Microwave Transistors Noise Modeling Using Noise Wave Temperatures

Olivera R. Pronić, Vera V. Marković

Abstract – **In this paper we propose the noise models of MESFET / HEMT transistors based on the wave representation of transistor intrinsic circuit. The noise wave temperatures are introduced as empirical parameters of the models. Besides the noise model based on the constant noise wave temperatures, the noise model based on the frequency dependent noise wave temperatures is also developed and the comparative analysis is done. These frequency dependences are modeled using second order polynomial regression. The results for transistor noise parameters obtained by the proposed procedures are verified by the comparison to experimental data.**

Keywords – **MESFET, HEMT, noise model, wave approach**

I. Introduction

The noise models of microwave transistors are based mostly on the well-known fact that a linear noisy two-port may be represented by a noiseless two-port and two additional noise sources, [1]. These noise sources are usually equivalent voltage and/or current sources.

In the last decade the Pospieszalski's approach to noise modeling of MESFETs / HEMTs has gained much attention in microwave community, [2]. The noise model he proposed is based on H representation of transistor intrinsic circuit with two uncorrelated noise sources, the voltage noise source at the gate side and the current noise source at the drain side. However, it has been found, [3], that in some cases the inaccuracy in transistor noise modeling caused by this assumption is not negligible. Therefore, the model including the correlation between noise sources has been developed and implemented into a standard microwave circuit simulator, [4].

In the microwave frequency region, a treatment of noise in terms of waves is more appropriate since it allows the use of scattering parameters for noise computations, [5]. It has been shown, [6], that the wave approach is useful for both noise modeling and measurement of microwave FETs. Using a similar approach, the new extraction formulas for the noise wave sources in the noise equivalent circuit of MESFETs / HEMTs, where the correlation between noise sources is included, are proposed in [7]. The noise parameter characteristics obtained by using that procedure are in better agreement with the measurements than the existing model, [6].

The noise wave modeling procedures of MESFETs, HEMTs and dual-gate MESFETs based on T representation of transistor intrinsic circuit are presented by the authors in previous papers, [8], [9]. Three noise wave temperatures are introduced as empirical parameters of those models. These temperatures, being constant over the whole frequency range, are obtained on the basis of some experimental noise data by applying standard optimization procedures.

However, it is shown that the noise wave temperatures are frequency dependent. Therefore, the microwave FETs' noise model based on variable noise wave temperatures will be also presented here. Thus, two different procedures for the noise wave modeling of packaged MESFETs / HEMTs are considered and compared in this paper. The verification of presented procedures is done by comparison with measured data.

II. Noise Modeling

We used a MESFET / HEMT small-signal equivalent circuit as shown in Fig. 1. This equivalent circuit represents the packaged devices very well.

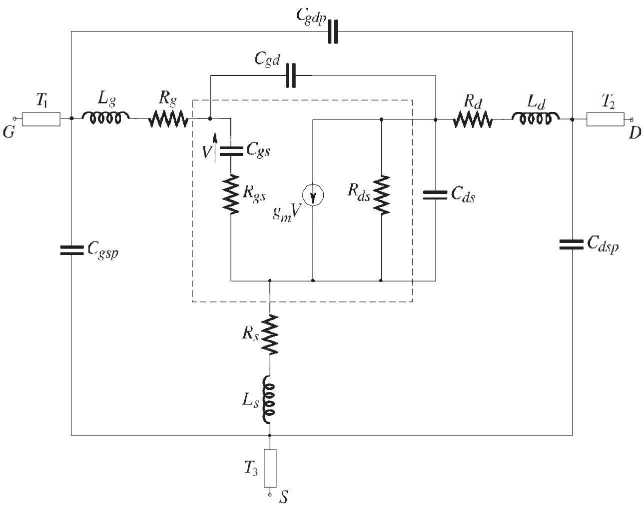

Fig. 1. A small-signal equivalent circuit for a MESFET / HEMT package ($g_m = g_{m0}e^{-j\omega\tau}$)

T representation of the intrinsic circuit with two correlated noise sources is considered. The intrinsic part of the circuit (denoted by a dashed line in Fig. 1), can be represented by a noiseless two-port defined by transfer scattering parameters, $[T]$, and two noise wave sources a_n and b_n referred to the input, as shown in Fig. 2.

