Sliding Mode Control of Third-Order Objects with Stable Finite Zero[1]

Čedomir Milosavljević[2], Milić M. Pejović[3], Goran Milosavljević[4]

Abstract – **In this paper we consider a new approach for sliding mode control of third-order objects with stable finite zero. The topology of proposed approach contains variable structure controller, object and model-observer intended to obtaining differentials of controlled variable. This traditional control scheme for objects without finite zero is completed by conventional PI part in observer's control channel, and with integral part in the controlled object control channel. Parameters of PI part are chosen in such way that the two control channels become identical for object's nominal parameters. For stability and disturbance rejection improvement a new control action is introduced to the controlled object input from detector of observation error. The described topology is very robust to the internal and external disturbances.**

Keywords – **Variable structure control systems, sliding modes, minimum phase systems**

I. Introduction

As it is well known, in the variable structure control systems (VSS) the control signal is formed as a discontinuous function of system state coordinates. The sign of control signal is determined by the system state with respect to some hyper-plane, which intersects the phase space origin. The control is determined in such way to provide, with no respect to the limited parameter or external disturbances, driving of the system state from any initial state toward the above mentioned hyper-plane, and that the further motion takes place on the hyper-plane in the sliding mode. After the sliding mode has been reached, the system motion is not function of control signal, object parameters and disturbance. It is only determined by parameters of sliding hyper-plane. If those parameters are chosen in such way to provide desired, stable system motion dynamics, then the process of system design is successfully achieved.

With respect to the fact that the control signal is discontinuous high frequency signal, the control of the object with the zeros in transfer function in this class of VSS is facing certain problems, caused by differential action upon the control signal. This problem was noticed, and has been studied at the beginning of VSS theory development [1,2].

Four methods are known. The first one is limited to second order systems [1]. It is based on appropriate state coordination transformation. The second method [2] is based on introduction of cascade of low pass commutation filters in the object control loop. The third method [4] is based on sliding mode organization in the subsystem of lower order. Above mentioned methods are not immune to unwonted motion phenomenon (chattering), caused by presence of non-modelled object's dynamics and high frequency switching control signal. Beside that, it was shown [4] that the conventional asymptotically stable sliding modes couldn't be organized in the systems with the right zeros, so the sliding modes based on generalized minimal variance [5] must be implemented in this case. That problem won't be explained in this paper, i.e. it will be considered only the problem of control of minimal phase objects.

In this paper it's made an attempt on eliminating the shortages of conventional methods as well as providing the quality control over one class of third order objects with the stable finite zero. The proposed way of control algorithm synthesis is based on combination of well-known VSS control laws with the observer and conventional linear control laws of PI type. The proposed way in fact represents combination of three above-mentioned methods [1,2,4], because it includes state coordinate transformation [1], filter implementation [2] and organization of sliding mode in the subsystem of lower order. Only single input single output systems have been considered.

In the second section it will be more preciously defined the structure of proposed solution for VSS, it will be stated some preliminary considerations with respect to synthesis of sliding control for systems without finite zero, with application of linear PI element in the control loop. The third section contains quality verification of proposed solution by simulation of concrete example.

II. Preliminary Considerations

Let us consider fully controllable and observable single input single output object, with bounded parameter perturbations, described by following transfer function

$$W_{ob}(s) = \frac{C(s)}{Us} = \frac{k_o(Ps+1)}{A(s)}, \qquad (1)$$

where $z = 1/P$ is finite zero ($z_{\min} \le z \le z_{\max}$), k_o is object's gain $k_{o\min} \le k_o \le k_{o\max}$, and $A(s)$ is n order polynomial de-

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[2]Čedomir Milosavljević is with the Faculty of Electronic Engineering, University of Niš, Beogradska 14, 18000 Niš, Serbia, E-mail: milosavljevic@elfak.ni.ac.yu

[3]Milić M. Pejović is with the Faculty of Electronic Engineering, University of Niš, Beogradska 14, 18000 Niš, Serbia.

[4]Goran Milosavljević is with The Advanced Technical School, Beogradska 20, 18000 Niš, Serbia.

scribed as

$$s^n + \sum_{i=1}^{n} a_i s^{i-1};\ a_{i\min} \le a_i \le a_{i\max}\ . \tag{2}$$

It is assumed that the object parameters are non-stationary, but the rate of their change is much slower then dynamics of control system process.

