IM/DD Optical System Performance in the Presence Timing Jitter and Gaussian Noise

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Abstract – **In this paper the IM/DD optical system is considered by treating the optical fiber as a linear and nonlinear medium, respectively. First, the conditions under which the dispersion effects dominate over the nonlinear effects are defined. The case of Gaussian pulse propagation is assumed. The error probability is calculated for unchirped incident pulses in the presence of timing jitter and Gaussian noise. The previous analysis repeats for a nonlinear and dispersive fiber. Also the detailed analysis of obtained results is performed.**

Keywords – **Fiber Optic Communications, IM/DD Optical System, Timing Jitter, Gaussian Noise, Bit Error Probability**

I. Introduction

The wavelength band and the optical communication system performance are determined by optical fiber characteristics. In this paper the attention is focused on the fiber dispersion and combined dispersion and nonlinear characteristics.

First, the optical system with Intensity Modulation and Direct Detection (IM/DD) is considered when the incident power and the fiber length are such that nonlinear effects can be neglected.

Dispersion is a consequence of the refractive index frequency dependence. Namely, different pulse spectral components travel with different speeds and have different time delays that lead to the pulse broadening with propagation through fiber. This generates the bit interference and the greater possibility of wrong detection. Such performance degradation is considered for Gaussian pulses.

Also, in this paper we mostly consider equal influence of both effects on pulse shape and also the case when exist self influence of dispersive effects. The main effect of dispersive influence of optical fiber is broadening an optical pulse as it propagates through the fiber. Size of these effects depends from values of GVD (group-velocity dispersion) parameter β_2.

GVD parameter can be positive or negative disregarding the light wavelength, which is below or above the zero-dispersion wavelength λ_D of fiber. Intensity dependence of the refractive index in nonlinear media occurs through SPM (self-phase modulation), a phenomenon that leads to spectral broadening of optical pulses. Size of nonlinear effect depends from values of parameter γ [1].

Direct detection is realized by sampling the photodiode current and by comparison the obtained sample with threshold. Because of timing jitter the sampling moment varieties. The error probability in the presence of timing jitter is also determined when the fiber is represented as a linear, i.e. nonlinear medium.

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II. Theoretical Basis of the Pulse Propagation

Propagation short pulse, which width is between 10 fs and 50 ps, along the nonlinear-dispersive optical fiber can be described by Schrödinger equation. It is [1-3]:

$$\frac{\partial A}{\partial z} = -\frac{1}{2}\alpha A - \frac{i}{2}\beta_2\frac{\partial^2 A}{\partial T^2} + i\gamma|A|^2 A \qquad (1)$$

where A is slowly varying amplitude of pulse envelope and $T = t - z/v_g$, v_g is group velocity, $\gamma = n_2\omega_0/(cA_{eff})$ is coefficient nonlinearity, A_{eff} is effective core area, GVD parameter is $\beta_2 = \partial^2\beta/\partial\omega^2\Big|_{\omega=\omega_0}$, n_2 is nonlinear-index refractive coefficient.

It is useful to observe eq. (1) in normalized form and then we can use following normalized parameters:

$$\tau = \frac{t-\beta_1 z}{T_0}, \quad \xi = \frac{z}{L_D}, \quad U = \frac{A}{\sqrt{P_0}} \qquad (2)$$

where T_0 is the half width (at $1/e$-intensity point) of pulse, P_0 is the peak power of the incident pulse and L_D is the dispersion length, i.e.

$$L_D = \frac{T_0^2}{|\beta_2|}. \qquad (3)$$

Than for $\alpha = 0$, eq. (1) takes normalized form:

$$i\frac{\partial U}{\partial \xi} = \text{sqn}(\beta_2)\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2} - N^2|U|^2 U \qquad (4)$$

where sgn(β_2) takes values +1 or -1 in dependence of dispersive regime ($\beta_2 > 0$ – normal and $\beta_2 < 0$ – anomalous dispersion regime). The eq. (5) is known as Nonlinear Shrödinger equation (NSE).

