Chaotic Signals Generated by Some Circuits – Comparative Study (Ljapunov Exponents)

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Abstract – **In this paper an algorithm, based on the algorithm of Eckmann, Kamphorst, Ruelle and Ciliberto (EKRC algorithm) for determination of the Ljapunov exponents of chaotic signals, obtained from some circuits, has been applied. These circuits have been developed in practice. The signals, obtained at the outputs of the circuits, have been digitized and analyzed. After the determination of the Ljapunov exponents, conclusions have been made.**

Keywords – **chaotic attractors, Ljapunov exponents**

I. Introduction

It is well known, that the estimation of the Ljapunov exponents is from great importance for the examination of the chaotic behaviour. In the presented paper an algorithm, based on the one of the most popular algorithms, used for determination of the Ljapunov exponents, has been applied. From the obtained results conclusions for the application of the analyzed circuits can be made.

II. Kolmogorov's Entropy and It's Relationship to the Ljapunov's Exponents[1,2]

The entropy is a basic physical parameter. The entropy gives the rate of randomness in an observed system. The Kolmogorov's entropy is one of the most important quantitative characteristics of chaotic motion in phase space with arbitrary high dimension. The Kolmogorov's entropy (K) shows, the rate of chaotic behaviour of the observed system [2,1].

In work [1] H.G.Schuster presents the basic relations, concerning the Kolmogorov's entropy and it's relationship with the Ljapunov's exponents. The following conclusions, concerning the Kolmogorov's entropy, have been presented in [1]:

- The Kolmogorov's entropy determines in time domain the rate of the loss of information about the state of the analyzed dynamical system.
- By one-dimensional maps the Kolmogorov's entropy is directly connected to the Ljapunov's exponent.
- For high-dimensional systems the Kolmogorov's entropy is a measure for the fragments deformation in the phase space.

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- The Kolmogorov's entropy is inversely proportional to the time interval, where a prediction for the state of the analyzed system can be made.
- The lower bound of the Kolmogorov's entropy can be obtained directly from the measurement in time domain of one of the components of the chaotic system.

After Schuster the Kolmogorov's entropy is this fundamental quantity, which characterizes the chaotic behaviour and the strange attractor can be treated as a attractor, characterized by positive entropy [1].

For determination of the Kolmogorov's entropy the Ljapunov's spectrum can be used. In [2] has been shown, that, by the assumption, that the Ljapunov's exponents are known, the Kolmogorov's entropy (K) can be determined by means of the following expression [2]:

$$K \leq \sum_{i=1}^{k} \lambda_i = \sum_{\lambda_i > 0} \lambda_i,$$

where k is the number of the positive Ljapunov exponent with the most little value.

III. Algorithms for Determination of the Ljapunov Exponents [2-5]

The Ljapunov exponents are very important by the examination of the chaotic attractors and by the estimation of the entropy of the analyzed circuits or systems.

There are several well known algorithms for determination of the Ljapunov exponents [2-5]:

- Algorithm of Alan Wolf, Jack B. Swift, Harry L. Swinney, John A.Vastano.
- Algorithm of Sano and Sawada.
- Algorithm of Eckmann, Kamphorst, Ruelle and Ciliberto (EKRC algorithm).

An algorithm for determination of the Ljapunov exponents from a time series has been presented by A. Wolf, J. Swift, H. Swinney and John A.Vastano in their work [3]. The authors have been presented the relation between the dynamic behaviour of the observed system and the Ljapunov's exponents.

In [3] has been shown, that from the point of view of the information theory, from the magnitudes of the Ljapunov's exponents, a quantitative estimate of the attractor's dynamic behaviour can be obtained, or i.e. the exponents measure the rate, at which system processes create or destroy information.

The relation between the information dimension d_f and the Ljapunov's spectrum has been given by the equation [3]:

$$d_f = j + \frac{\sum_{i=1}^{j} \lambda_i}{|\lambda_{j+1}|},$$

where: $\sum_{i=1}^{j} \lambda_i > 0$ and $\sum_{i=1}^{j+1} \lambda_i < 0$ [3].

Two another well known algorithms for determination of the Ljapunov's exponents are the algorithm of Sano and Sawada and the algorithm of Eckmann, Kamphorst, Ruelle and Ciliberto (EKRC algorithm).

