Use of Space Correlation of Satellite Move in GPS

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Abstract – **This paper sets out an approach for improving the estimation accuracy of the coordinates of an object that uses the global positioning system GPS.**

Keywords – **GPS, distance estimation, least squares. .**

I. Introduction

The satellite-based navigation system GPS is increasingly used as personal equipment, providing position information, thus it exceeds the bounds of the truly professional equipment. In the case of this new application the user defines its coordinate mostly as a fixed point. The alteration of the distance between the user and the GPS satellites is only determined by the satellite orbital movement and the Earth rotation.

This paper presents an approach where data processing of the distance to the satellite groups takes place, taking into consideration the parameters, known in advance, of the satellite movement in their orbits. This approach is developed for an immobile or slow-moving object. In order to estimate the distance, the least squares approach is used, aiming at an increase in the accuracy while specifying the coordinates of the object.

II. Distance Alteration Model

Each of the GPS satellites transmits ephemeris data, some of the data is related to the orbital parameters. Apart from this parameter type, orbital perturbation corrections are transmitted too. This data provide the necessary precision within the range of an hour. The basic parameters, which are being transmitted, are set out in Table 1.

These parameters can help defining the satellite coordinates, in Earth-Centered-Earth-Fixed coordinate system (ECEF), using the following algorithm Eqs. (1) to (17), [3]:

- Computed mean motion (rad/s)

$$n_0 = \sqrt{\mu A^{-3}} \tag{1}$$

- Time from ephemeris reference epoch (s)

$$t_k = t - t_{oe} \tag{2}$$

- Corrected mean motion (rad/s)

$$n = n_0 + \Delta n \tag{3}$$

Table 1. Ephemeris parameter definition

Param.	Definition
M_0	Mean anomaly at reference time
Δn	Mean motion difference from computed value
e	Eccentricity
$(A)^{1/2}$	Square root of the semi-major axis
Ω_0	Longitude of ascending node of orbit plane at weekly epoch
i_0	Inclination angle at reference time
ω	Argument of perigee
$\dot{\Omega}$	Rate of right ascension
IDOT	Rate of inclination angle
C_{uc}	Amplitude of the Cosine harmonic correction term to the argument of latitude
C_{us}	Amplitude of the Sine harmonic correction term to the argument of latitude
C_{rc}	Amplitude of the Cosine harmonic correction term to the orbit radius
C_{rs}	Amplitude of the Sine harmonic correction term to the orbit radius
C_{ic}	Amplitude of the Cosine harmonic correction term to the angle of inclination
C_{is}	Amplitude of the Sine harmonic correction term to the angle of inclination
t_{oe}	Reference time Ephemeris
IODE	Issue of Data (Ephemeris)

- Mean anomaly (rad)

$$M_k = M_0 + nt_k \tag{4}$$

- Kepler's formula (solved by iteration)

$$E_k = M_k + e\sin E_k \tag{5}$$

- True anomaly (rad)

$$v_k = arctg\left[\sqrt{1-e^2}\frac{\sin E_k}{\cos E_k - e}\right] \tag{6}$$

- Eccentric anomaly (rad)

$$E_k = \arccos\left(\frac{e+\cos v_k}{1+e\cos v_k}\right) \tag{7}$$

- Argument of latitude (rad)

$$\Phi_k = v_k + \omega \tag{8}$$

- Argument of latitude correction (rad)

$$\delta u_k = C_{us}\sin 2\Phi_k + C_{uc}\cos 2\Phi_k \tag{9}$$

- Radius correction (m)

$$\delta r_k = C_{rc}\cos 2\Phi_k + C_{rs}\sin 2\Phi_k \tag{10}$$

- Correction to inclination (rad)

$$\delta i_k = C_{ic}\cos 2\Phi_k + C_{is}\sin 2\Phi_k \tag{11}$$

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- Corrected argument of latitude (rad)

$$u_k = \Phi_k + \delta u_k \tag{12}$$

- Corrected radius (m)

$$r_k = A\,(1 - e\cos E_k) + \delta r_k \tag{13}$$

- Corrected inclination (rad)

$$i_k = i_0 + \delta i_k + (IDOT)\ t_k \tag{14}$$

- Position in orbital plane (m)

$$\left.\begin{array}{l} x'_k = r_k\cos u_k \\ y'_k = r_k\sin u_k \end{array}\right\} \tag{15}$$

