Bit Error Probability of the QPSK System in the Presence of Ricean Fading and Pulse Interference

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Abstract – **The purpose of this paper is to provide the theoretical approach for determining the bit error probability in detecting a coherent quaternary phase shift keyed signal in the presence of the Ricean fading, Gaussian noise and pulse interference, as well as the noisy carrier reference signal. Phase locked loop, as the constituent part of the receiver, is used in providing the synchronization reference signal extraction, which is assumed to be imperfect in this paper. The determined results are based on the non-linear phase locked loop model with primary emphasis on the degradations in the system performance produced by the imperfect carrier signal extraction.**

Keywords – **QPSK, PLL, Pulse interference, Ricean fading**

I. Introduction

The performance evaluation of binary and M-ary ($M > 2$) phase-shift-keying communication systems have been analyzed in a great variety of papers, which have appeared in the literature [1-7]. In the majority of this papers, the system performance has been determined under the condition of the perfect signal extraction. Quaternary phase-shift-keying (QPSK or 4-PSK) systems have the greatest practical interest of all nonbinary (multiposition) systems of digital transmission of messages by phase modulated signals. Currently, QPSK is one of the prevalent modulations in use for digital communication systems (since bandwidth efficiency). The only significant penalty factor is an increased sensitivity to carrier phase synchronization error.

Any successful transmission of information through a digital phase-coherent communication system requires a receiver capable of determining or estimating the phase and frequency of the received signal with as few errors as possible. In practice, quite often the phase locked loop (PLL) is used in providing the desired reference signal [1-4]. Frequently, a PLL system must operate in such conditions where the external fluctuations due to the additive noise are so intense that classical linear PLL theory neither characterizes adequately the loop performance, nor explain the loop behavior [5]. Numerical results for QPSK system is presented so that this results combined with the characteristic of the phase recovery circuit will enable the best practical design of a QPSK system.

It is well known that the certain components that appear in telecommunication channels are very often with a pulse characteristics, i.e. noise can be described as a sum of peaks of large amplitudes in comparison with the common noise level. This channels are often narrowband, so it follows narrowband systems are considered. Poisson pulse noise model is used for modelling. Samples consist of a random delta functions. This model gives a very good approximation of the most important natural pulse noise features [6,7].

The error probability, as a measure of systems quality, is an important issue. Noise influence and pulse interference are often fundamental limiting factors in digital transmission systems. An expression for the bit error probability was calculated when the signal and Gaussian noise are applied at the input of the QPSK system [4]. QPSK system performance when the signal, Gaussian noise, pulse interference, Ricean fading and imperfect carrier phase recovery are considered as source of degradation, are determined in this paper.

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II. System Features

The model for the communication system to be considered in this paper is shown in Fig. 1. Let the input signal at QPSK receiver consists of the signal, pulse interference and Gaussian noise:

$$r(t) = A\cos(\omega_0 t + \phi_0) + A_1 n_i \cos(\omega_0 t + \theta) + m(t), \quad (1)$$

where A is a signal amplitude, ϕ_0 can be $\pi/4$, $3\pi/4$, $-\pi/4$ or $-3\pi/4$ depending upon which symbol is transmitted, ω_0 is a constant carrier frequency, $m(t)$ is a Gaussian noise, A_1 is a pulse interference amplitude, θ is the uniformly distributed phase with the probability density function $p(\theta) = 1/2\pi$, $\{-\pi \leq \theta \leq \pi\}$. Cos wave is modulated by the pulses with the form:

$$n_i = \sum_{i=-\infty}^{\infty} a_i \sqrt{\frac{2}{T}} \cos\omega_1(T - t_i), \quad (2)$$

where a_i represents a random area where i pulses appears at the random time t_i, $\omega_1 = 2\pi n/T$, n is an integer and T is a pulse duration. Moments of pulses appearing is presented as a Poisson process.

It is assumed that the signal is corrupted by the pulse interference with the following probability density function:

$$p(n_i) = (1-\gamma)\delta(t) + \left(\frac{\gamma}{\pi}\right)^{3/2} \int_0^{\pi/2} \frac{e^{-\frac{\gamma n_i^2}{4\sigma_i^2 \cos^2(y)}}}{\sigma_i^2 \cos(y)} dy \quad (3)$$

where γ represents average number of pulses which appears in the signal time duration T, σ_i is the spectral density of the pulse modulated interference.

