Diversity Systems Performance in the Presence of Shadowing

Milan Živković[1], Nenad Milošević[2], Zorica Nikolić[3]

Abstract – **In this paper we compare several diversity reception techniques in an additive white Gaussian noise channel in the presence of Rayleigh fading and log-normal shadowing employing coherent (BPSK) and noncoherent (DPSK) digital signaling. Dependence to the required SNR over MR combining of the number of branches to produce the same bit error rate (BER) is used as the measure of the performances. It is shown that the effects of log-normal shadowing and Reyleigh fading affect almost identically both of the signaling cases, coherent (BPSK) and noncoherent (DPSK).**

Keywords – **BPSK, DPSK signaling, Rayleigh fading, log-normal shadowing, diversity combining**

I. Introduction

Binary digital signaling is often followed by presence of fading and shadowing. Fading is the term used to describe the rapid fluctuations in the amplitude of the received radio signal over a short period of time caused due to the interference between two or more versions of the transmitted signals which arrive at the receiver at slightly different times. The resultant received signal can vary widely in amplitude and phase, depending on various factors such as the intensity, relative propagation time of the waves, bandwidth of the transmitted signal etc... In mobile environments transmitted signal can be also affected by effect of shadowing which results in the long-term attenuation of received signal due to specific propagation environment (vegetation, buildings). Therefore, the mobile communication channel can be modelled as additive white Gaussian noise channel subject to Rayleigh fading (received amplitudes has Rayleigh distribution) and log-normal shadowing (the mean of signal-to-noise ration has log-normal distribution). A powerful communication receiver technique that provides wireless channel improvement at relatively low cost is a well-known as diversity reception. Diversity techniques are based on the notion that errors occur in reception when the channel attenuation is large (when channel is in a deep fade). Supplying to the receiver several replicas of the same information signal transmitted over independently fading channels, the probability that all the signal components will fade simultaneously is reduced considerably [1] and therefore, instant and mean SNR can be increased. The diversity reception can be categorized as microscopic and macroscopic.

Microscopic diversity is a method for reducing the effect of instantaneous fading in which several uncorrelated faded signals are received at a radio port. There are several techniques for evaluating transmitted signal at the receiver. For the coherent digital signaling (CFSK, BPSK) with independent branch fading, achieved by separating receiver antennas at least 10 wavelenghts, the optimum diversity technique is known as Maximal Ratio Combining (MRC). In Maximal Ratio Combining (MRC), the signals from all the branches are co-phased and individually weighed by fading factor to provide the optimal SNR at the output. But it is seldom implementable in a multipath fading channel because the receiver complexity for MRC is directly proportional to the number of branch signals L available at the receiver. Since L may vary with location as well as time, it is undesirable to have receiver complexity dependent on a characteristic of the physical channel from a production and implementation point of view. Similarly, for the noncoherent digital signaling (NCFSK, DPSK) the commonly used technique is Equal Gain Combining (EGC), where all available branches are equally weighted and then added incoherently. It is clear that this technique is analogous to MRC in the sense that all available branches are used, therefore it has the same undesirable feature of having receiver complexity dependent on L. So it is very desirable to implement some other suboptimal diversity techniques in order to evaluate transmitted signal. The simplest suboptimal technique is the Traditional Selection Diversity Model (SC) that selects, among the L diversity branches, the branch providing the largest signal-to-noise ratio (or largest fading amplitude). Clearly, SC and MRC represent the two extremes in diversity combining strategy with respect to the number of signals used for demodulation. Consequently, other techniques representing compromise between this two were developed. One of them is S + N Selection Model, where S + N denotes a signal-plus-noise sample (i.e., not a power measurement), and noise is treated as random variable. The selective technique, which selects the branch providing the largest LLR (log-likelihood ratio) developed for BPSK signaling, has the closest performance to MRC. Also, combining diversity techniques that use two (SC2) or three (SC3) branches with largest amplitudes (or signal-to-noise ratio) for getting transmitted signal were developed. In this paper we compare this several diversity combining techniques for a Rayleigh faded channel in the presence of white Gaussian noise (AWGN) employing one coherent (BPSK) and one noncoherent (DPSK) digital signaling.

Macroscopic diversity, again, is a method for reducing the

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effect of shadowing, in which several signals are received at different radio-ports, with differently experienced long-term shadowing. The most common technique of macroscopic diversity is macrodiversity selection where selected signal originates from the port where the smallest long-term shadowing (the largest mean SNR) is present. In this paper we consider different systems with implemented macroscopic and microscopic techniques and compare different microscopic selection methods where dependence to the required SNR over MR combining of the number of branches to produce the same bit error rate (BER) is used as the measure of the performances.

