Antenna Miniaturization Using Fractal Geometry

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Abstract – **The expected benefit of using a fractal as a dipole antenna is to miniaturize the total height of the antenna at resonance, where resonance means having no imaginary component in the input impedance. In this paper three types of fractals are investigated as dipole wire antennas. They include two planar structures, Koch curve and a fractal tree, and a three dimensional fractal tree. These three types of fractals are compared among each other and to a straight dipole. The starting structure for each of these fractal geometries is straight dipole that is resonant in the PCS band, at 1900 MHz. In the simulation, the antenna height is held constant and the frequency is swept. It can be seen that the resonant frequency decreases as the number of fractal iterations increases. The decrease in resonant frequency can correlate to a miniaturized antenna, if the resonant frequency would be held fixed.**

Keywords – **Fractal antenna, antenna miniaturization, Koch curve, fractal tree**

I. Introduction

Although fractals are mainly discussed in mathematical forums, they exist in all parts of nature. For example Mandelbrot [1] discusses the basics of fractal theory as applied to the characteristics of a coastline (Fig. 1.). The length of a coastline depends on the size of the measuring yardstick. As the yardstick we use to measure every turn and detail decreases in length, the coastline perimeter increases exponentially. As the view of a coastline is brought closer, we discover that within the coastline there lie miniature bays and peninsulas. As we examine the coastline on a rescaled map, we discover that each of the bays and peninsulas contain sub-bays and sub-peninsulas. There is a self-similar trait observed as we look at the coastline at various resolutions. The number of microscopic structures begin to approach infinity. In fact, because of the immense number of irregularities, the physical length of a coastline is virtually infinite. Self similarity (seen in the coast example above) is defined by structures that look the same at variable magnifications. This recurring self-similarity is one of the many attributes of many fractals. Much like the coastline described above, any small part in a self-similar fractal is going to look exactly like the fractal as a whole.

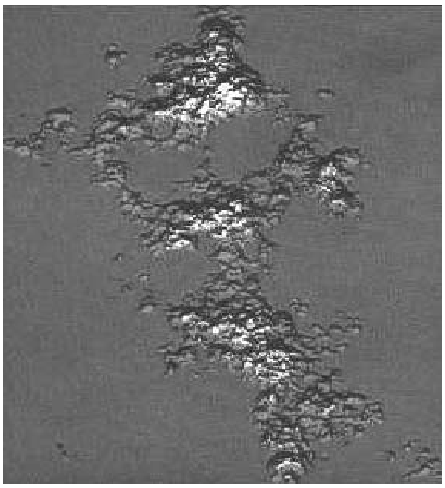

Fig. 1. Fractal-generated coastline

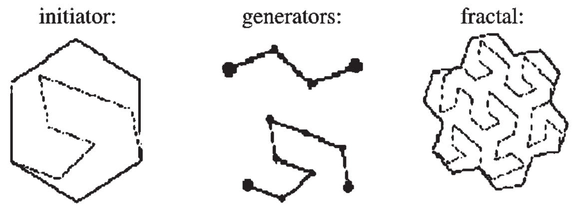

Fig. 2. Initiator/generator fractal

Another type of figure uses a generator/initiator relationship system of construction. This construction begins by placing an "initiator", which will be the base format for the figure. The initiator is then divided into a collection of lines upon which the generator(s) will be placed. Fig. 2. shows the initiator and its first stage of growth where the lines are replaced, or added to, by one of the two generators. Once the generators replace the lines belonging to the initiator, the generator may repeat "n" number of time, or a different generator may begin growth upon the one already in place.

II. Koch Curve

The first fractal shape that is investigated as a dipole antenna is Koch curve [2,3,6]. The geometry of how this antenna could be used as a dipole is shown in Fig. 3.

A Koch curve is generated by replacing the middle third of each straight section with a bent section of wire that spans the original third. Each iteration adds length to the total curve. This can be seen from the figure depicting the generating process (Fig. 3.). Each iteration results in a total length that is 4/3 times the original geometry. However, the original overall height of the fractal does not change from one iteration to the next. Therefore, if the process is carried out for an infinite number of times, the curve would have an infinite length while the overall height would not change.