In the input of control object we will introduce integral part, what leads to transfer function of extended control object in the form of

$$W_{ab}^{p}(s) = \frac{C(s)}{U_{kl}(s)} = \frac{(Ps+1)}{s}\frac{k_o}{A(s)} = \\ = \left(P + \frac{1}{S}\right)\frac{k_o}{A(s)}\ . \tag{3}$$

Extended control object can be considered as object without finite zero with additional PI part (PI regulator) with determined parameters. Beside full object model let us introduce reduced object model with nominal parameters without finite zero. Relation describing such model is

$$W_{ob}^{s} = \frac{k_o}{A(s)}\ . \tag{4}$$

The reduced object model can be implemented by computer. From that model, all canonical state coordinates (x_i) are available, and its mathematical model is

$$\begin{aligned} \dot{x}_i &= x_{i+1}\ , \\ \dot{x}_n &= -\sum_{i=1}^{n} a_i x_i + k_o u\ . \end{aligned} \tag{5}$$

For such object-model it can be synthesized variable structure control law, for example in the following form,

$$u = \sum_{i=1}^{n-1} \Omega_i x_i\ , \tag{6}$$

$$\Omega_i = \begin{cases} \omega_{i1} & for \quad g x_i > 0\ , \\ \omega_{i2} & for \quad g x_i < 0\ , \end{cases} \tag{7}$$

$$g = \sum_{i=1}^{n} c_i x_i;\ c_i = \text{const} > 0,\ c_n = 1\ , \tag{8}$$

which will provide realization of continuous sliding mode on the hipper-plane $g = 0$ for any variation of parameters a_i and k_o in predefined boundary.

In the monograph [1] it was shown that choice of commutation function parameters (7) in the form of

$$\begin{aligned} \omega_{i1} &> \sup_{a_i, k_o} \frac{-a_i + c_{i-1} + c_i a_n - c_i c_{n-1}}{k_o}\ , \\ \omega_{i2} &< \min_{a_i, k_o} \frac{-a_i + c_{i-1} + c_i a_n - c_i c_{n-1}}{k_o}\ , \\ & i = 1, 2, \ldots, n-1\ , \end{aligned} \tag{9}$$

provides stable sliding mode in the system (5) on $g = 0$, after short reaching period.

In such systems, if object is of zero type, sliding mode could be lost in the small vicinity of equilibrium point, when static error occurred, and some unwonted self-oscillations are possible. Introducing proportional-integral (PI) action in control law can successfully solve that problem. In that case instead of (6), the following control law should be introduced

$$u = \sum_{i=1}^{n-1} \Omega_i x_i + \int_0^t \left(\sum_{i=1}^{n-1} \Omega_i x_i\right) \mathrm{d}t\ . \tag{10}$$

It was shown [6] that by such control sliding conditions (9) are more easier to fulfill. System has zero error and control becomes smooth in the steady state. Chattering exists only in time interval from the reaching moment until the moment when system enters steady state. The system (5) with control (10) can be threaten as a system with finite zero, for which sliding mode is organized in subsystem of lower order [4], because system output signal and its differentials are available for measurement, what is the basic assumption for third above mentioned method.

In the theory and practice of VSS beside control algorithm (6), often is used algorithm in the form

$$u = U_o \text{sgn}(g)\ , \tag{11}$$

which is characterized with powerful switching signal in the reaching mode of sliding hipper-plane as well as in sliding mode itself. From one point of view, it makes system more robust, especially to external disturbances. In the meantime, such signal leads to strong excitation of non-modeled system dynamics. Besides that, for systems with discrete-time data processing, static error might occur. That error could be eliminated by implementation of control algorithm in the form

$$u = U_o(\text{sgn}(g) + \int_0^t \text{sgn}(g)\mathrm{d}t)\ , \tag{12}$$

i.e. by introducing PI variable structure control.

So, for the object-model, with added PI action (10) or (12), sliding mode on hipper-plane (8) can be organized in model space (5) of n-order. Under conditions that object parameters are known and unchangeable, and that the model parameters are the same and that there are no external disturbances affecting the object, obtained control will be at the same time also valid for the extended object. Because on the input of extended object is simple integrator, switching control signal in the input of real object will be continuous and chattering will be eliminate.

As the object parameters, according to assumption, are changing within the known boundary and the object is exposed to external disturbance, in order to provide stability and desired accuracy it is necessary to introduce closed loop from error signal between outputs of object and model.

Above stated considerations lead to the structure block diagram of control system for regulation of considered class of objects, Fig. 1. SMC is sliding mode controller, designed in above described, or some other known methods for reduced object. I-term stands for integral action, which is added to the object input. PI stands for proportional-integral action on the input of objects model (observer).

At this point a remark should be made. We are considering object-model. However, it is *de facto* observer in its basic

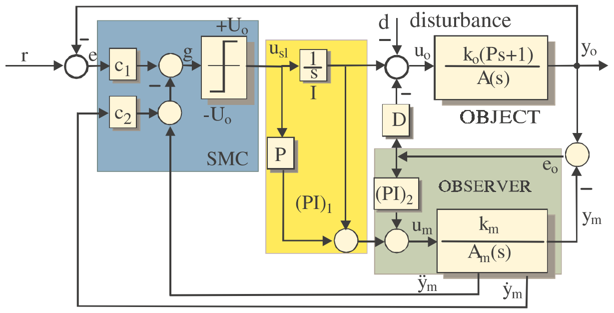

Fig. 1. Proposed control scheme.

