Functional Precision and Work Ability of the Communication System

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Abstract – **The article discusses the communication system precision and work ability problems in their designing, manufacturing and exploitation. It is connected with the defining the fields of functioning, when techno-economical factors are given. In the article, it is proposed the using of Shapiro-Wilk test for express estimation of the normal density of distribution. The method is illustrated by an analysis of the precision and work ability of UHF radio transmitters.**

Keywords – **Work ability, Shapiro-Wilk test**

I. Introduction

Every task for communication system design, exploitation, and manufactory is related to the defining of the fields of functioning, when techno-economical factors are given. For example they could be: data errorless transmission, noise performance, reliability, electromagnetic compatibility etc.

In the general case the system S is assumed, that functions if the condition is true:

$$a_i \leq y_i \leq b_i, \qquad i = 1, 2, 3, ...$$

where y_i is the nominal value of the system indicator (power magnitude, frequency etc.). The interval $[a, b]$ defines the field limits – the deviation from the nominal value of y_i. It is expressed by the reliability probability:

$$P_D = p(a < \bar{y} < b) = \int_a^b W(y)dy. \qquad (1)$$

Let the following probabilities are given:

$$Pa_i = p(y < a_i) = \int_{-\infty}^{a_i} W(y)dy,$$
$$Pb_i = p(y < b_i) = \int_{-\infty}^{b_i} W(y)dy. \qquad (2)$$

The precision and work ability estimation of the system S, is made by checking two alternative hypothesis:

H_1 – system indicators are in the field of limits;

H_2 – system indicators are out of the $[a, b]$ interval.

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II. Precision and Work Ability

The problem for defining the work ability is in obtaining the one dimensional distribution law $W(y_i)$, as a function of the parameters $x_1,\ x_2, ..., x_n$. Because of the great number of factors $x_i\ i = 1, 2, ..., n$, no one among them cannot be dominating. Consequently, the random parameters are distributed in a Gaussian law $W(y_i)$ according to the Central Limit Theorem

$$W(y_i) = \frac{1}{\sqrt{2\pi}\sigma_y} \exp\Big(- \frac{(y_i - \bar{y})^2}{2\sigma_i^2} \Big), \qquad (3)$$

where $\bar{y}$ is the mean value of y_i. If the parameters $\{x_1, x_2, ..., x_n\}$ are measured the check if the probability density has a normal distribution may be done through the Shapiro-Wilk algorithm [3,7]:

1. From a sample of N elements a variation sequence is build: $\{x_1, x_2, ..., x_n\}$;
2. The weight sum b is defined by the coefficients a_{n-i+1}:

$$b = a_n(x_n - x_1) + a_{n-1}(x_{n-1} - x_2) + \cdots + a_{n-k+1}(x_{n-k+1} - x_k)$$
$$= \sum_{i=1}^{k} a_{n-i+1}(x_{n-i+1} - x_i), \qquad (4)$$

   where $k = N/2$ for even N and $k = (N - 1)/2$ for odd N.

3. The statistic criterion is:

$$V = \frac{b^2}{S_x}, \quad S_x = \sum_{i=1}^{N} x_i^2 - \frac{1}{N}\Big(\sum_{i=1}^{N} x_i\Big)^2; \qquad (5)$$

4. The reliability probability P_D is given and it is compared to the critical value $P_V(N, P_D)$. If $V > P_V(N, P_D)$ it is assumed the normal distribution; for $V < P_V(N, P_D)$ – the hypothesis is denied.

For the given sample, it is computed the statistic estimations – mean value μ_y and dispersion σ_y.

The allowable boundaries of the parameters with the reliability P_D are determined. For a normal distribution law the probability for falling of the y_i in the field of limits $[a, b]$ is:

$$P_D = P(a \leq \bar{y} \leq b)$$
$$= \frac{1}{2}\Big|\Big[\mathrm{erf}\Big(\frac{b - \mu_y}{\sqrt{2}\sigma_y}\Big) - \mathrm{erf}\Big(\frac{a - \mu_y}{\sqrt{2}\sigma_y}\Big)\Big]\Big| \qquad (6)$$

for $\text{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-x^2)dx$.

If the changing of the output parameters is symmetrical $b - \mu_y = \mu_y - a = \Delta$ for probability is derived:

$$P(a \le \bar{y} \le b) = \text{erf}\Big(\frac{\Delta}{\sqrt{2}\sigma_y}\Big). \quad (7)$$

For a given σ_y and P_D, the unknown quantities Δ are computed:

$$\begin{aligned} \Delta_1 &= b_1 - \mu_y = \sigma_y \quad a_1 = \mu_y - \sigma_y, \ \text{for } P_D = 0.68 \\ \Delta_2 &= b_2 - \mu_y = 2\sigma_y \quad a_2 = \mu_y - 2\sigma_y, \text{for } P_D = 0.95 \end{aligned} \quad (8)$$