In their work [5] the authors (Eckmann, Kamphorst, Ruelle and Ciliberto) presented and analyzed an algorithm for computing Ljapunov exponents from an experimental time series. The algorithm includes the following steps [5]:

a) reconstructing the dynamics in a finite dimensional space.
b) obtaining the tangent maps to this reconstructed dynamics.
c) deducing the Ljapunov exponents from the tangent maps.

In [4] a comparison between the algorithm of Sano and Sawada and the EKRC algorithm has been made. After the conclusions, made there, the following properties have been observed [4]:

- The EKRC algorithm better approximates the zero and negative exponents in most cases and it seems more promising for the accurate determination of the whole spectrum.
- The EKRC algorithm gives stable Ljapunov exponents for a larger region of parameters and thus more easily determines the Ljapunov spectrum.

On other hand [4]:

- The Sano and Sawada algorithm is easy to implement and can determine the Ljapunov exponents for smaller values of the embedding dimension.

IV. Experimental Results

In what follows by the computation of the Ljapunov's exponents an algorithm, based on the well known EKRC algorithm, has been used. This algorithm has been applied by the investigations, carried out upon several circuits, designed to produce chaotic signals. The obtained values of the Ljapunov's exponents permit a qualitative and quantitative comparison between the considered circuits to be made. Based on the obtained results an estimate about the suitability of the investigated circuits for generating of chaotic signals can be made.

The determination of the time delay is from great importance for the accurate calculation of the Ljapunov exponents. The algorithm, used in the presented paper, is based on analysis of the autocorrelation functions (ACF) of the obtained chaotic signals. The time value, where the ACF for the first time obtains value near to zero, has been chosen as basis for the other necessary calculations [2].

In the presented work the Ljapunov exponents for signals, generated from the following circuits, have been computed:

1) H-generator [7];
2) 4-D generator [7];
3) Circuits, based on the canonical realization of Chua's circuit [8].

These circuits have been developed in practice. The signals, obtained at the outputs of the circuits, have been digitized by means of oscilloscope interface and have been analyzed. The obtained signals in time domain and the related autocorrelation functions have been presented in an another work [6].

Following the methodology, proposed in [2], the value for the time delay (τ) in each case has been determined.

The dynamic behaviour of the observed circuits has been analyzed in 3 or in 4-dimensional phase space. The phase space has been formed after the proposal, given in an example in [2].

An algorithm, based on the EKRC algorithm, has been implemented for determination of the Ljapunov's exponents and the Ljapunov's dimension.

A. H-generator [7]

Different signals have been obtained by the experiments. Some of them, displayed in the phase plane, have been shown on Fig. 1 [7].

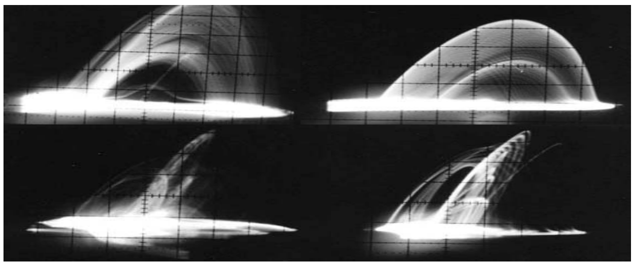

Fig. 1.

The Ljapunov's exponents for different sets of values of the parameters have been obtained:

A) By a first state of the parameter set of the presented in [7] H-generator, a chaotic signal has been obtained. The values of τ, the Ljapunov's exponents and the Ljapunov's dimension have been computed, as follows:

$$\tau = 6, \quad \lambda_1 = 0.0426, \quad \lambda_2 = -0.0137,$$
$$\lambda_3 = -0.0463, \quad \lambda_4 = -0.0864, \quad D_{Ljap} = 4.1175$$

B) By a second state of the parameter set an another chaotic signal has been obtained. The values of τ, the Ljapunov's exponents and the Ljapunov's dimension have been computed, as follows:

$$\tau = 2, \quad \lambda_1 = 0.1623, \quad \lambda_2 = 0.0117,$$
$$\lambda_3 = -0.1538, \quad \lambda_4 = -0.3167, \quad D_{Ljap} = 3.1319$$

C) By a third state of the parameter set a different chaotic signal has been obtained. The values of τ, the Ljapunov's exponents and the Ljapunov's dimension have been computed, as follows:

$$\tau = 2, \quad \lambda_1 = 0.2536, \quad \lambda_2 = 0.0135,$$
$$\lambda_3 = -0.1398, \quad \lambda_4 = -0.3084, \quad D_{Ljap} = 3.9105$$

The values, obtained for the Ljapunov's exponents, show a variation, depending on the set of the values of parameters. As expected, the magnitudes of the Ljapunov's dimension are significant. The results show, that on base of the H-generator, presented in [7], chaotic signals with different properties can be produced.