The linear matrix equation describing this noisy two-port

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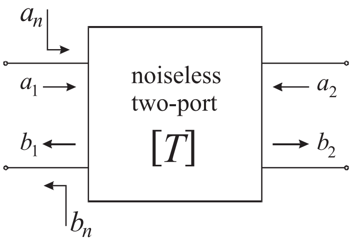

Fig. 2. T representation of a noisy two-port

is:

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix} + \begin{bmatrix} a_n \\ b_n \end{bmatrix} , \qquad (1)$$

where a_i and b_i, $i = 1, 2$, are incident and output waves at the i-th port.

The elements of the noise source vector are correlated and characterized by the correlation matrix C_T given by

$$C_T = \begin{bmatrix} \langle |a_n|^2 \rangle & \langle -a_n b_n^* \rangle \\ \langle -b_n a_n^* \rangle & \langle |b_n|^2 \rangle \end{bmatrix} \qquad (2)$$

where the brackets $\langle\rangle$ indicate time average of the quantity inside and * indicates complex conjugation.

It is very convenient to use the noise wave temperatures as empirical noise model parameters, [10]. In this way, the correlation matrix C_T can be expressed by

$$C_T = k\triangle f \begin{bmatrix} T_a & |T_c| e^{j\varphi_c} \\ |T_c| e^{-j\varphi_c} & T_b \end{bmatrix} , \qquad (3)$$

where k is the Boltzmann's constant and $\triangle f$ is the noise bandwidth (it is assumed that $\triangle f$=1Hz). In this way the noise performance of a two-port network is completely characterized by two real temperatures T_a and T_b and a complex correlation temperature $T_c = |T_c| e^{j\omega \tau_c}$.

Using this representation of noise, the noise wave temperatures can be expressed in term of intrinsic circuit noise parameters - minimum noise figure F_{min}, optimum reflection coefficient $\Gamma_{opt} = |\Gamma_{opt}| e^{j\varphi_{opt}}$ and noise resistance R_n, as

$$T_a = T_0(F_{min} - 1) + \frac{4R_n T_0 |\Gamma_{opt}|^2}{Z_0 |1 + \Gamma_{opt}|^2} , \qquad (4)$$

$$T_b = \frac{4R_n T_0}{Z_0 |1 + \Gamma_{opt}|^2} - T_0(F_{min} - 1) , \qquad (5)$$

$$T_c = \frac{4R_n T_0 \Gamma_{opt}}{Z_0 |1 + \Gamma_{opt}|^2} , \qquad (6)$$

where Z_0 is the normalization impedance (Z_0=50Ω) and T_0 is standard reference temperature (T_0=290K).

III. Numerical Example

The numerical results presented in this paper are related to small-signal and noise modeling of Siemens HEMT packaged device, type CFY65A. All simulations are performed with microwave circuit simulator Libra, [11].

The parameters of the proposed MESFET / HEMT noise models are three noise wave temperatures. Generally, besides S parameter data, the noise parameters measured at several frequency points are needed to predict the transistor noise parameters over the whole operating frequency range.

The noise modeling procedures are performed in the following way:

At the beginning, the small-signal equivalent circuit elements are extracted from the scattering parameters measurements. The extracted values are given in Table 1. The transmission lines segments (T_1, T_2 and T_3) are characterized by the characteristic impedances (Z_1, Z_2 and Z_3) and electrical lengths (e_1, e_2 and e_3) at frequency f=10 GHz.

Table 1. Equivalent circuit element values

intrinsic circuit elements	*values*	*parasitics*	*values*
C_{gs} (pF)	0.176	C_{ds} (pF)	0.046
R_{gs} (Ω)	3.29	C_{gd} (pF)	0.028
g_m (mS)	37.4	R_g (Ω)	0.75
τ (ps)	2.82	L_g (nH)	0.112
R_{ds} (Ω)	591.95	R_d (Ω)	0.52
		L_d (nH)	0.262
		R_s (Ω)	0.34
		L_s (nH)	0.001
		C_{gdp} (pF)	0.011
		C_{gsp} (pF)	0.01
		C_{dsp} (pF)	0.099
		Z_1 (Ω)	51.21
		Z_2 (Ω)	68.62
		Z_3 (Ω)	31.8
		e_1 (°)	35.78
		e_2 (°)	24.49
		e_3 (°)	14.1

In order to determine all model parameters, after the extraction of the small-signal equivalent circuit elements, it is also necessary to determine the noise temperatures.