- Corrected longitude of ascending node (rad)

$$\Omega_k = \Omega_0 + \left(\dot{\Omega} - \dot{\Omega}_e\right)\ t_k - \dot{\Omega}_e t_{oe} \tag{16}$$

- ECEF coordinates (m)

$$\left.\begin{array}{l} x_k = x'_k\cos\Omega_k - y'_k\cos i_k\sin\Omega_k \\ y_k = x'_k\sin\Omega_k + y'_k\cos i_k\cos\Omega_k \\ z_k = y'_k\sin i_k \end{array}\right\} . \tag{17}$$

As Marinov, Stanchev [4,5] claim, the change in the satellite coordinates for short periods of time (up to a second) can be approximated with a great precision (probable error - less than 10 cm.), by means of linear function. Thus it's possible to simplify the calculation operations when calculating the distance between GPS receiver and a satellite.

The alteration of the satellite coordinates is set by the following expressions:

$$\left.\begin{array}{l} x_k(t) = x_k(t_0) + \alpha_{k,x}t \\ y_k(t) = y_k(t_0) + \alpha_{k,y}t \\ z_k(t) = z_k(t_0) + \alpha_{k,z}t \end{array}\right\} t_0 \leqslant t \leqslant t_0 + T\ , \tag{18}$$

where k is the satellite number, T is the processing interval, and t_0 is the initial moment of the processing interval. The coefficients $\alpha_{k,x}$, $\alpha_{k,y}$ and $\alpha_{k,z}$are calculated by the ephemeris parameters through the approximation of the alteration of the coordinates to a segment.

The distances between the user and the satellite are set by the relations:

$$D_k(t) = \sqrt{[x_k(t)-x_0]^2+[y_k(t)-y_0]^2+[z_k(t)-z_0]^2}\ , \tag{19}$$

where x_0, y_0, z_0 are the user's exact coordinates.

After replacing Eq. (18) in Eq. (19) one can get the expression:

$$D_k(t) = \\ = D_k(t_0)\left\langle \begin{array}{l} 1 - \dfrac{2}{D_k^2(t_0)}\left\{\begin{array}{l} \alpha_{k,x}[x_k(t_0)-x_0] \\ +\alpha_{k,y}[y_k(t_0)-y_0] \\ +\alpha_{k,z}[z_k(t_0)-z_0] \end{array}\right\} t \\ +\dfrac{\alpha_{k,x}^2 + \alpha_{k,y}^2 + \alpha_{k,z}^2}{D_k^2(t_0)} t^2 \end{array}\right\rangle^{\frac{1}{2}} \tag{20}$$

The square root in Eq. (20) is represented as a sequence:

$$D_k(t) = D_k(t_0) + \frac{\alpha_{k,x}^2 + \alpha_{k,y}^2 + \alpha_{k,z}^2}{2D_k(t_0)}\, t^2 + \\ + \frac{\alpha_{k,x}[x_k(t_0)-x_0] + \alpha_{k,y}[y_k(t_0)-y_0] + \alpha_{k,z}[z_k(t_0)-z_0]}{-D_k(t_0)}\, t. \tag{21}$$

Estimating the distance to the satellites, errors always occur due to different factors [3]. These errors are according to the Gaussian distribution with zero mean. In order to apply the least squares approach, Eq. (21) is reduced to the matrix form:

$$\begin{array}{l} D_k(t) = \mathbf{H}_k(t).\lambda_k \\ \mathbf{H}_k(t) = [1,\ t,\ \dfrac{\alpha_{k,x}^2 + \alpha_{k,y}^2 + \alpha_{k,z}^2}{2}\, t^2] \\ \lambda_k = [D_k(t_0),\ \beta_k,\ \gamma_k]^T \end{array}\ , \tag{22}$$

where β_k and γ_k are distance model coefficients, estimated by the least squares approach.