The parameter N is defined as

$$N^2 = \frac{\gamma P_0 T_0^2}{|\beta_2|} = \frac{L_D}{L_{NL}} \qquad (5)$$

and it represents nondimensional combination of the pulse and fiber parameters. Dispersion dominates for $N \ll 1$, while SPM dominates for $N \gg 1$. In eq. (5) parameter L_{NL} is the nonlinear length and it is defined as

$$L_{NL} = \frac{1}{\gamma P_0}. \quad (6)$$

Also, in this paper we shall consider the effect of GVD on pulse propagation in linear dispersive medium by setting $N = 0$ (i.e. $\gamma = 0$) in eq. (4):

$$i\frac{\partial U}{\partial \xi} = \mathrm{sgn}(\beta_2)\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2}. \quad (7)$$

This paper considers the pulse-propagation problem in IM/DD optical system with timing jitter by treating the fiber as a linear and nonlinear and dispersive medium.

III. The System Performance

For pulse widths greater than 0.1 ps, in linear dispersive medium by setting $\gamma = 0$ where the fiber loss is periodically compensated, $U(z, T)$ – a length normalized complex amplitude satisfies a linear partial differential equation given by [1].

$$i\frac{\partial U}{\partial z} = \frac{1}{2}\frac{\partial^2 U}{\partial T^2}. \quad (8)$$

The case of a Gaussian pulse is considered. The incident field is given by

$$U(0, T) = \exp\Big(-\frac{T^2}{T_0^2}\Big). \quad (9)$$

The linear partial differential equation is solved by using the Fourier method [1] and the solution, i.e. the normalized complex amplitude at any point z along the fiber is

$$U(z, T) = \frac{1}{2\pi}X$$
$$X = \int_{-\infty}^{\infty} \tilde{U}(0, \omega)\exp\Big(\frac{i}{2}\beta_2\omega^2 z - i\omega T\Big)d\omega \quad (10)$$

Then, we consider incident pulses which propagated along the nonlinear and dispersive propagation regime. The power normalized eq. (1) represents nonlinear partial differential equation and we can use a great number of numerical methods for its solving. One of those methods is *"Split-step" Fourier method* [1,3], which represents pseudospectral methods. In this paper, this method is used for solving nonlinear Schrödinger equation, when incident pulse has considered Gaussian form.

The change of the pulse width due to propagation through the fiber can lead to appearance of the interference. So, the equivalent complex amplitude for the fiber length $z = L$ is

$$U_{eq}(L, T) = U(L, T) + \sum_{n=1}^{m/2} b_{\pm n}U(L, T \mp n2T_0) \quad (11)$$

if "1" is sent, or

$$U_{eq}(L, T) = \sum \_n = 1^{m/2} b_{\pm n}U(L, T \mp n2T_0) \quad (12)$$

if "0" is sent, where m is the number of the neighborhood pulses which interfere and $b_{\pm n}$ is the coefficient from set $\{0, 1\}$ depending if 0 or 1 is transmitted.

In IM/DD optical system [3] the photodiode current sample compares with the threshold level determined for ideal case. Since the photodiode is the quadrate detector, the complex photodiode current I is

$$I = R|U_{eq}(L, T)|^2 + n(T) \quad (13)$$

where R is a conversion coefficient and $n(T)$ is a Gaussian noise. The sampling moment due to timing jitter [4] is not at $T = 0$, i.e. $T = T_J$. Then the complex amplitude I at the sampling point T_J for a given combination of bits $b_{\pm n}$, $n = 1, m/2$ is a Gaussian variable with a mean value $R|U_{eq}(L, T_J)|^2$ and variance σ^2

$$p_1(I/b_{-m/2}, ..., b_{-1}, b_1, ..., b_{m/2}) =$$
$$= \frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{\left[I - R\left|U(L,T_J) + \sum_{n=1}^{m/2} b_{\pm n}U(L,T_J \mp n2T_0)\right|^2\right]^2}{2\sigma^2}\right) \quad (14)$$
$$p_0(I/b_{-m/2}, ..., b_{-1}, b_1, ..., b_{m/2}) =$$
$$= \frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{\left[I - R\left|\sum_{n=1}^{m/2} b_{\pm n}U(L,T_J \mp n2T_0)\right|^2\right]^2}{2\sigma^2}\right)$$