The intrinsic circuit noise parameters needed for the noise wave temperatures calculation can be obtained by applying the deembeding procedure in Libra. The deembedding of device parasitics is done by adding parasitic elements with negative values and in reverse order to the complete circuit. In that way we got the noise parameters of an intrinsic circuit. After that the noise wave temperatures could be calculated by applying Eqs. (4)-(6).

Since the calculated values of the noise temperatures vary with the frequency, it is useful to model this frequency dependence by some mathematical relationships. Here, a polynomial regression model of the second order is stated to model the frequency dependence of the noise wave temperatures,

$$T_i = a_i + b_i f + c_i f^2, \quad i = a, b, c\,. \tag{7}$$

The parameters for polynomial fit of the noise wave temperatures are given in Table 2.

Table 2. Parameters for polynomial fit

	a	b	c
T_a	-3.69	31.67	-1.70
T_b	-214.95	111.03	-6.79
$\|T_c\|$	-78.19	60.26	-3.68
φ_c	4.88	8.52	-0.25

The noise wave temperatures could be converted to standard noise parameters with the aim to perform a comparison with the measured data. Equations for the conversion between these parameter sets are:

$$\Gamma_{opt} = \left(\frac{T_a + T_b}{2|T_c|} - \sqrt{\left(\frac{T_a + T_b}{2|T_c|}\right)^2 - 1} \right) e^{j\omega\tau_c}\,, \tag{8}$$

$$R_n = Z_0 \frac{|T_c|}{4T_0|\Gamma_{opt}|} \left[1 + 2|\Gamma_{opt}|\cos\phi_{opt} + |\Gamma_{opt}|^2 \right]\,, \tag{9}$$

$$F_{min} = 1 + \frac{T_a - T_b}{2|T_0|} + \frac{1}{2T_0}\sqrt{(T_a + T_b)^2 - 4|T_c|^2}\,. \tag{10}$$

The frequency dependence of standard noise parameters of the intrinsic circuit is obtained by replacing the parameters for polynomial fit of the noise wave temperatures in Eqs. (8)-(10). In order to obtain the noise parameters of the complete transistor model it is necessary to include all remaining elements of the equivalent circuit that represent the parasitics (as shown in Fig. 1).

The second approach is based on approximation that the noise wave temperature values are constant over the whole frequency range. The constant noise wave temperatures can be extracted by using the optimization capabilities of a powerful microwave circuit simulators like Libra, ADS, etc, in the following way: First, the expressions for the intrinsic circuit noise parameters (Eqs. (8)-(10)) are programmed using the ”equation” capability of the circuit simulator and assigned to the intrinsic circuit by the corresponding statement. After that, all parasitics are connected and the topology of the entire transistor is described. Finally, all small-signal circuit elements and the noise wave temperatures are optimized with the aim that the complete model fits the measured S parameters and noise parameters as well as possible.

The frequency dependences of the transistor noise parameters obtained by the proposed models are presented in Figs. 3-6. The characteristics for the minimum noise figure are presented in Fig. 3. Real and imaginary parts of the optimum reflection coefficient are shown in Fig. 4 and Fig. 5, respectively. Finally, equivalent noise resistance, normalized with respect to 50 Ω, is presented in Fig. 6.

The curves obtained by using the first approach - fitting by the polynomial regression - are denoted by MOD1. The characteristics based on the second approach - using of constant noise wave temperatures and optimization procedure, are denoted by MOD2. The referent values, based on the data measured by manufacturer, are denoted by REF.