The measured values of the distances to the satellites are sum of the true values and errors of different nature:

$$R_k(t) = D_k(t) + \Delta D_k(t). \tag{23}$$

The observation equation, to which we could apply the least squares algorithm, can be drawn from Eqs. (22) and (23)[1]:

$$\begin{array}{l} \mathbf{Z}_{k,n} = \mathbf{H}_{k,n}\lambda_k + \mathbf{\Delta D}_{k,n}; \\ \mathbf{H}_{k,n} = \begin{bmatrix} \mathbf{H}_k(t_0) \\ \mathbf{H}_k(t_0+\Delta t) \\ \vdots \\ \mathbf{H}_k(t_0+n.\Delta t) \\ \vdots \\ \mathbf{H}_k(t_0+N.\Delta t) \end{bmatrix}; \\ \mathbf{\Delta D}_{k,n} = \begin{bmatrix} \mathbf{\Delta D}_k(t_0) \\ \mathbf{\Delta D}_k(t_0+\Delta t) \\ \vdots \\ \mathbf{\Delta D}_k(t_0+n.\Delta t) \\ \vdots \\ \mathbf{\Delta D}_k(t_0+N.\Delta t) \end{bmatrix}; \\ 0 \leqslant n \leqslant N; N.\Delta t = T; \Delta t = 2.10^{-2}s. \end{array} \tag{24}$$

The parameter Δt^{-1} is the rate of measuring the distance in the GPS receivers.

The estimation, according to the standard least squares approach, is given through the equation

$$\hat{\lambda}_k = (\mathbf{H}_{k,n}\mathbf{K}_k^{-1}\mathbf{H}_{k,n})^{-1}\mathbf{H}_{k,n}^T\mathbf{K}_k^{-1}\mathbf{Z}_{k,n} \tag{25}$$

where $\mathbf{K}_k$ is covariance matrix of $\Delta\mathbf{D}_{k,n}$. The elements of this covariance matrix are of the type $\exp\{-n\Delta t/t_{corr}\}$.

The algorithm given by Eq. (26) is non-recursive but it can be turned into a recursive one [1,2].

III. Research Results

A model of the distance estimation errors is created for the needs of this research work. For the satellite positioning and movement, true ephemeris data is used in the simulation, and using the algorithms mentioned above the measured and estimated distances to the satellites are modelled. The results are obtained for different error variances.

Fig. 1 shows the results how efficient the suggested algorithm about distance estimation to the satellite is, with error

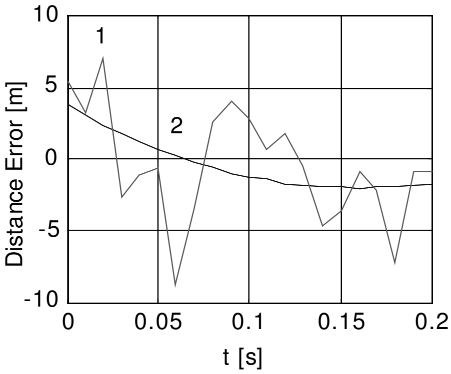

Fig. 1. Distance error for $\sigma^2 = 9\text{m}^2$ and T=200 ms.

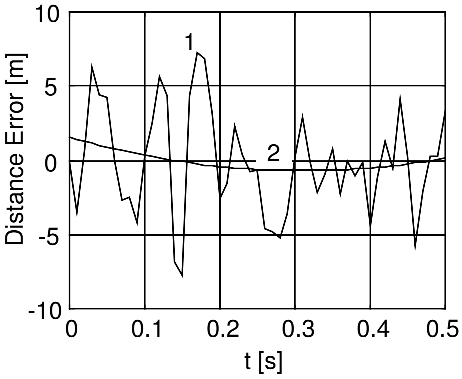

Fig. 2. Distance error for $\sigma^2 = 9\text{m}^2$ and T=500 ms.

variance 9 m^2, geometrical delusion of position GDOP=2.45 and processing interval T=200 ms. Fig. 2 presents the results when the processing interval is 500 ms.

It's obvious that the error in estimating the distance is changing from -8 to +7 meters for the observed values (curve 1), while after applying the suggested algorithm, for the estimated values (curve 2) the error is within the bounds of -2 to +4 meters.

The error in estimating the distance is changing from -7 to +7 meters for the observed values (curve 1), while after apply-ing the suggested algorithm, for the estimated values (curve 2) the error is within the bounds of -1 to +2 meters.