II. System Model

Consider the system with K different radio-ports forming the microscopic diversity group. In order to mitigate the effect of shadowing (long-term attenuation) one can select signal originated from those port where the largest mean SNR is present. Consider, again, L independent microdiversity branches at every radio-port employ one of considered microdiversity techniques. If the transmitted signal is $x(t)$, the low-pass equivalent received signal l-th branch of the k-th port [2]

$$\omega_{kl} = \alpha_{kl} e^{j\phi_{kl}} x(t) + \eta_{kl} \quad k = 1, ..., K \quad l = 1, ..., L \quad (1)$$

where α_{kl} – fading amplitude (factor) in the l-th branch of the k-th port (nonnegative number); ϕ_{kl} – fading phase in the l-th branch of the k-th port; η_{kl} – additive complex Gaussian noise in the l-th branch of the k-th port; E_b – bit energy.

Corresponding signal in the l-th branch of the k-th port after cophasing is

$$r_{kl}(t) = \mathrm{Re}\{\omega_{kl} e^{j\phi_{kl}}\} = \alpha_{kl} x(t) + n_{kl} \quad (2)$$
$$k = 1, ..., K \quad l = 1, ..., L$$

where $n_{kl} = \mathrm{Re}\{\eta_{kl} e^{j\phi_{kl}}\}$. We assume that $E\{n_{kl}^2\} = N_0/2$ for every k and l.

Let's $\alpha_{k1}, \alpha_{k2}, ..., \alpha_{kL}$ denote fading amplitudes correspondent to microdiversity branches of the k-th port. We assume that they are statistically independent with Rayleigh probability density function (pdf) of the instant SNR, $\gamma_{kl} = \alpha_{kl}^2 \frac{E_b}{N_0}$, which has the form

$$p_{\gamma_{kl}}(\gamma_{kl}/\gamma_k) = \frac{1}{\gamma_k} e^{\frac{\gamma_{kl}}{\gamma_k}} \quad (3)$$

which is conditioned on the local mean SNR at the k-th port $\gamma_k = E\{\alpha_k^2\}\frac{E_b}{N_0} = z_k \frac{E_b}{N_0}$, where $z_k = E\{\alpha_k^2\}$ denotes mean-square amplitude values at the k-th port. We assume, also, that, the local mean-square amplitude values at given radio-ports are statistically independent and have log-normal pdf [2]

$$p_{z_{k\ \max}}(z_k) = \frac{10/\ln 10}{\sqrt{2\pi}\sigma_s z_k} \exp\Big(-\frac{(10\log_{10} z_k - \mu_k)^2}{2\sigma_s^2}\Big) \quad (4)$$

where μ_k (dB) denotes mean, and σ_s (dB) denotes standard deviation of the quantity $10\log_{10} z_k$. Let $z_{k\ \max}$ be the largest local mean-square value selected from the K radio-ports, that is $z_{k\ \max} = \max\{z_1, z_2, ..., z_k\}$. One can show that the pdf of $z_{k\ \max}$ has the form

$$p_{z_{k\ \max}}(z_{k\ \max}; K) = \sum_{j=1}^{K} \frac{10/\ln 10}{\sqrt{2\pi}\sigma_s z_{k\ \max}} \times \exp\Big(-\frac{(10\log_{10} z_{k\ \max} - \mu_j)^2}{2\sigma_s^2}\Big) \times \prod_{k\neq j, k=1}^{K} \Big(1 - \frac{1}{2}\mathrm{erfc}\Big(\frac{10\log_{10} z_{k\ \max} - \mu_k}{\sqrt{2}\sigma_s}\Big)\Big) \quad (5)$$

where the mean μ_k (dB) is generally dependent on the distance between the port and the user's location. For any given arrangement of the macrodiversity ports, an average probability density function can be calculated by averaging (5) over all possible locations within the serving cell. At the point which is equidistant from the serving ports which make up a macrodiversity group, area mean of each port can be assumed identical, that is $\mu_k = \mu$ for $k = 1, 2, ..., K$. Therefore (5) becomes [2]

$$p_{z_{k\ \max}}(z_{k\ \max}; K) = \frac{K.10/\ln 10}{\sqrt{2\pi}\sigma_s z_{k\ \max}} \times \exp\Big(-\frac{(10\log_{10} z_{k\ \max} - \mu)^2}{2\sigma_s^2}\Big) \times \Big(1 - \frac{1}{2}\mathrm{erfc}\Big(\frac{10\log_{10} z_{k\ \max} - \mu}{\sqrt{2}\sigma_s}\Big)\Big)^{K-1}. \quad (6)$$