As bits are independent, each having a 0.5 probability of being 1 and 0, the conditional probability density functions are averaged with $1/2^m$. The error is made if 0 is sent and the current sample is greater than threshold

$$I_T = \frac{R|U(L, 0)|^2}{2} \quad (15)$$

and vice versa. Then the error probability is [4,5]

$$BER = P(D_1, H_0) + P(D_0, H_1) =$$
$$= \frac{1}{2}P(D_1/H_0) + \frac{1}{2}P(D_0/H_1) =$$
$$= \frac{1}{2}\sum_{b_{-m/2},\dots,b_{m/2}=0\dots0}^{1\dots1} \frac{1}{2^m}\frac{1}{2}\mathrm{erfc}\left(\frac{I_T - R\left|\sum_{n=1}^{m/2} b_{\pm n}U(L,T_J \mp n2T_0)\right|^2}{\sqrt{2}\sigma}\right) +$$
$$+ \frac{1}{2}\sum_{b_{-m/2},\dots,b_{m/2}=0\dots0}^{1\dots1} \frac{1}{2^m}\frac{1}{2}\mathrm{erfc}\left(\frac{R\left|U(L,T_J) + \sum_{n=1}^{m/2} b_{\pm n}U(L,T_J \mp n2T_0)\right|^2 - I_T}{\sqrt{2}\sigma}\right) \quad (16)$$

IV. Numerical Results

The obtained results for bit error probability as a function of the normalized fiber length L/L_D for signal to noise ratio $SNR = \{20, 25\}$ dB and the system without timing jitter are shown in Fig. 1. Fiber length is always normalized in accordance to L_D when dispersion in fiber is not higher than order two. For initially unchirped Gaussian pulse and linear dispersion propagation medium, the error probability monotonically increases with the increase of the fiber length. Also, we can see that BER increases when SNR decreases.

Fig. 2. represents BER as a function of L for $SNR = \{20, 25\}$ dB and the system without timing jitter. The

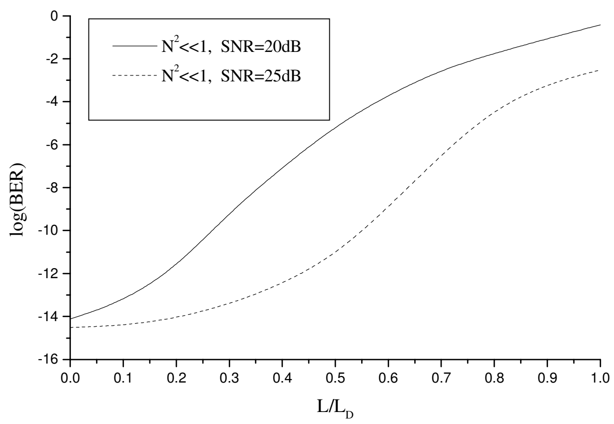

Fig. 1. BER as a function of the fiber length L for different SNR for system without timing jitter when the dispersion is dominant along fiber; $\lambda = 1.55$ μm, $\beta_2 = -20$ ps^2/km, $\gamma = 20$ (Wkm)$^{-1}$, $B = 20$ Gbit/s, $P_0 = 0.08$ mW

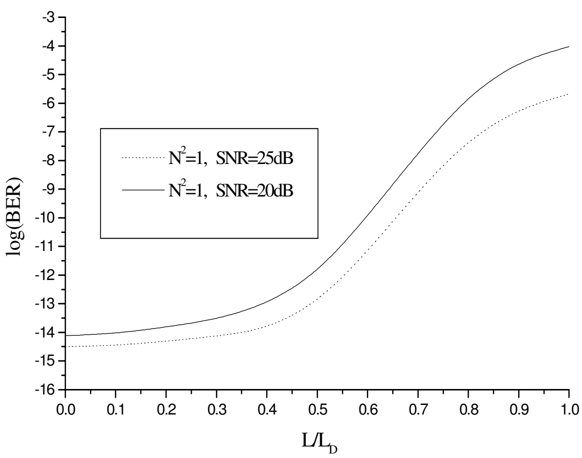

Fig. 2. BER as a function of the fiber length L for different SNR for system without timing jitter when is considered equal influence of dispersion and nonlinearity along fiber; $\lambda = 1550$ nm, $\beta_2 = -20$ ps^2/km, $\gamma = 20$ (Wkm)$^{-1}$, $B = 20$ Gbit/s, $P_0 = 1.6$ mW

unchirped Gaussian pulses are propagated along the nonlinear and dispersive medium. BER monotonically increases with the increase of the fiber length and the decrease of SNR, too. System performance is better in this propagation regime, looking BER as a function of fiber length in both cases.