The starting structure that is used is a half of a resonant PCS dipole, which is 3.75 cm in length. The overall length

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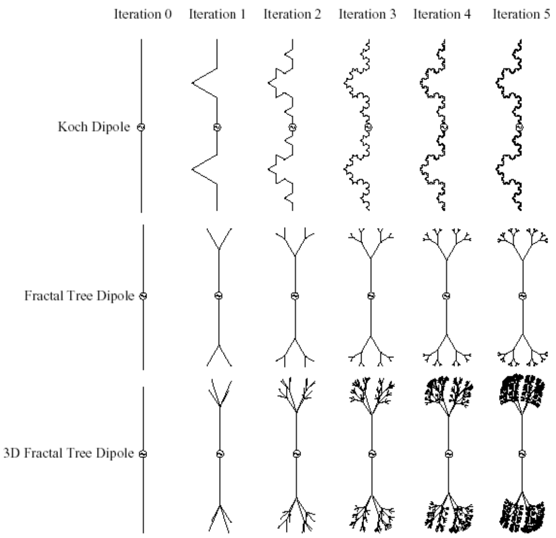

Fig. 3. The various fractal geometries are configured as dipoles, including a Koch fractal, a fractal tree and a three dimensional fractal tree. The starting size of the geometries are identical PCS band dipoles.

of the resonant dipole is 7.5 cm, which is slightly smaller than $\lambda/2$ at 1900 MHz.

The total length of the Koch curve is given by:

$$l_{Koch} = l\left(\frac{4}{3}\right)^n , \tag{1}$$

where n is the number of iteration and h is the height of the straight starting generator.

These fractals are analyzed as resonant dipole antennas using WIPL-D software [4]. The input match, compared to 50Ω, of the fractal dipoles and straight dipoles as calculated are shown in Fig. 4. It can be seen how the resonant frequency drops as the number of generating iterations for the fractal is increased. Also, it is interesting to note that the resonant frequency approaches an asymptotic limit. This limit gives an insight into where the resonance of an ideal Koch fractal curve as a dipole would lie, if such a structure were manufacturable. The simulated input impedance plots are shown in Fig. 5.

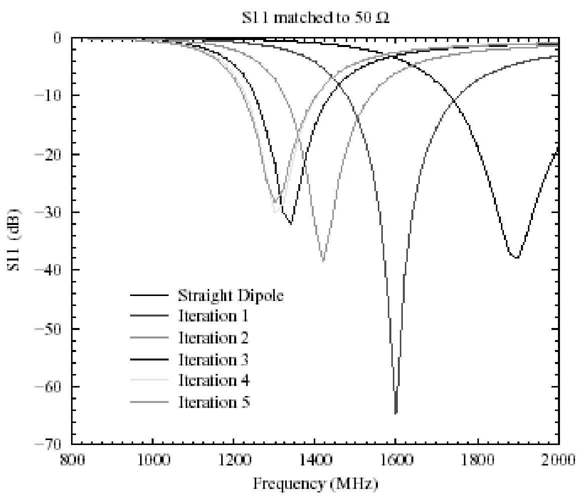

Fig. 4. Simulated input match of the straight dipole and the first five iteration for the Koch dipole antennas matched to 50 Ω

