Evaluation of Optimum Orthogonal Code Allocation in CDMA Downlinks

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Abstract – **This paper presents an optimum orthogonal code allocation in case of the multiple-scrambling-code approach. The multiple-scrambling-code approach that was used in the aforementioned capacity results for the code limitation cases fully utilizes the soft capacity feature of CDMA. If the orthogonal code is short in a cell, then a new orthogonal code set is made appending a new scrambling code. However, we can no longer expect to avoid intracell interference between signals assigned different scrambling codes, and therefore the assignment of the orthogonal codes to the respective signals is an important study item if we continue to consider the multiple-scrambling-code approach.**

I. Introduction

The orthogonal code limitation affects CDMA (Code-division multiple access) performance: The fewer the orthogonal codes, the smaller the capacity. This code limitation problem is also discussed extensively in [1], which takes a theoretical approach.

Two general approaches are used to cope with the code limitation: the call-blocking approach and the multiple-scrambling-code approach. In call blocking, the arrival of a new call is blocked if the number of MSs (mobile stations) connected exceeds the number of orthogonal codes available. This approach sacrifices some of the soft capacity feature of CDMA because a call will be rejected in case of orthogonal code shortage even though the interference has not reached the specified limit. However, the multiple-scrambling-code approach that was used in the aforementioned capacity results for the code limitation cases fully utilizes the soft capacity feature of CDMA; if the orthogonal code is short in a cell, then a new orthogonal code set is made appending a new scrambling code. However, we can no longer expect to avoid intracell interference between signals assigned different scrambling codes, and therefore the assignment of the orthogonal codes to the respective signals is an important study item if we continue to consider the multiple-scrambling-code approach.

In cellular CDMA systems, downlink signals are spread by orthogonal spreading codes in order to minimize the interference between the signals [2]. However, because the number of orthogonal codes is limited, the downlink capacity is also limited once the number of MSs connected exceeds the number of orthogonal codes available. The impact of code limitation depends on such factors as the data transmission rate and the forward error control coding rate and is especially significant when soft handoff is used. This is because soft handoff accelerates the consumption of the orthogonal codes in accordance with the number of base stations connected to an MS. The number of downlink channels can be increased by enabling multiple scrambling codes to be allocated to a single BS. But a downlink signal is then subject to strong interference from the other signals assigned different scrambling codes. In this paper we discuss, with the help of a general genetic algorithm toolkit implemented in Java , an optimum code allocation maximizing the average SIR(Signal-to-interference power ratio) measured at MSs within a cell. A genetic algorithm is a search/optimization technique based on natural selection. Successive generations evolve more fit individuals based on Darwinian survival of the fittest. The genetic algorithm is a computer simulation of such evolution where the user provides the environment (function) in which the population must evolve [3].

II. Average SIR

The term *code allocation* throughout this section means an allocation of codes given as products of the multiplication of a basic orthogonal code set and multiple scrambling codes. We denote as S the number of basic orthogonal codes and denote as N the number of orthogonal code sets provided for a cell. The transmission power for downlink signals is fixed, and the orthogonal code occupancy ratio k_i is the ratio of the number of MSs assigned the i-th orthogonal code set to the number n of all MSs within a cell, where the sum of $k_1, k_2, ...,$ and k_N is 1. Defining the code occupancy ratio vector of $k = (k_1, k_2, ..., k_N)$, we can write the following equation for the average SIR measured at MSs within a cell:

$$\bar{\Gamma}(k) = \sum_{i=1}^{N} k_i \frac{P_g}{\varepsilon k_i n + (1-k_i)n} = \frac{P_g}{n} \sum_{i=1}^{N} \frac{1}{\varepsilon + 1/k_i - 1} \quad (1)$$

where P_g and ε denote the processing gain and the interference figure. The value of ε ranges from 0 to 1, and $\varepsilon = 0$ denotes no multipath distortion in the downlink channel. In (1), $\varepsilon = 1$ is assumed between downlink signals assigned different scrambling codes, hence the signals with different scrambling codes interfere completely with each other. We also assume in (1) a single-cell environment, that is, we exclude the intercell interference in measurement of the average SIR.

