Precision and Noise Immunity of the Communication Systems

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Abstract – **The article proposed an estimation of the functional precision and noise immunity of a communication system, taking in to account the instability factors. Graphic dependencies for error probability, as a function of the reference signal phase dispersion in coherent demodulation of 2PSK, are derived. The results can be used in designing, exploitation and adjusting of communication systems.**

Keywords – **Noise immunity, Communication system**

I. Introduction

The main factor impacting on the precision and the noise immunity of the communication system are: schematics, production tolerance and the operating conditions [1,4]. The system precision is defined as the ability of the system to execute the given functions at a definite proximity to its ideal parameters. The functional dependence of the system may be show as a function of:

$$y = \varphi(S, X) \tag{1}$$

where S are the input signals, X – the system parameters. Through introducing the concept of ideal system, an estimation of the reproduction accuracy of (1) may be done. Formula (1) is modelled when absolute accuracy is set:

$$\Delta y = y_n - y = \varphi(S_n, X_n) - \varphi(S, X) \tag{2}$$

where $y_n = \varphi(S_n, X_n)$ is the nominal value of y. The functional precision is determined in the field of limit $[a, b]$ of y:

$$a \leq y \leq b. \tag{3}$$

If the function $\varphi(x_1, x_2, ..., x_n)$ is derivate, the absolute error Δy of the function φ is:

$$\Delta y = \sum_{i=1}^{n} \left| \frac{\partial y}{\partial x_i} \right| \Delta x_i \tag{4}$$

where Δx_i usually in the design process is given the boundary value of Δy, that may be done by determining the boundary values of Δx_i, satisfying the inequality:

$$\Delta y \leq \sum_{i=1}^{n} \Delta x_i. \tag{5}$$

If it is accepted that the influence of all x_i is equal and for a given Δy:

$$\Delta x_i = \Delta y \Big/ \sum_{i=1}^{n} \left| \frac{\partial y}{\partial x_i} \right|, \qquad \Delta x_i = \Delta y \Big/ n \left| \frac{\partial y}{\partial x_i} \right|. \tag{6}$$

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II. Calculation of the System Parameters Limits

The calculation of the limits of the system parameters may be done by using experimental-statistic modelling. The following problem is solved:

$$y = y_n + \Delta y. \tag{7}$$

For a given Δy and known nominal characteristics, from equation (7) are derived the necessary changes of the factors Δx_i $(i = 1, ..., n)$.

There could be the following cases:

A.) The output index (signal magnitude, energy) depends on only one factor by a linear law:

$$y = \varphi(x_1) \rightarrow y = b_0 + b_1 x_1 \rightarrow \Delta y = b_1 \Delta x_1. \tag{8}$$

B.) Linear dependence of two factors:

$$y = \varphi)x_1, x_2) \rightarrow y = b_0 + b_1 x_1 + b_2 x_2 \rightarrow \rightarrow \Delta y = b_1 \Delta x_1 + b_2 \Delta x_2. \tag{9}$$

Equation (10) is transformed in:

$$\frac{\Delta y}{b_1 b_2} = \frac{1}{b_2} \Delta x_1 + \frac{1}{b_1} \Delta x_2. \tag{10}$$

The parameter Δx_2 is defined with:

$$\Delta x_2 = \frac{b_2 \Delta y - b_1 \Delta x_1}{b_2}. \tag{11}$$

By analogy, Δx_1 is determined. It is necessary to remark that the computed values of the parameters must satisfy the technology limits:

$$-c_1 \leq \Delta x_1 \leq c_1, \quad -c_2 \leq \Delta x_2 \leq c_2.$$

For the general case for n factors, the equation is:

$$\Delta y = \sum_{i=1}^{n} b_i \Delta x_i. \tag{12}$$

Solution of (12) is possible, when it is used the method of limit increasing or the method of succession approximation where are computed repeatedly the values of b_i. These are the values of partial derivatives and define the order of impact every variable Δx_i.

III. The Noise Immunity as a Criterion for Functional Precision

The system noise immunity may be used as a criterion for defining its functional precision. In this case the main parameters is the relation between the average power of the signal and noise power – signal-to-noise ratio (SNR) at the which is obtained a given error probability $Pe = \varphi(q)$. At the practical realization and the exploitation of the communication system is observed worsening of the noise immunity in comparison with the "ideal" (theoretical) system. The reason for this, are divergence of the real characteristics from the ideal under the influence of instability factors. These differences between the ideal and the real system lead to an energy and reliability loss:

$$\gamma_E = \frac{E_R}{E_N} > 1 \qquad \gamma_P = \frac{P_R}{P_N} > 1 \quad \text{for}$$
$$h_R^2 = \frac{E_R}{N_0} \qquad h_n^2 = \frac{E_n}{N_0}. \tag{13}$$

E_R and E_n are the signal energies of the real and ideal system, and N_0 is the power spectral density of the noise. If K_R and K_n are transmission coefficients of the real and ideal channel with additive white Gaussian noise (AWGN), the error probability is given by:

$$Pe_R = 0.5\text{erfc}\Big[K_R\sqrt{0.5h_R^2(1-\rho)}\Big] \tag{14}$$

for a real communication system and

$$Pe_N = 0.5\text{erfc}\Big[K_N\sqrt{0.5h_N^2(1-\rho)}\Big] \tag{15}$$

for an ideal communication system and

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}}\int_u^\infty \exp(-x^2)dx.$$

If $Pe_R = Pe_N$, the necessary SNR is:

$$q = \frac{K_R}{K_N}\Big[\sqrt{0.5h_N^2(1-\rho)}\Big].$$

ρ is a correlation coefficient of the modulated signals and error probability is:

$$Pe_N = 0.5\text{erfc}\Big[\frac{K_R}{K_N}\sqrt{0.5h_N^2(1-\rho)}\Big]. \tag{16}$$

Figure 1 shows relation $Pe = \varphi\Big(\frac{K_R}{K_N}\Big)$ for $\rho = -1$ 2PSK.