B. Four-dimensional chaotic generator [7]

In [7] a four-dimensional chaotic generator with modified external driven nonlinearity has been presented. The trajectories in the phase plane have been presented on Fig. 2 [7].

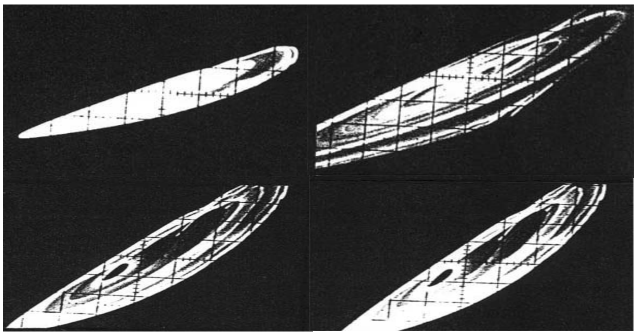

Fig. 2.

A) For a first state of the parameter set of the 4-D generator, presented in [7], the following values for τ, for the Ljapunov's exponents and for the Ljapunov's dimension have been obtained:

$$\tau = 4, \quad \lambda_1 = 0.0642, \quad \lambda_2 = 0.0105,$$
$$\lambda_3 = -0.0399, \quad \lambda_4 = -0.1452, \quad D_{Ljap} = 3.8722$$

B) By an another set of values of the system parameters the following values for τ, for the Ljapunov's exponents and for the Ljapunov's dimension have been obtained:

$$\tau = 9, \quad \lambda_1 = 0.0378, \quad \lambda_2 = 0.0085,$$
$$\lambda_3 = -0.0134, \quad \lambda_4 = -0.0537, \quad D_{Ljap} = 5.4536$$

The values of the Ljapunov's dimensions are significant. This is a logical consequence from the type of the system of differential equations, describing the processes in the 4-D generator, analyzed in [7].

The obtained results show, that the 4-dimensional generator, discussed in [7], is appropriate for generating of chaotic signals.

A first version of Chua's circuit [8]. The signals, concerning the discussed version, presented in [8], have been observed in the phase plane. They have been presented on Fig. 3 [8].

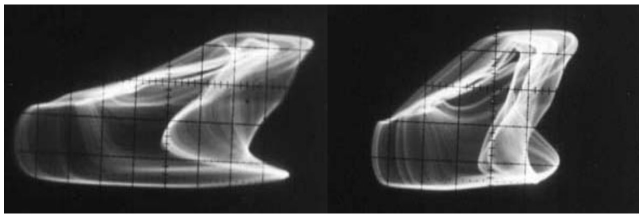

Fig. 3.

For the signal, which characteristic in time domain and the related ACF have been presented in [6], has been computed: $\tau = 2$. This signal has been obtained by a fixed set of values of the parameters. For determination of the Ljapunov exponents an algorithm, based on the EKRC algorithm, has been implemented.

Here the analysis has been conducted for a 3-dimensional system and the number of the corresponding Ljapunov exponents is 3.

The following values have been obtained:

$$\lambda_1 = 0.0998, \quad \lambda_2 = -0.0802, \quad \lambda_3 = -0.3151,$$
$$D_{Ljap} = 2.2435$$

The value of the Ljapunov's dimension is not as significant, as the values, obtained by the H-generator and the 4-dimensional generator, presented respectively in cases A and B. Nevertheless, from the final results becomes obvious, that the first version of Chua's circuit, discussed in [8], is appropriate for generating of chaotic signals.

A second version of Chua's circuit [8]. Another circuit, discussed in [8], includes external driven nonlinearity. By defined conditions (i.e by some set of values of the parameters) the presence of chaotic behaviour of the signal at the output of the circuit has been established.