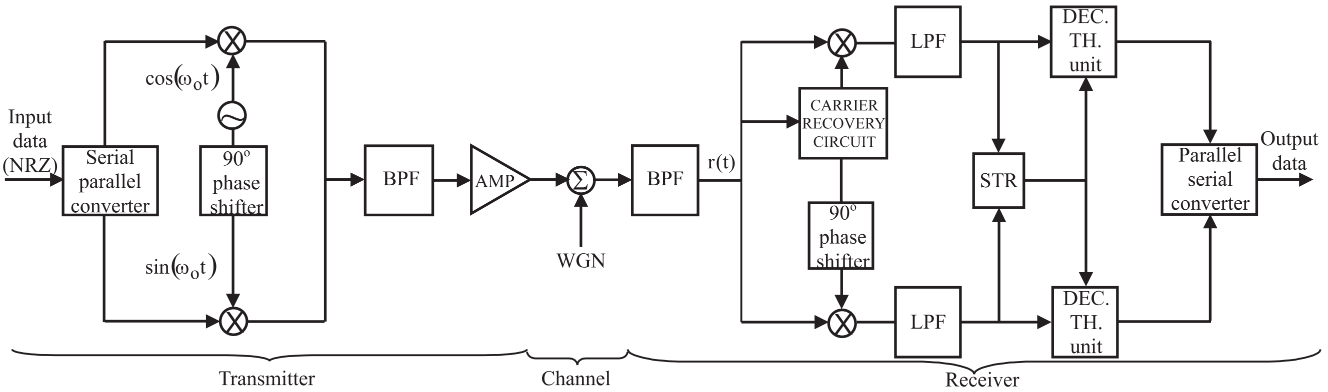

Fig. 1. QPSK communications modem

It is assumed that the signal is affected by the Ricean fading:

$$p(A) = \frac{A}{\sigma_f^2} e^{\frac{A^2+B^2}{2\sigma_f^2}} I_0\left(\frac{AB}{\sigma_f^2}\right), \quad A \geq 0,\ B = 2\sigma_f. \qquad (4)$$

Now, input signal can be also written with in the form:

$$r(t) = AR\cos(\omega t + \psi) + m(t), \quad \eta = \frac{A_1}{A},$$
$$R = \sqrt{1+\eta^2+2\eta\cos\theta}, \quad \psi = \text{arctg}\frac{\eta\sin\theta}{1+\eta\cos\theta} \qquad (5)$$

From now on, pulse interference, additive Gaussian noise Ricean fading and imperfect phase carrier recovery, are taken into account in our detection analysis. All other functions are considered ideal. The block diagram of a QPSK receiver would be adopted is shown in Figure 1. The recovered carrier signal is assumed to be in the form of the sin wave. Also, it would be adopted that a original message is in binary form.

Under the assumption of a constant phase in the symbol interval, the conditional error probability for the given phase error ϕ (the phase error ϕ is the difference between the receiver incoming signal phase and the voltage controlled oscillator output signal phase) can be written as [9],

$$P_{e/\phi}(\phi) = \frac{1}{4}\text{erfc}\Big\{\Big[\sqrt{2R_b}(\cos\phi - \sin\phi)\Big] + \text{erfc}\Big[\sqrt{2R_b}(\cos\phi + \sin\phi)\Big]\Big\} \qquad (6)$$

where the function $\text{erfc}(x)$ is the well known complementary error function defined as:

$$\text{erfc}(x) = \frac{1}{\sqrt{2\pi}}\int_x^{+\infty} e^{-z^2/2}dz. \qquad (7)$$

The received signal to noise spectral density ratio in the data channel (demodulator) denoted by R_b, is given by $R_b = E/N_0$, where E is a signal energy per bit duration T. N_0 represents the normalized noise power spectral density in W/Hz, referenced to the input stage of the demodulator, since the signal to noise ratio is established at that point. The signal detection in receiver is accomplished by cross-correlation-and-sampling operation. The effect of filtering due to $H(f)$ in Figure 1 is not considered here.