The error in the recovering the phase of the reference signal at the coherent demodulation impacts on the noise immunity. If $\Delta\varphi$ is the phase error of the reference signal; the phase of received symbol $\varphi_i \in [0.\pi]$ and the phase of reference signal φ_0, consequently the signal after the correlator of the ideal receiver is:

$$Z_s(\varphi) = \frac{2}{N_0}\int_0^T S(t,\varphi_i)S_0(t,\varphi_0+\Delta\varphi)dt.$$

The error probability as a function of the phase error is equal to:

$$Pe(\Delta\varphi) = 0.5\text{erfc}\Big(\sqrt{h^2}\cos(\Delta\varphi)\Big). \tag{17}$$

The average error probability for all values of $\Delta\varphi$ in $-\pi \leq \Delta\varphi \leq \pi$ is:

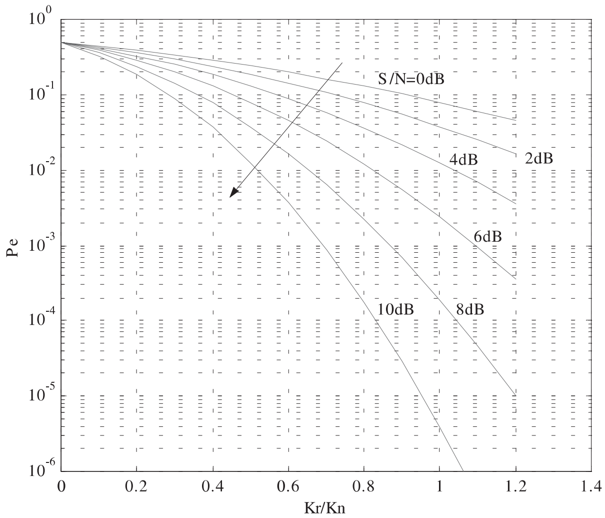

Fig. 1. Another example is related to the error correction coding. The code parameters (n, k) and the SNR is reduced in $h^2 = \frac{k}{n}h_N^2$. In that way the equivalent error probability is computed

$$Pe = \int_{-\infty}^{+\infty} W(\Delta\varphi)Pe(\Delta\varphi)d\Delta\varphi =$$
$$= \int_{-\pi}^{\pi} W(\Delta\varphi)Pe(\Delta\varphi)d\Delta\varphi. \tag{18}$$

The distribution $W(\Delta\varphi)$ of $\Delta\varphi$ may by uniform or normal. Because of the filtering used in optimal receiver, the distribution density approximates with the Gaussian low with dispersion σ_φ^2 and mean value $m_\varphi = 0$:

$$W(\Delta\varphi) = \frac{1}{\sqrt{2\pi}\sigma_\varphi}\exp\Big(-\frac{\Delta\varphi}{2\sigma_\varphi^2}\Big). \tag{19}$$

The mean value of $\cos(\Delta\varphi)$ is:

$$\overline{\cos(\Delta\varphi)} = \int_{-\infty}^{\infty}\cos(\Delta\varphi)W(\Delta\varphi)d\Delta\varphi = \exp\Big(\frac{\sigma_\varphi^2}{2}\Big). \tag{20}$$

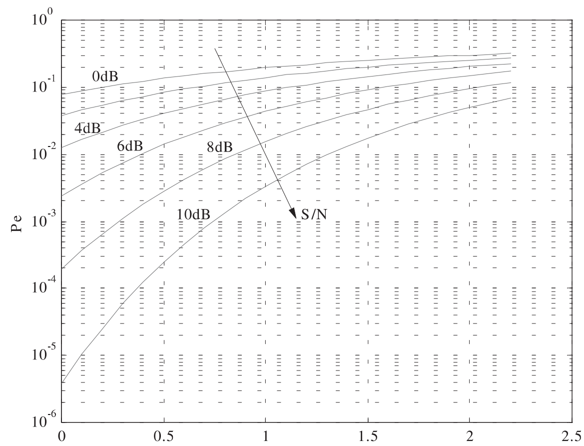

Fig. 2.

Figure 2 show the error probability by demodulation of 2PSK modulated signal in relation to the phase error dispersion by a parameter SNR.

If it is assumed that the signal magnitude after the receiver correlator and $\Delta\varphi$ has a Gaussian distribution, consequently the probability for falling into the interval $E_1 < E < E_2$ and $\varphi_1 < \Delta\varphi < \varphi_2$ is:

$$P(E, \Delta\varphi \in R) = \frac{1}{4}\left[\mathrm{erf}\left(\frac{E_2 - \bar{E}}{\sqrt{2}\sigma_E}\right) - \mathrm{erf}\left(\frac{E_1 - \bar{E}}{\sqrt{2}\sigma_E}\right)\right] \times \left[\mathrm{erf}\left(\frac{\varphi_2 - \varphi}{\sqrt{2}\sigma_\varphi}\right) - \mathrm{erf}\left(\frac{\varphi_1 - \varphi}{\sqrt{2}\sigma_\varphi}\right)\right].$$

IV. Conclusion

The article proposed an estimation of the functional precision and noise immunity of a communication system, taking in to account the instability factors. Graphic dependencies for error probability, as a function of the reference signal phase dispersion in coherent demodulation of 2PSK, are derived. The results can be used in designing, exploitation and adjusting of communication systems.

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