The obtained signals have been observed in the phase plane and some of them have been presented on Fig. 4 [8].

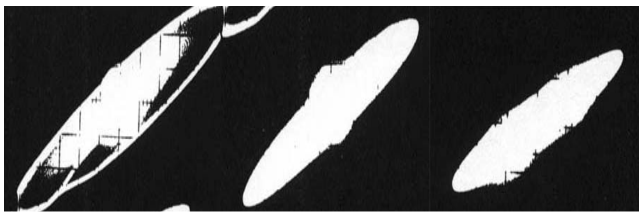

Fig. 4.

The following values for τ, for the Ljapunov's exponents and for the Ljapunov's dimension have been obtained:

$$\tau = 3, \quad \lambda_1 = 0.0885, \quad \lambda_2 = -0.0179, \quad \lambda_3 = -0.2162,$$
$$D_{Ljap} = 5.935$$

The value of the Ljapunov's dimension is significant. This fact is favourable to the implementation of the discussed variant, presented in [8], for generation of signals with chaotic behaviour.

A third version of Chua's circuit [8]. Another version of Chua's circuit, designed to generate chaotic signals, has been

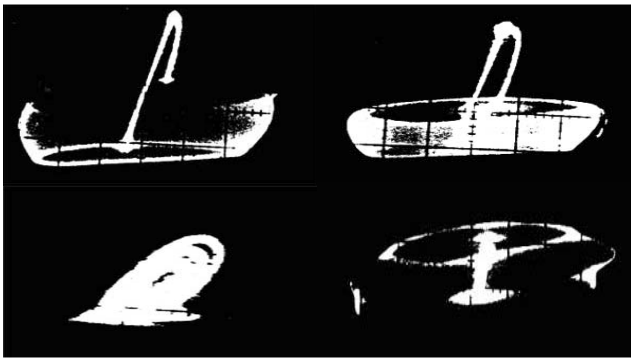

Fig. 5.

presented in [8]. At the output of the circuit a variety of signals have been obtained. Photos of the trajectories, observed in the phase plane, have been shown on Fig. 5 [8].

A) For a first state of the parameter set of the discussed circuit, presented in [8] , the following values for τ, for the Ljapunov's exponents and for the Ljapunov's dimension have been obtained:

$$\tau = 2, \quad \lambda_1 = 0.1645, \quad \lambda_2 = -0.0609, \quad \lambda_3 = -0.3401, \quad D_{Ljap} = 3.6997$$

B) By another conditions the following values for τ, for the Ljapunov's exponents and for the Ljapunov's dimension have been obtained:

$$\tau = 8, \quad \lambda_1 = 0.0419, \quad \lambda_2 = -0.0061, \quad \lambda_3 = -0.0411, \quad D_{Ljap} = 7.8468$$

C) By different values of the circuit parameters the following values for τ, for the Ljapunov's exponents and for the Ljapunov's dimension have been computed:

$$\tau = 2, \quad \lambda_1 = 0.2333, \quad \lambda_2 = -0.0324, \quad \lambda_3 = -0.3331, \quad D_{Ljap} = 8.21$$

Compared to the values of the Ljapunov's dimensions, computed for the first variant of Chua's circuit, the values, obtained here, are more significant.

It is obvious, that the magnitudes of the Ljapunov's dimension change to a considerable extent. The reason for the change in the values is the corresponding change in the set of the values of the parameters in the analyzed circuit. In this way, by means of the discussed circuit, presented in [8], by appropriate choice of the system parameters, different types of signals can be produced.

V. Conclusions

- Signals, obtained from some versions of circuits, designed to generate chaotic signals, have been studied.
- On the lab prototypes of these circuits experiments have been made.
- The signals, obtained at the outputs of the circuits, have been digitized and analyzed.
- By means of the autocorrelation functions, related to the signals, investigated in time domain, for each case the parameter has been computed and the necessary for the space construction calculations have been made.
- The Ljapunov's exponents have been computed by means of an algorithm, based on the EKRC algorithm. The algorithm, used in the presented work, will be improved and will be investigated more properly in future experiments.
- The obtained results have been compared and conclusions about the suitability for the implementation of each variant have been made.

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