{\text{clk}} - f_{\text{out}})}{f_{\text{clk}}}\right)}{\frac{2\pi (f_{\text{clk}} - f_{\text{out}})}{f_{\text{clk}}}} \right) \tag{6}$$

The choice of a proper filter depends on its behavior in the frequency and time domains. Usually when the parameters of filters are analyzed the stress is put on frequency domain but there are some special cases where the time domain behavior is more important. The knowledge of the relation between the time and frequency characteristics of the filter is very important for the DDS, because depending on the application's demand one of them could be optimized. It is impossible to reach at the same time good selectivity and good pulse response from one filter. Typical filter parameters are the cut off frequency fc, the frequency where the minimum specified attenuation is reached fs, maximum ripple in the passband ($A_{\max}$) and minimal attenuation in the stopband ($A_{\min}$).

An important parameter of the filter is the group delay time or GDT, which is the first derivation of the phase and is a measure for the delay of different frequencies, when passing through the filter. GDT could be comparatively easily measured and used for criteria for the time domain characteristics of the filter.

There are a lot types of filters known from the theory [3] and practice but the most popular are Butterworth, Chebyshev and Kauer (elliptic) approximations.

On Fig. 2 from [3] a comparison can be made for fifth order filters of different types. It can be seen that only the

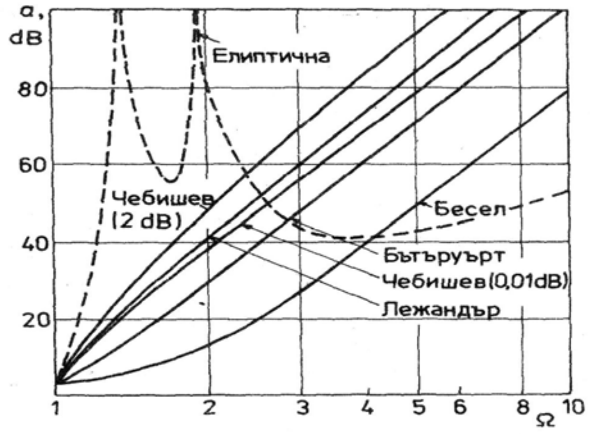

Fig. 2.

elliptic approximation of fifth order is near the needed characteristics for the filter for DDS applications.

IV. Filter Design and Simulations

On the basis of these analysis and consideration a program in Matlab was designed where the input parameters are the $(f_{\text{clk}} - f_{\text{out}}/f_{\text{out}}) - \Omega$ ratio, the ripple in the passband and the minimal attenuation in the stopband. The definition of the limits for all these parameters depends exclusively on the specific application of the synthesizer but for the start a ripple of 1 dB, Ω of 1,2 and attenuation of 60 dB and 80 dB were chosen. Parts of the results are given in Table I (1).

Table 1.

	type	Elliptic	Chebishev 1	Chebishev 2	Butterworth
60dB	order	5	9	9	14
	GDT_{100M}	2÷18	2÷14	2÷14	2÷23
	GDT_{80M}	3	5	5	7
	ripple dB	1	1	0	0
80dB	order	7	13	13	18
	GDT_{100}	2÷40	2÷45	2÷15	2÷30
	GDT_{80}	4	7	6	10
	ripple	1	1	0	0

The results from the simulation confirm the most of the intuitive made assumption and practical knowledge on this matter. As can be seen from the results for the attenuation of 60 dB filters with Chebyshev approximation still can be used with some concern on the complexity of the circuit, but for 80 db attenuation the only reasonable choice is the elliptic filter of seventh and higher order.

From the table could be seen two values for the GDT -one for the cut off frequency of 100MHz and one for 80MHz. One conclusion can be done that even the high order elliptic filter can be used near the cut off frequency with comparatively low level of GDT distortion. Another conclusion is that for equal attenuation and different order, the GDT fluctuations of the elliptic filters are similar those of Chebishev 1 and Butterworth filters.

A seventh order practical filter design was synthesized with the help of program Serenade7.5 for the outputs of the quadrature DDS AD 9854 . The circuit diagram of the filter is shown on Fig. 3.

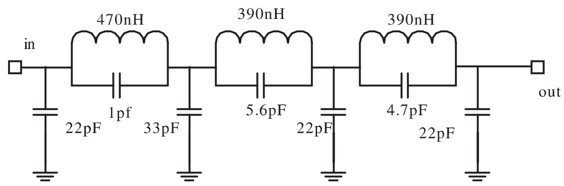

Fig. 3.

When the requirements to the purity of the output spectrum of the signal of DDS are more stringent there is a possibility for further improvement with the use of tuneable lowpass filters.

The analysis of the requirements for the characteristics of the output filters for other two DDS applications shows that good improvement can be achieved with the addition of narrow band bandpass filter. A quartz or SAW filter can filter the parasitics and improve the carrier to noise ratio at the output enough for wireless application.

References

[1] A.V. Oppenhaim, and R.W. Schafer, “Digital Signal Processing”, Prentice Hall Englewood Cliffs, New Jersey, 1975.

[2] Y .C. Jeng, “Digital Spectra of Nonuniformly Sampled Signals – Digital Look-Up Tunable Sinusoidal Oscillators”, *IEEE Trans. On ins. And Meas.*, Vol. 37, No.3, pp 358- 362, Sept. 1988.

[3] G. Stojanov, “Theoretical Foundations of telecommunication technics”, Sofia, Technica, 1993.