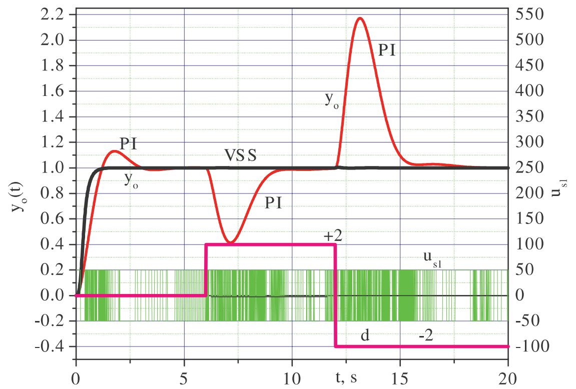

Fig. 2. Step response of the controlled variable y_o for the object with nominal parameters, sliding control signal u_{sl} and disturbance d.

sense, which is formed by feed backing the object model by $(PI)_2$ feedback. Integral action in the observation error loop is introduced in order to increase the observer ability to detect slowly varying disturbances $d(t)$. At the object input it could be introduced additional action from the observation error signal over the element D. That action could be P or PI type for second order objects, while for third order objects it must be of the D type. Without such action, system very slowly rejects external disturbances or oscillations might occur.

III. Verification of Proposed Approach by Digital Simulation

In order to verify efficiency of proposed approach computer simulation was performed for randomly chosen example of third order object, which nominal parameters are

$$a_1 = 10,\ a_2 = 17,\ a_3 = 8,\ k_o = 10,\ P = 0.25;$$
$$d(t) = 2h(t-6) - 4h(t-12)\ .$$

The sliding control was chosen in the following form

$$u_{sl} = 50\mathrm{sgn}(g);\ g = 50x_1 + 15x_2 + 2\ . \tag{13}$$

$(\mathrm{PI})_2$ is $200(1 + 1/s)$ while the differential part is $D = 50$ s. A real differentiator with time constant of 1 ms was used in the simulation.

Fig. 2 shows: step response of controlled variable y_o for nominal parameters when conventional PI and proposed VSS controller are used, sliding mode control signal and load disturbance. The PI controller is approximately optimally tuned by the technical (magnitude) optimum method ($W_{\mathrm{PI}}(s) = 2(1 + 1/s)$). As it can be seen, the system with the proposed VSS controller is invariant to the given disturbance, while the system with the PI controller doesn't have that ability.

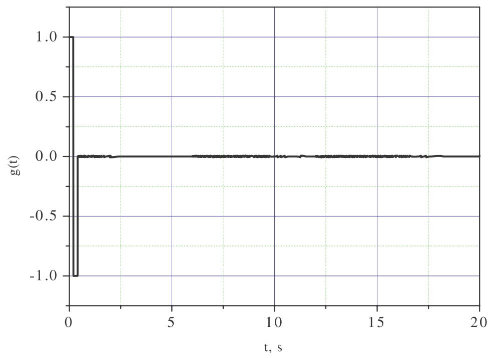

Fig. 3. Switching function $g(t)$.

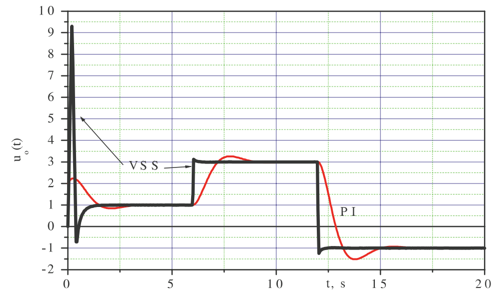

Fig. 4. Control signals u_o for nominal parameters.

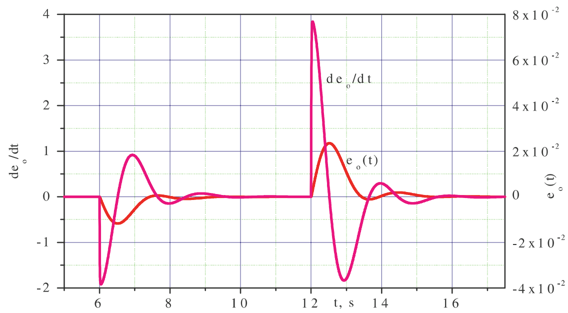

Fig. 5. The observation error e_o and its differential.

Fig. 3 represents switching function $g(t)$ which (as well as the control signals u_{sl} in Fig. 2) indicates existence of sliding mode in the proposed system ($g(t)$=0) after short reaching time.