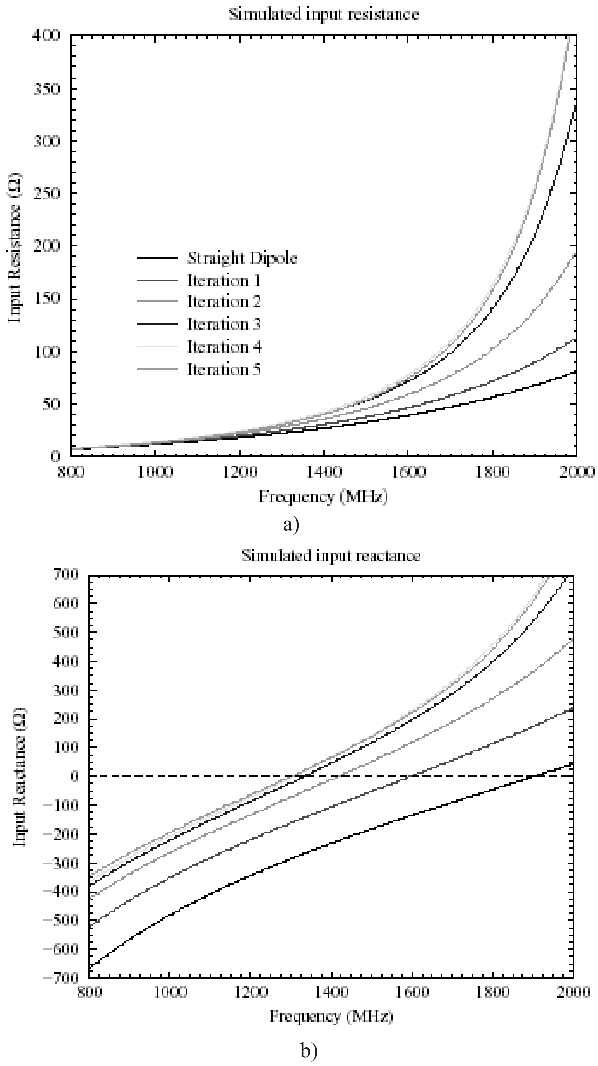

Fig. 5. Simulated input impedance for the first five fractal iterations of Koch dipoles plus a straight dipole for comparison. a) input resistance b) input reactance

III. Fractal Tree

Another type of fractal that can be utilized as a dipole is a fractal tree. The geometry of how the fractal is used is shown in Fig. 3. This deterministic fractal is a simple model of branching found in nature. Again, the goal of using this type of fractal is to reduce the height of a resonant dipole antenna.

The fractal is generated by applying an iterative sequence to the starting structure. The fractal is started with a simple monopole. The top segment of this monopole is then split at a pre-determined angle, $\theta = 60^\circ$, to form the first two branches. As the iterative process continues, the end segment of each branch splits into two more branches. The total electrical length of the conductor, l, remains constant throughout the iterative process. The total electrical length can be defined as the shortest length from base of the fractal to any other end. The lengths of each straight section in the first five iterations are shown in Table 1. It can be seen from the section lengths that the total conductor length, l, always adds up to unity for each iteration.

Table 1. Length of each straight section of the fractal tree and 3D fractal tree for the first five iterations

Iteration	0	1	2	3	4	5
	1	1/3	1/7	1/15	1/31	1/63
		2/3	2/7	2/15	2/31	2/63
			4/7	4/15	4/31	4/63
				8/15	8/31	8/63
					16/31	16/63
						32/63

The first five iteration plus a straight dipole were analyzed. In the previous section describing the Koch dipole antenna, the overall height was maintained from iteration to iteration. For the tree fractal, the total length of the conductor path is maintained among iterations. The subsection size for each iteration of the antenna is the same.

The input match, compared to 50Ω, of the fractal dipoles as calculated are shown in Fig. 6.

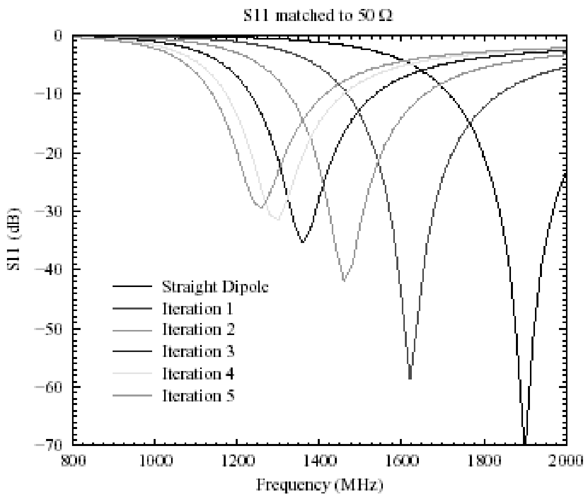

Fig. 6. Simulated input impedance matched to 50Ω for the first five iterations of a fractal tree dipole with a split angle of 60° and for a straight dipole

It can be seen that the resonant frequency drops as the fractal iteration is increased. The ratio of miniaturization versus the fractal iteration is very similar to that of the Koch dipole. As the fractal iteration increased, the resonant frequency decreases in a saturating manner. At each iteration the extra number of branches top loads the antenna. Even though the electrical length of a single conductor path from the generator port of the antenna to the top of a branch is identical for all antennas, there are more branches after each iteration. This adds more conduction paths at the top of antenna serving as a top-loaded device. This, in turn, lowers the resonant frequency at every iteration. It can be seen that the top loadings effect diminishes as the number of iterations is increased. The length of wire that branches out during each iteration is almost half as small as the previous iteration, thus the effect it has on the input characteristics of the antenna diminishes.