III. Results

The task consists of defining the precision and work ability of UHF radio transmitters. They are controlled in two basic parameters – output power P and carrier frequency f. If the two dimensional distribution of P and f is normal, so it is equal to:

$$W(P,f) = \frac{1}{2\pi\sigma_P\sigma_P\sqrt{1-r^2}} \times \exp\left[-\frac{\frac{(P-P_0)^2}{\sigma_P^2} - \frac{2r(P-P_0)(f-f_0)}{\sigma_P^2\sigma_f^2} + \frac{(f-f_0)^2}{\sigma_f^2}}{2(1-r^2)}\right],$$

where r is a correlation coefficient. If the frequency changes are in small interval in which the output power stays unchanged, consequently these two parameters are independent and $r = 0$. Then the probability of the power and frequency for being in the field R (Fig. 1) is:

$$P(E, f \in R) = \frac{1}{4}\Big|\Big[\text{erf}\Big(\frac{P_h - P_0}{\sqrt{2}\sigma_P}\Big) - \text{erf}\Big(\frac{P_l - P_0}{\sqrt{2}\sigma_P}\Big)\Big] \times \Big[\text{erf}\Big(\frac{f_h - f_0}{\sqrt{2}\sigma_f}\Big) - \text{erf}\Big(\frac{f_l - f_0}{\sqrt{2}\sigma_f}\Big)\Big]\Big|.$$

The results from the power and frequency measurements of the sets of $N = 10$ are given in Table 1.

The distribution of $y(P, f)$ is checked through Shapiro-Wilk algorithm. For $N = 10$ and $a_{10} = 0.574$, $a_9 = 0.33$, $a_8 = 0.21$, $a_7 = 0.12$, $a_6 = 0.04$, the relation of $P_V(N, P_D)$ is shown in Fig. 2 [3].

Making the computation in the algorithm, given in the previous section, for the parameter V is derived $V_P = 0.9345$.

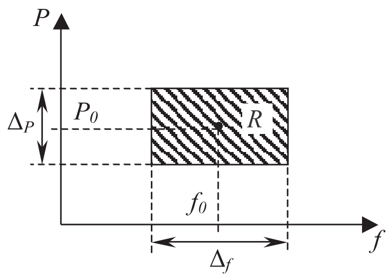

Fig. 1.

Table 1.

No	P [W]	f [MHz]
1	2.16	53.3257
2	1.65	53.3262
3	2.31	53.3259
4	2.7	53.3258
5	1.76	53.3261
6	2.39	53.3267
7	2.55	53.3248
8	1.38	53.3264
9	1.5	53.3256
10	2.2	53.3265
	$\mu_P = 2.06$ $\sigma_P = 0.4576$	$\mu_f = 53.3260$ $\sigma_f = 5.4507\text{e}-04$

In the conditions $V_P > P_V(N, P_D)$ and power values distributed in a Gaussian law, from Fig. 2 is found that the reliability probability must be greater than $P_D \ge 0.605$.

The power deviation from the nominal value $P_n = 2$ W is equal to ± 1 dB. Then $P_l = 1.59$ and $P_h = 2.41$ W. The reliability interval $\Delta_P = 0.41$. For the view of the mean value of the power is approaching to the nominal value so that the formula (7) is used. The reliability probability of the power is equal to $P_D = 0.6297$, when the dispersion $\sigma_P = 0.4576$ (Table 1).

In the same way is computed $V_f = 0.9473$. If the frequency accuracy is ± 100 Hz and the nominal value is $f_n = 53.325$ MHz, then the reliability interval is $\Delta_f = 1\text{e-}4$ MHz. For a dispersion $\sigma_f = \Delta_f$ (formula 8), the reliability probability is $P_D = 0.68$ and $P_V(N, P_D) = 0.9234$(Fig. 2). The condition $V_f > P_V(N, P_D)$ is true, so that it is assumed that the frequency values have normal distribution.

For this parameter, the mean value and nominal value of the frequency are different. The upper and bottom limit of the frequency precision are equal to $f_h = 53.3251$ MHz and $f_l = 53.3249$ MHz. The reliability probability is the defined by (6) for $f_0 = \mu_f = 53.326$ MHz $P_D = 0.0308$.

The results show the following:

1. The probability of the output power of transmitters sample for falling in the limits interval is equal to $P_D =$

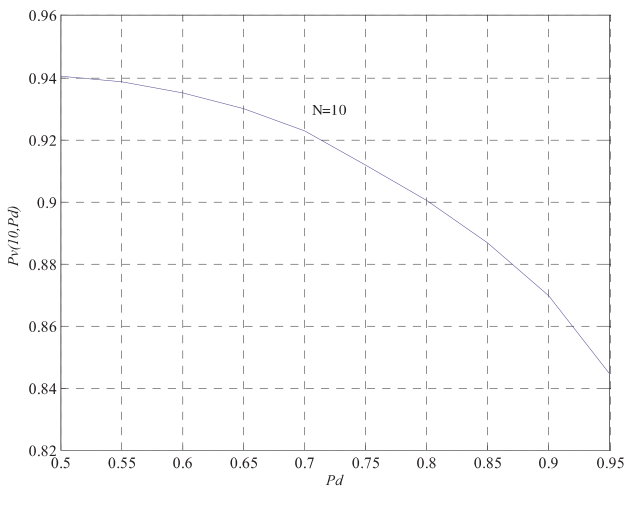

Fig. 2.

0.6297 – 62% from manufacturing devices will have the necessary work ability with pre-adjustment of the amplification;

2. The carrier frequency must be adjusted additionally because of the deviation from the nominal value is more than 1 KHz. The reliability probability is less than the necessary $P_D = 0.68$ – $P_D = 0.0308$.

IV. Conclusion

The proposed algorithm allows express statistic estimation of the precision and the work ability of communication systems and devices. It is used Shapiro-Wilk's method for defining the type of distribution of the controlled parameters. The algorithm is illustrated thought a study of an UHF radio transmitters' sample. The given algorithm can be used in design, exploitation and adjustment of communication devices and systems

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