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III. Extreme Value of Average SIR

When $k^* = (k_1^*, k_2^*, ..., k_N^*)$ is the code occupancy ratio vector giving an extreme value of $\bar{\Gamma}(k)$ and when $d = (d_1, d_2, ..., d_N)$ is the variation vector, the equation

$$\left.\frac{d\bar{\Gamma}(k^* + \alpha d)}{d\alpha}\right|_{\alpha=0} = 0 \quad (2)$$

is satisfied for an arbitrary variation vector d because $\bar{\Gamma}(k)$ takes an extreme value at $k = k^*$. From (1) and (2), we can derive

$$\frac{P_g}{n}\sum_{i=2}^{N} d_i \left[\frac{-1}{\left((\varepsilon-1)\left(1-\sum_{j=2}^{N}k_j^*\right)+1\right)^2} + \frac{1}{\left((\varepsilon-1)k_i^*+1\right)^2} \right] = 0 \quad (3)$$

which is an identical equation with respect to d_i because the equation must be satisfied for an arbitrary variation vector d, and hence all coefficients of d_i must be 0. Eventually, simultaneous equations with respect to k_i are given as follows:

$$\frac{-1}{\left((\varepsilon-1)\left(1-\sum_{j=2}^{N}k_j^*\right)+1\right)^2} + \frac{1}{\left((\varepsilon-1)k_i^*+1\right)^2} = 0 \quad (4)$$

Solving simultaneous equations led by (4), we have

$$k_i^* = 1 - \sum_{j=2}^{N} k_j^* = k_1^* \quad \because \quad i = 2 \sim N \quad (5)$$

which leads to a unique solution to the simultaneous equations of interest, $k^* = (1/N, 1/N, ..., l/N)$. To determine which $k^* = (1/N, 1/N, ..., l/N)$ gives a minimum or maximum SIR, we use Figure 1, which is a plot of the average SIR against k_1 and k_2 in the case for which $N = 3$ is assumed. The assumed case gives $k^* = (k_1, k_2, k_3) = (1/3, 1/3, 1/3)$. From Figure 1, we can see that the point at $(k_2, k_3) = (1/3, 1/3)$ shows the lowest SIR. The sequence developed so far tells us that the average SIR is smallest when each orthogonal code set is equally used.

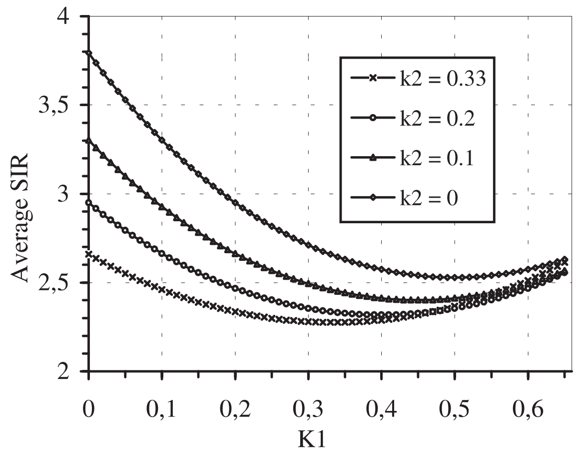

Fig. 1. The average SIR for $N = 3$

IV. Optimum Code Allocation

Figure 1 and Table 1 also shows that the average SIR $\bar{\Gamma}(k^* + \alpha d)$ becomes larger as the absolute value of α increases. There is, however, a limit to the value of α because all elements of the vector $k^* + \alpha d$ have to be between 0 and S/n $(n \geq S)$. Defining the limit of α as α_1, we can write the code occupancy vector $k^* + \alpha_1 d$ at α_1 as

$$k^* + \alpha_1 d = \left(\frac{S}{n}, \frac{S}{n}, ..., \frac{S}{n}, r_1, r_2, ..., r_R, 0, 0, ..., 0\right) = k \quad (6)$$

where $0 < r_1 < r_2 < \cdots < r_R < S/n$. We call k_r a code occupancy bound vector, and the code occupancy vector maximizing $\bar{\Gamma}$ is one of these code occupancy bound vectors.

From Figure 1 and Table 1 we can see that as the number of fully occupied orthogonal code sets increases, the average SIR can be larger and also as the number of empty orthogonal code sets increases, and hence as the use of scrambling codes is reduced, the average SIR can be larger.

Finally, an optimum code allocation can be described as follows: Optimum code allocation is an allocation scheme that maximizes the number of code sets fully occupied and minimizes the use of scrambling codes.

Table 1. Average SIR against N for $S = 128$, $n = 270$, $P_g = 512$ and $\varepsilon = 0.5$

N	2	3	4	5	8	10
Average SIR, dB	4.03	3.57	3.36	3.24	3.06	3.00

V. Evaluation of Optimum Code Allocation

The average SIRs for the optimum code allocation were compared with those for the worst code allocation in which every orthogonal code set enabled by multiple scrambling codes is equally occupied. When $N = 5$, $S = 128$, $n = 270$, $P_g = 512$, and $\varepsilon = 0.5$, the average SIR was found to be 3.24 dB for the worst allocation and 3.90 dB for the optimum allocation. That is, the optimum code allocation improved the average SIR by about 0.7 dB. And when $N = 2$, $S = 128$, $n = 150$, $P_g = 512$, and $\varepsilon = 0.5$, the average SIRs given by the optimum and worst code allocations, respectively, were 6.58 and 7.50 dB; the average SIR was about 1 dB better with the optimum allocation.

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