IV. Three Dimensional Fractal Tree

A three dimensional fractal tree has a similar geometry as the fractal tree. However, instead of branching in one plane, the fractal branches out in three dimensions. The resulting antenna exhibits similar benefits as the two dimensional case to a greater degree. The geometry of how this type of fractal can be utilized as a dipole is shown in Fig. 3.

The three dimensional fractal tree is generated in a similar fashion as the two dimensional case. The top of a straight monopole is split into four branches. The branches split off at a set angle in two orthogonal planes. The angle used in this case is 60°. The resulting four branches then split in a similar manner. The ratio of the sizes of each of the branches at each iteration is outlined in Table 1. For the purpose of studying this fractal as an antenna, the first five iteration are used. As before, this shows us the trends of the benefits of using a fractal within the computational limitations of the simulations. The fractal generated is mirrored at the base. These antennas are simulated in a dipole configuration.

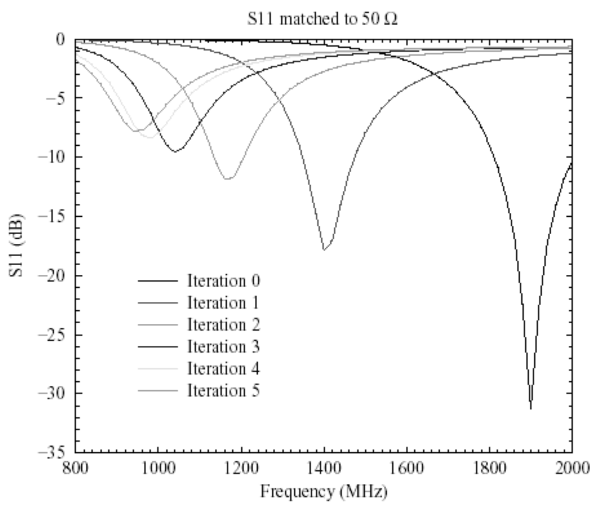

Fig. 7. Input match for various iterations of a three dimensional fractal tree matched to 50 Ω

The simulated input match for the antennas is shown in Fig. 7. It can be seen how the resonant frequency decreases as the fractal iteration is increased. In a similar fashion as the previous fractal dipoles studied, the input resistance decreases as the fractal iteration is increased, resulting in a poorer input match.

V. Fractal Dipole Comparison

The benefits of the various fractal geometries can be compared. All of the dipoles that are compared have the same starting height. The starting geometry is a resonant dipole

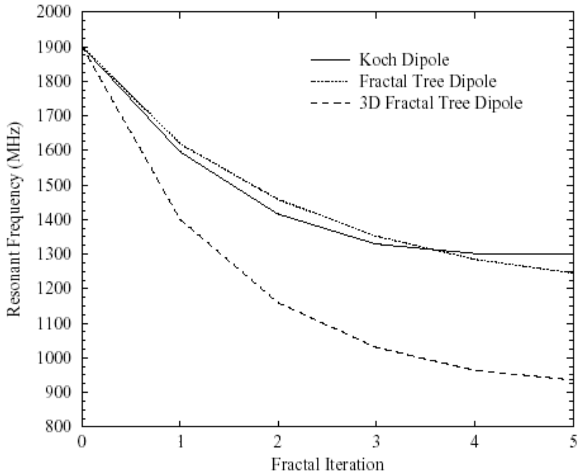

Fig. 8. The resonant frequency for each of the fractal antennas versus the number of iterations for a Koch tree, a fractal tree, and a 3D fractal tree in a dipole configuration as simulated with WIPL

that is 7.5 cm in length, resonant in the PCS band at 1900 MHz. The relative geometry of all of the compared dipoles is shown in Fig. 3.

The benefits of using a fractal geometry are dependent on the type of fractal that is chosen. A comparison of the miniaturization of the antennas by increasing the number of generating iterations is depicted graphically in Fig. 8.