III. The System Performance

The bit error probability (BER) is derived by determining $P_b(\gamma_b; L/z_{k\ \max})$, the bit error probability conditioned on $z_{k\ \max}$, and averaging it over the pdf given in (6)

$$P_b = \int_0^\infty P_b(\gamma_b; L/z_{k\ \max}) p_{z_{k\ \max}}(z_{k\ \max}; K) dz_{k\ \max}. \quad (7)$$

For BPSK, MR Combining is the optimal microdiversity combining technique for fading overcoming. The matched filter outputs at every branch of the k-th port are multiplied with corresponding factor $\alpha_{kl} e^{-j\phi_{kl}}$ (cophasing and weighting branch signals) and then summed at the combiner. The BER conditioned on $\gamma_{k\ \max}$ has the form [2]

$$P_{bmrc}(\gamma_b; L/z_{k\ \max}) = = \Big(\frac{1-m}{2}\Big)^L \sum_{i=0}^{L-1} \binom{L-1+i}{i} \Big(\frac{1+m}{2}\Big)^i \quad (8)$$

where $m = \sqrt{\frac{\gamma_{k\ \max}}{1+\gamma_{k\ \max}}}$ and $\gamma_{k\ \max} = z_{k\ \max}\frac{E_b}{N_0}$. This microdiversity technique is optimum if it is assumed that channel parameters $\alpha_{kl} e^{-j\phi_{kl}}$ is estimated perfectly. Otherwise, if fading fluctuations is sufficiently fast to preclude the implementation of coherent detection, the implementation of noncoherent detection or selection diversity techniques may be more adequate.

In the case of the traditional selection diversity technique (SC), the one branch with the largest SNR is selected on each radio-port. BER conditioned on $\gamma_{k\ \max}$ has the form [2]

$$P_{bSC}(\gamma_b; L/z_{k\ \max}) = \\ = \frac{L}{2}\sum_{i=0}^{L}\binom{L}{i}(-1)^i\sqrt{\frac{\gamma_{k\ \max}}{i+\gamma_{k\ \max}}} \quad (9)$$

for BPSK signaling, and [3]

$$P_{bSC}(\gamma_b; L/z_{k\ \max}) = \\ = \frac{L}{2}\sum_{k=0}^{L-1}\binom{L-1}{i}(-1)^k\frac{1}{1+i+\gamma_{k\ \max}} \quad (10)$$

for DPSK signaling.

For practical implementations, however, measurement of SNR may be difficult or expensive, especially for high signaling rates. For this reason, the branch with the largest signal-*plus*-noise is often chosen. We use S + N to denote a signal-plus-noise sample (i.e., not a power measurement). When physically realizing this technique, by sampling the output of a matched filter, the noise is a random variable (SC assumes that noise is constant in all branches). Consequently, this model perform better than traditional SC model because there is opportunity for at least one sample to be better (less noisy) than the average of the samples.The exact expressions for BER conditioned on $\gamma_{k\ \max}$ can be found in [4].

One of the latest modification of the selective microscopic combining technique employing with BPSK signaling is “ar” selection where the signal from the microdiversity branch with the largest value of log-likelihood ratio (LLR) is selected at the each port. This value is equal to the product of fading amplitude α_{kl} and output of the matched filter r_{kl}. The exact expression for BER conditioned on $\gamma_{k\ \max}$ is provided in earlier author’s work [2].

Combining microscopic diversity techniques that use two (SC2) or three (SC3) branches with largest amplitudes (or signal-to-noise ratio) for getting transmitted signal were developed. This techniques, denoted as second or third order selection combining is a compromise between MR or EG Combining and traditional SC model and requires a less complex receiver than MR or EG EG Combining, therefore may be implemented regardless of the number of resolvable branch signals available and, consequently, offer better performance (BER) than traditional SC model. The exact expressions for BER conditioned on $\gamma_{k\ \max}$ for SC2 and SC3 microscopic diversity techniques can be found in [3].

The average BER for proposed systems consisted of K radio ports with L microdiversity branches can be obtained by substituting derived conditioned BERs and (5) in (7).