The conditional steady state probability density function, for the non-linear PLL model with a known signal and noise at the PLL input, of modulo 2π reduced phase error is given by the following approximation [10]:

$$p(\phi) = \frac{e^{\beta\phi+\alpha\cos\phi}}{4\pi^2 e^{-\pi\beta}|I_{j\beta}(\alpha)|^2}\int_\phi^{\phi+2\pi} e^{-\beta x+\alpha\cos x}dx, \qquad (8)$$

$I_{j\beta}(\alpha)$ is the modified Bessel function of complex order $j\beta$ and real argument α. The range of definition for ϕ in the previous equation is any interval of width 2π centered about any lock point $2n\pi$, with n an arbitrary integer. The parameters α and β, that characterize Eq.(8), for the first order non-linear PLL model in this case are:

$$\alpha = \alpha_0 R, \qquad \beta = \beta_0\Omega, \qquad (9)$$

where α_0 and β_0 are constants [10,11]. The parameter α is a measure of the loop signal to noise ratio in the sense that the larger the value of α, the smaller are the deleterious effects due to noise reference signal. The parameter β is a measure of the loop stress. Ω is the loop detuning, i.e. the frequency offset defined by:

$$\Omega = \frac{d}{dt}(\omega_0 t + \psi) - \omega_0 = \frac{\eta(\eta+\cos\theta)}{R^2}\frac{d\theta}{dt}. \qquad (10)$$

Since $(d\theta/dt) = 0$, it follows $\Omega = 0$, i.e. $\beta = 0$ and Eq.

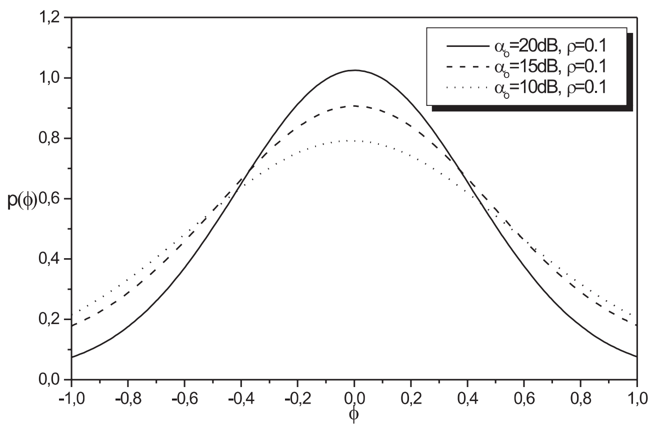

Fig. 2. Probability density function of phase error for the nonlinear first order PLL model

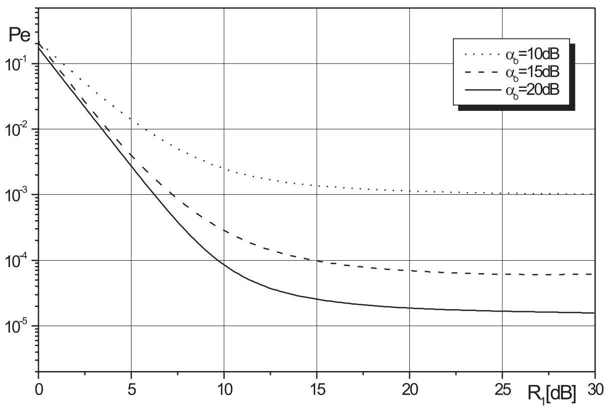

Fig. 3. Average bit error probability performance of a QPSK coherent system with noisy carrier synchronization reference signal in the presence of both, pulse interference and Ricean fading

(8) takes the form

$$p(\phi) = \iiint\limits_{A\ \theta\ n_i} p(\phi/A, n_i, \theta)p(A)p(n_i)p(\theta)dAd\theta dn_i. \quad (11)$$

Calculating the previous equations yields the probability density function of the phase error shown in Fig. 2. In order to obtain numerical results the following is supposed $\gamma = 1$, $A_1 = 1, \sigma_i = 1, \sigma_f = 1$.