Fig. 3 shows the bit error probability as a function of the signal to noise ratio SNR for fiber length $L = 0.5L_D$ and $T_J = \{T_0, T_0/10, T_0/5\}$. For initially unchirped Gaussian pulses and linear dispersion propagation medium, the bit error probability monotonically decreases with the increase of SNR and the decrease of parameter T_J.

In Fig. 4. we can see the bit error probability as a function of the signal to noise ratio SNR for fiber length $L = 0.5L_D$ and $T_J = \{T_0, T_0/10, T_0/5\}$, when signal propagated along the nonlinear and dispersive fiber. The same conclusions, regarding the bit error probability, being valid for Fig. 3 are

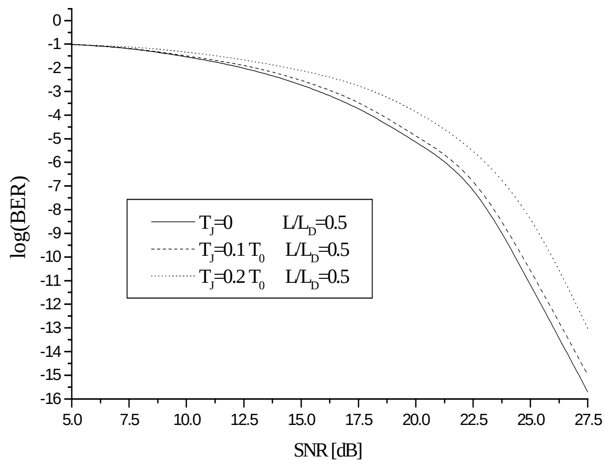

Fig. 3. BER as a function of SNR for different SNR for different values of T_J when the dispersion is dominant along fiber; $\lambda = 1.55$ μm, $\beta_2 = -20$ ps^2/km, $\gamma = 20$ (Wkm)$^{-1}$, $B = 20$ Gbit/s, $P_0 = 0.08$ mW

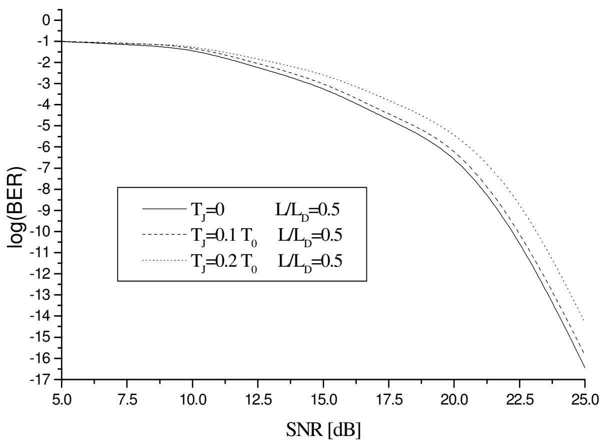

Fig. 4. BER as a function of SNR for different SNR for different values of T_J when a signal propagated along a nonlinear and dispersive fiber; $\lambda = 1.55$ μm, $\beta_2 = -20$ ps^2/km, $\gamma = 20$ (Wkm)$^{-1}$, $N^2 = 1$, $B = 20$ Gbit/s, $P_0 = 1.6$ mW

valid for Fig. 4, too. The system performance is better, when we have equal influence of dispersion and nonlinearity, looking BER as a function of SNR.

V. Conclusion

Dispersion-induced broadening of the unchirped pulse is undesirable since it interferes with the detection process leading to errors in decision. Dispersion limits the bit rate $B = 1/(2T_0)$ and the transmission distance L of a fiber-optic communication system. Timing jitter and Gaussian noise have negative influence of system detection performance in both propagation regimes, but that influence is significantly for linear dispersion propagation regime, considering equal product BL in the both cases.

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