Fig. 5 shows observer error signal $e_o(t)$ and its derivative $(50de_o/\mathrm{d}t)$, obtained by real differentiator (D) with time constant of 1 ms .

Fig. 6 represents simultaneously process of load rejection for the proposed VSS and the conventional PI controller, while the process for VSS is amplified 100 times.

Fig. 7 shows control process for the real zero changes of $\pm 50\%$ with respect to the nominal value of 0.25.

Fig. 8 gives regulation processes when object gain is changed $\pm 50\%$ with respect to the nominal value (k_o=10).

Fig. 9 presents control signals for the case when object have non modeled inertial mode with time constant of 0.02 s,

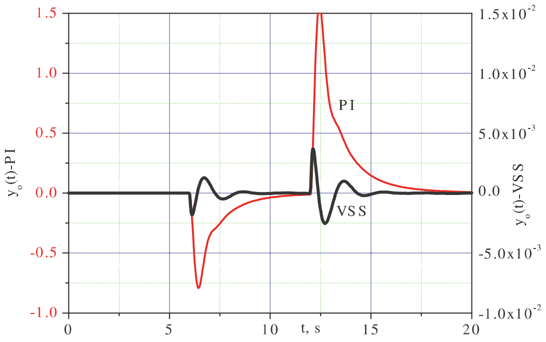

Fig. 6. Load rejection.

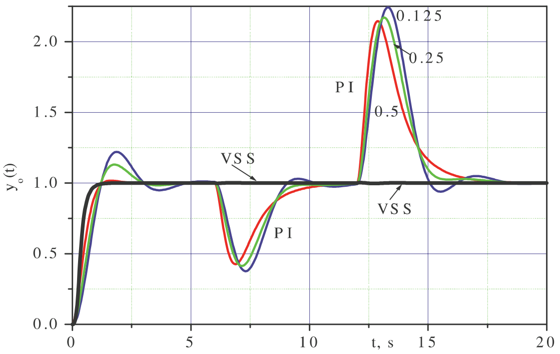

Fig. 7. Step response of the controlled variable y_o for different values of finite zero (D).

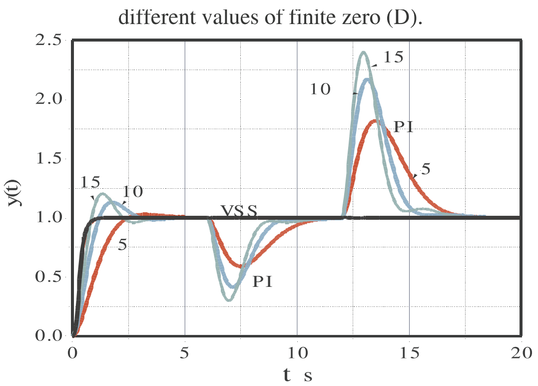

Fig. 8. Step response of the controlled variable y_o for different gain k_o.

(ten times lower then the smallest time constant of object model). In this case, temporary control oscillations occurs for VSS controller. Inertial mod is placed on the object input. In the steady state VSS control signal is smooth and therefore chattering is eliminated.

All computer simulations were performed with integration time of $5 \cdot 10^{-4}$ s with Euler integration method. The proposed system preserves his characteristics with sampling data processing by sampling time lower or equal 1 ms.

IV. Conclusions

The basic properties of proposed solution is considerable robustness to the variations of internal and external disturbances, i.e. practical invariance, chattering elimination, because the control signal is free from high frequency discontinuities. System is also robust to the non-modelled inertial dynamics if the non-modelled time constant is for the order of magnitude smaller then the smallest modelled time constant.

On the basic of this research it can be concluded that the proposed solution for the mentioned class of objects is worth to investigate in detail in order to establish the sliding mode existence conditions, stability and robustness for systems of general case, for single input single output, multiple input multiple output systems, as well as possibility of using this approach to the systems without finite zeros.

For the final verification of proposed control algorithm, it should be implemented on the adequate real object.

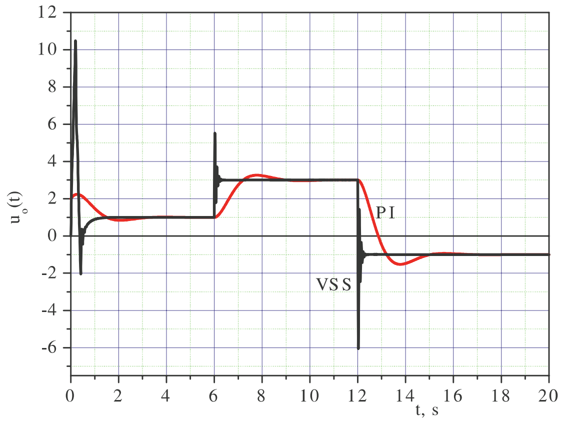

Fig. 9. Control signals with unmodelled inertial dynamics T_i=0.02 s.

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