III. System Performance

Substituting $R_b = R_1 R^2$ in Eq. (6), where R_1 corresponds to the case when there is no interference, the conditional bit error probability is determined. The total error probability is determined by averaging the conditional error probability over random variable ϕ:

$$P_e = \iiiint\limits_{A\ \theta\ n_i\ \phi} P_{e/\phi} p(\phi/A, \theta, n_i)p(A)p(\theta)p(n_i)dAd\theta dn_i d\phi \quad (12)$$

The total error probability is computed on the basis of the Eq. (12) and is plotted versus signal to noise ratio (R_1 [dB]) in Fig. 3. The values are given in figures.

The total bit error probability, when the signal, Gaussian noise and pulse interference are applied at the input of the receiver, as a function of the signal to noise ratio for the Ricean fading, is shown in Fig. 3 and Fig. 4. From figures follows that the system error probability decreases with the increase of the signal to noise ratio.

Figure 3 shows the influence of the PLL parameter α_0 on the bit error probability of the observed QPSK system. On the basis of the numerical results and shown figures it can be seen that the bit error probability increases with the decrease of the PLL parameter α_0. Also, in the ideal case, when the fading and pulse interference are absent, system has better performance rather than in the presence of both, pulse interference and Ricean fading.

The influence of the pulse interference and Ricean fading is evident from Fig. 4. The following observation is significant. One can see that the bit error probability has

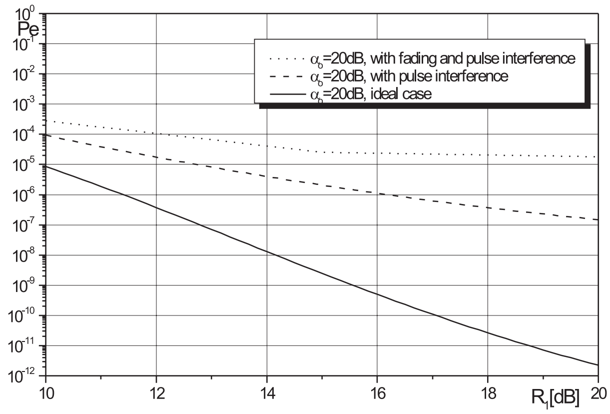

Fig. 4. Comparation of the QPSK system performances

changed 809.7, for SNR = 15 dB, if at the receiver input is present pulse interference. But, if the fading appeared together with interference the bit error probability has changed $11.298\,10^3$ times. On the basis of the above discussion can be concluded that the pulse interference and Ricean fading have a great influence on the QPSK system performances. Therefore it is justified to increase the parameter α_0 to a certain limit when the error probability decreases considerably with its increase, but there is no justification for the parameter α_0 increase then that value because certain great variations can cause the insignificant error probability reductions.

IV. Conclusion

The QPSK system is analyzed by means of the system error probability, in this paper. Noise influence, interference, fading and imperfect carrier phase recovery are the limiting factors in the observed system performance. The interference is represented by cosinusoidal signal with the uniform distributed phase. The influence of the imperfect reference signal extraction is expressed by the probability density function of the PLL phase error.

The detailed analysis of the obtained numerical results is performed in this paper. Case when the signal, Gaussian noise and pulse interference are applied at the input of the receiver, as a function of the signal to noise ratio for the Ricean fading, is considered in this paper. On the basis of the shown analysis can be concluded that the system has better performance if both, the PLL parameter α_0 and signal to noise ratio has a greater value. The influence of the pulse interference as well as the influence of fading on the system error probability are especially considered. On the basis of the shown analysis one can conclude that the system has better performances if both, pulse interference as well as fading have a smaller values.

However, from Fig. 3, for the large signal to noise ratio system error tends to a constant value (BER floor). In the BER floor area, the signal to noise ratio is relatively large with respect to parameter α_0 and has therefore a small influence on the system error probability. It is seen from figures that this BER floor can be reduced by increasing the parameter α_0 which depends on the applied PLL loop. On the basis of the shown analyze it is possible to determine the QPSK system parameter α_0 and useful signal power necessary to compensate the imperfect carrier extraction. This means that the presented conclusions can be useful in QPSK system design.

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