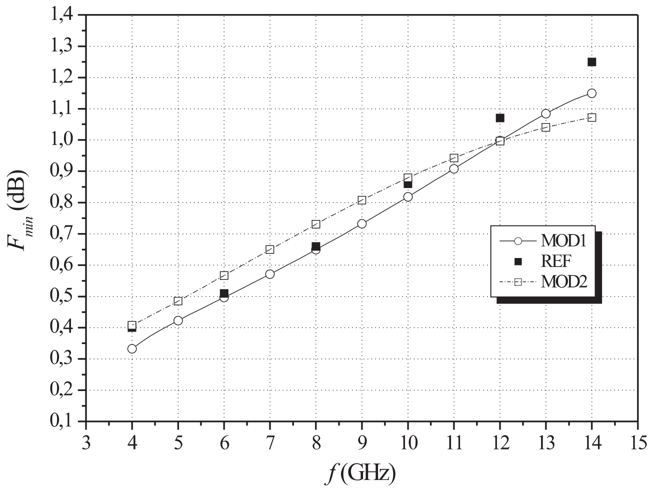

Fig. 3. Minimum noise figure

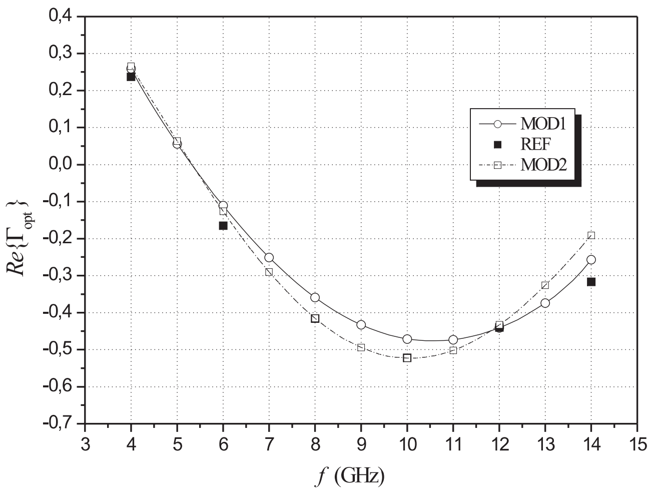

Fig. 4. Real part of the optimum reflection coefficient

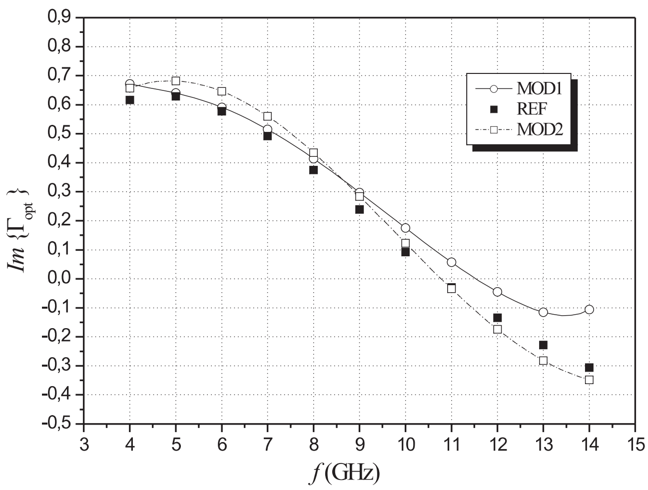

Fig. 5. Imaginary part of the optimum reflection coefficient

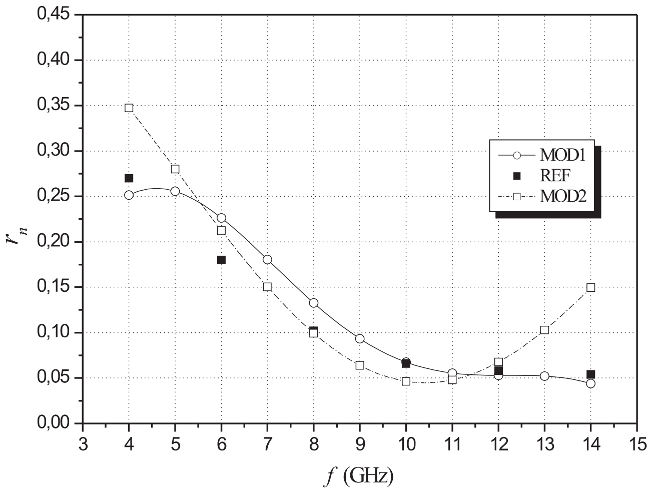

Fig. 6. Normalized equivalent noise resistance

Generally, a good agreement between measured and modeled parameters can be observed in both cases. However, as expected, the better modeling of referent data is achieved when the first approach, MOD1, is applied (especially for the noise resistance).

Several other microwave FETs in packaged form have been analyzed and modeled with similar results.

IV. Conclusion

The modeling of packaged MESFET / HEMT devices presented in this paper is based on a noise wave representation of transistor intrinsic circuit. Two noise models, one based on the frequency dependent, and the other based on the constant noise wave temperatures are presented. The frequency dependences of noise wave temperatures are modeled using second order polynomial regression. In that way frequency extrapolation of noise parameters is enabled and on-line optimization in circuit simulator is omitted. The example of packed HEMT noise modeling is presented. A good agreement with the measured noise parameters is observed in both cases, but the model including frequency dependant noise temperatures enables at some degree more accurately prediction of noise parameters, in comparison to the model with constant noise wave temperatures.

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