Fig. 3 and Fig. 4 show the results with the error variances 100 m^2.

It's obvious that the error in estimating the distance is changing from -16 to +38 meters for the observed values (curve 1), while after applying the suggested algorithm, for the estimated values (curve 2) the error is within the bounds of -5 to +12 meters.

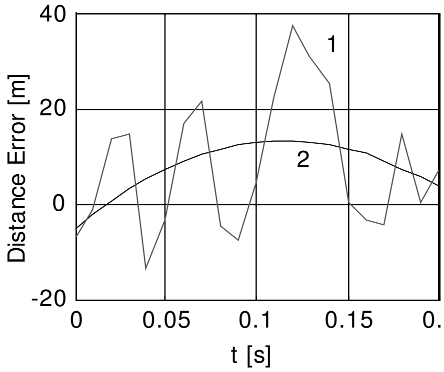

Fig. 3. Distance error for $\sigma^2 = 100\text{m}^2$ and T=200 ms.

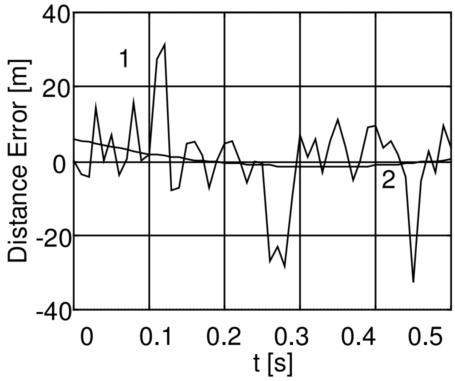

Fig. 4. Distance error for $\sigma^2 = 100\text{m}^2$ and T=200 ms.

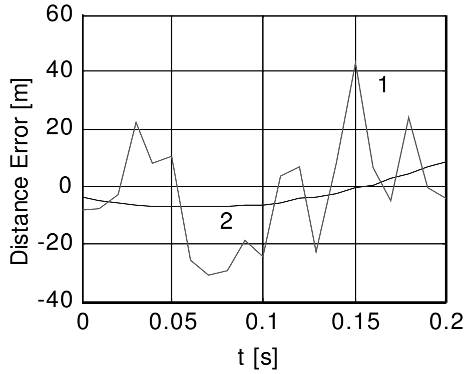

Fig. 5. Distance error for $\sigma^2 = 400\text{m}^2$ and T=200 ms.

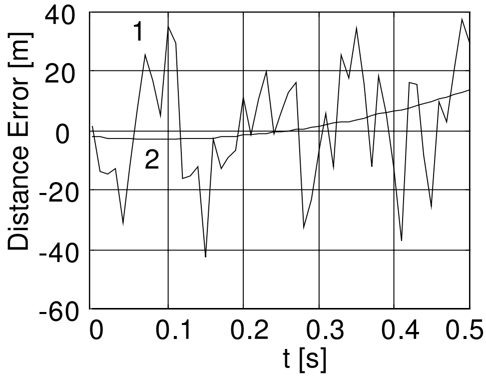

Fig. 6. Distance error for $\sigma^2 = 400\text{m}^2$ and T=500 ms.

It's clear that the error in estimating the distance is changing from -32 to +30 meters for the observed values (curve 1), while for the estimated values, after applying the suggested algorithm, the error is within the bounds of -2 to +7 meters (curve 2).

Fig. 5 and Fig. 6 show the results with the error variances 400 m^2.

It's obvious that the error in estimating the distance is changing from -30 to +42 meters for the observed values (curve 1), while for the estimated values, after applying the suggested algorithm, the error is within the bounds of -8 +10 meters (curve 2).

It's clear that the error in estimating the distance is changing from -42 to +39 meters for the observed values (curve 1), while for the estimated values, after applying the suggested algorithm, the error is within the bounds of -3 to +11 meters (curve 2).

IV. Conclusion

The attained results prove that the accuracy in estimating the distance increases if the suggested algorithm is used. The improvement of the accuracy in estimating the distances leads to improvement of the accuracy in estimating the geographical coordinates of the user. Implementing the algorithm in its recursive form would further more reduce the requirements to the processors.

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