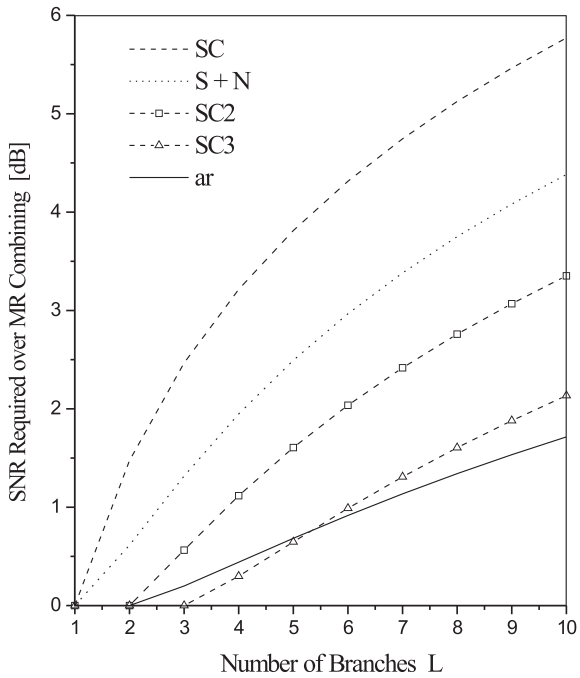

Fig. 1. Required SNR per bit over MR combining for various microdiversity techniques and specified number of diversity branches for average BER of $P_b = 10^{-3}$ employing BPSK signaling

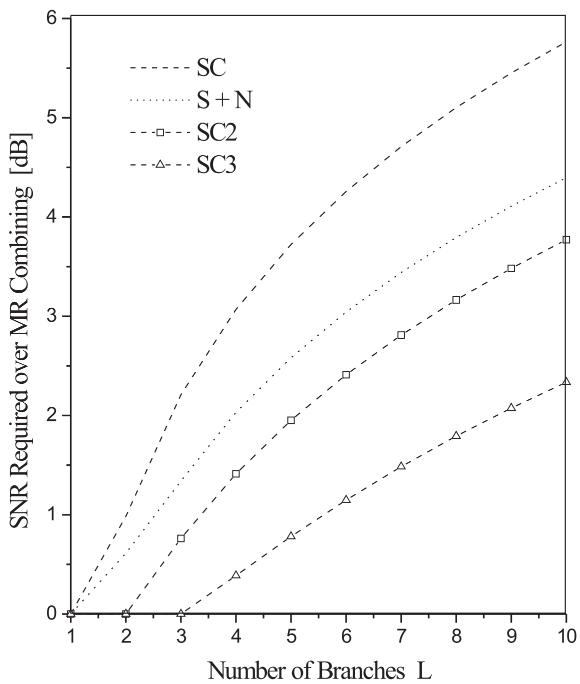

Fig. 2. Required SNR per bit over EG combining for various microdiversity techniques and specified number of diversity branches for average BER of $P_b = 10^{-3}$ employing DPSK signaling

IV. Numerical Results

We compare different systems with macrodiversity selection technique and different microscopic diversity techniques considering the two cases, employing coherent BPSK signaling and noncoherent DPSK signaling. The impact of different values of long-term attenuation (effect of shadowing), as well as different microdiversity techniques, is considered in both cases. We use the value of required SNR over MR combining, which correspond to the different number of branches that produce the same bit error rate (BER), as the measure of the performances. Fig. 1 and Fig. 2 compare different microscopic selection combining techniques to the optimum combining technique, MR combining in the case of BPSK and DPSK signaling, respectively, where an average BER of $P_b = 10^{-3}$ is chosen. The MRC curve is effectively the horizontal axis. As the number of microdiversity branches increases, the selection combining curves are shown to deviate. The SC technique gives the worst performance while the modified selection (SC, S + N, SC2, SC3, "ar") curves fall intermediately between the SC and MRC. In the case of BPSK signaling it is shown that SC3 has the best performance if the number of branches is 5 or less. As the number of microdiversity branches increases "ar" technique outperforms other selection techniques. The same performance trend is present at different magnitudes of shadowing, as it is shown in [3]. It is also shown that the same performance trend is present in the case of DPSK signaling (barring "ar" selection, which is purely coherent technique). Therefore, the effects of log-normal shadowing and Reyleigh fading affect almost identically both of the signaling cases, coherent (BPSK) and noncoherent (DPSK).

V. Conclusion

In this paper we compare several diversity reception techniques in an additive white Gaussian noise channel in the presence of Rayleigh fading and log-normal shadowing employing coherent (BPSK) and noncoherent (DPSK) digital signaling. Dependence to the required SNR over MR combining of the number of branches to produce the same bit error rate (BER), $P_b = 10^{-3}$, is used as the measure of the performances. The SC technique gives the worst performance while the modified selection (SC, S + N, SC2, SC3, "ar") curves fall intermediately between the SC and MRC. In the case of BPSK signaling it is shown that SC3 has the best performance if the number of branches is 5 or less. As the number of microdiversity branches increases "ar" technique outperforms other selection techniques. The same performance trend is present at different magnitudes of shadowing, as it is shown in [3]. It is also shown that the same performance trend is present in the case of DPSK signaling (barring "ar" selection, which is purely coherent technique). Therefore, the effects of log-normal shadowing and Reyleigh fading affect almost identically both of the signaling cases, coherent (BPSK) and noncoherent (DPSK).

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