It can be seen that the miniaturization benefits of both two dimensional structures, the Koch fractal and the fractal tree, are very similar. The benefits of the three dimensional fractal tree, however, is more pronounced.

Even thought the three dimensional fractal miniaturizes the antenna at resonance to a greater degree than the other fractals, the input resistance is lowered by a significant amount, as well.

It can be seen from Fig. 9. that the input resistance of the Koch and fractal tree dipoles drops to near 30 Ω at resonance for the fifth iteration. Likewise, the input resistance of the three dimensional fractal tree drops to 20 Ω due to the increased amount of conducting branches. This would decrease the match to a 50 Ω feed line. The fractal geometry chosen for a particular application would have to weight the trade-off between increased miniaturization versus input resistance.

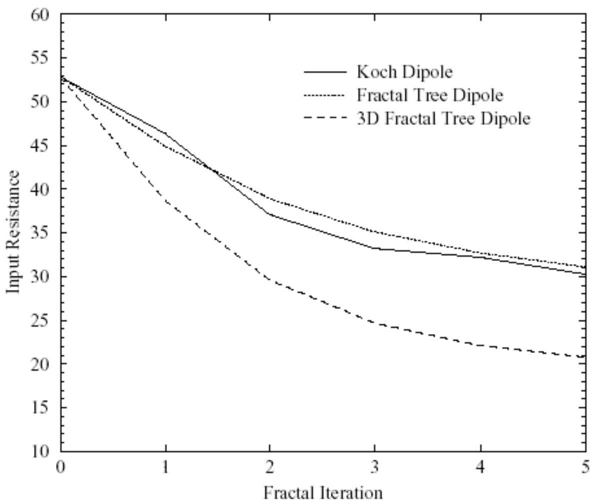

Fig. 9. Simulated input resistance versus the number of generating iterations for three fractal antennas

It can be seen from the plots of the simulated input match for the various dipoles that they are all narrow band antennas. The simulated 3 dB bandwidth of the dipole antenna is about 2.4%. This can be compared with the 3 dB bandwidth of the simulated fractals generated from the highest number of iterations, which have the lowest resonant frequency. The simulated bandwidth for the highest iteration of the Koch dipole is around 3.1%. For the fifth iteration of the fractal tree dipole, the simulated bandwidth is 4.2%. The simulated bandwidth of the fifth iteration of the three dimensional fractal tree is 12.7%, but only has a -7.75 dB input match at resonance.

VI. Conclusion

Fractal dipole antennas have shown the possibility to miniaturize antennas and to improve input matching. There are three distinct advantages which are reached by using fractal antennas. First, fractal geometries can be implemented to miniaturize dipole antenna. Also, designing with fractal geometries can overcome limitations to improve the input resistance of antenna that are typically hard to match to feeding transmission lines. Furthermore, the self-similar nature in the fractal geometry can be utilized for operating a fractal antenna at various frequencies.

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References

[1] Mandelbrot, B.B., *The Fractal Geometry of Nature*, New York, W. H. Freeman and Company, 1983.

[2] C. Puente, J. Romeu, R. Pous, J. Ramis, A. Hijazo, ”Small but long koch fractal monopole”, *Electronic Letters*, 34(1):9-10, January, 1998.

[3] C. Puente-Baliarda, J. Romeu, A. Cardana, ”The Koch Monopole: A Small Fractal Antenna”, *IEEE Transaction Antennas and Propagation*, Vol. 48, No. 11, 2000, pp. 1773-1781

[4] B.M.Kolundzija, J.S.Ognjanovic, T.P.Sarkar, *WIPL-D: Electromagnetic Modeling of Metallic and Dielectric Structures*, Artech House, Inc., Boston, 2000.

[5] N. Cohen, (Unsigned), *Fractal Antenna White Paper*, Fractal Antenna Systems, Inc., 1999.

[6] J.P. Gianvittorio, Y. Rahmat-Samii, ”Fractal Antennas: A Novel Antenna Miniaturization Technique and Applications”, *IEEE Antennas and Propagation Magazine*, Vol. 44, No. 1, February 2002, pp. 20-36

[7] J.J. Carr, *Practical Antenna Handbook*, 2nd edition, TAB Books, New York, 1994

[8] W.L. Weeks, *Antenna Engineering*, McGraw-Hill Book Company, New York, 1969