Comparative Analysis of Some Discrete-Time Sliding Mode Chattering-Free Control Algorithms

Bojan T. Milosavljević[1]and Čedomir Milosavljević[2]

Abstract – **This paper performs a comparative analysis of three discrete-time sliding mode control algorithms that tend to reduce chattering in the controlled variables. First, a brief review of each algorithm is given, based on the original papers. In order to compare presented algorithms, computer simulations have been carried out prior to verify their robustness features. The robustness tests are not limited to parameter uncertainties and external disturbances only, but robustness to the existence of unmodelled dynamics is also considered. Simulation results reveal the best performance algorithm.**

Keywords – **discrete-time sliding mode, chattering-free**

I. Introduction

Variable structure systems with sliding mode theoretically possess very desirable features, such as robustness to controlled plant parameter variations and to external disturbances in very wide range, simple definition of requested motion dynamics, being described by differential equations of lower order than one of the controlled object, high compatibility of modern electronic components and devices to the requests of such a control etc. However, in a real system, there exists parasitic high frequency motion around the sliding surface, the so-called chattering. This phenomenon exists due to discontinuities, high gains, sampling effects and finite switching speed in the system. It can cause damage to actuators or the plant. There are essentially two ways to overcome this problem. One way is to use higher order sliding mode, and the other way is to add a boundary layer around the switching surface and use continuous control inside the boundary. The problem with the first method is that the derivative of the certain state variable is not available for measurement, and therefore methods have to be used to observe that variable. A modification of this method was presented in [1], where the control algorithm is still based on state- and control input derivatives, but combining the equivalent control method and Lyapunov theory, direct use of those variables was avoided. Therefore, it was possible to achieve a continuous control input and thus reduce chattering without observing any variable. In the second method, it is important that the trajectories inside the boundary layer do not try to come outside the boundary after entering the boundary layer.

A number of algorithms can be found in papers based on one of those techniques. Most of them attempt to ensure robustness of the system to parameter uncertainties and external disturbances only. Incomplete knowledge of the system dynamics is very common in engineering practice. Therefore it is very important to provide robust algorithms to the existence of unmodelled dynamics. Then it would be possible to simplify the control design being applied to lower-order system. This paper provides a comparative analysis of three control algorithms regarding robustness properties.

II. Algorithm 1

The following discrete-time system is considered in [3]:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \triangle\mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k) + \mathbf{f}(k) \\ y(k) &= \mathbf{h}^T\mathbf{x}(k) \end{aligned} \tag{1}$$

$\mathbf{x}$ is the $n\times 1$ state vector, $\mathbf{A}$ is an $n\times n$ matrix, $\mathbf{b}$ and $\mathbf{h}$ are $n\times 1$ vectors, u is the system input and y is the system output. In this equation, the $n\times n$ matrix $\triangle\mathbf{A}$ represents parameter uncertainties and the $n\times 1$ vector $\mathbf{f}$ denotes external disturbances, satisfying matching conditions. The time-varying switching surface is defined as follows:

$$s(k) = \mathbf{c}^T\mathbf{x}(k)\ . \tag{2}$$

Disturbances and parameter uncertainties are bounded so that the following relation holds:

$$d_l \le d(k) = \mathbf{c}^T\triangle\mathbf{A}\mathbf{x}(k) + \mathbf{c}^T\mathbf{f}(k) \le d_u\ . \tag{3}$$

The lower and upper bounds d_l and d_u are known constants. The average value of $d(k)$ (d_0) and its maximum admissible deviation (δ_d) are introduced as follows:

$$d_0 = \frac{d_l + d_u}{2}, \quad \delta_d = \frac{d_u - d_l}{2}\ . \tag{4}$$

First, the required evolution of the time-varying switching surface $s(k)$ is specified:

$$s(k+1) = d(k) - d_0 + s_d(k+1) - \sum_{i=0}^{k}\left[s(i) - s_d(i)\right]\ . \tag{5}$$

The evolution of the time-varying hyperplane is

$$s_d(k) = \begin{cases} \dfrac{k^* - k}{k^*}s(0) & k = 0, 1, .., k^*, k^* < \dfrac{s(0)}{2\delta_d} \\ 0 & k > k^* \end{cases} \tag{6}$$

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[1]Bojan T. Milosavljević is a postgraduate student at the Faculty of Electronic Engineering, University of Niš, Beogradska 14, Serbia, Serbia and Montenegro, E-mail: mbojanks@ptt.yu

[2]Čedomir Milosavljević is with the Faculty of Electronic Engineering, University of Niš, Beogradska 14, Serbia, Serbia and Montenegro, E-mail: milosavljevic@elfak.ni.ac.yu

[3]Dimitar Stoyanov is with the Institute of Electronics at the Bulgarian Academy of Sciences, 72, Tzarigradsko chausee, 1784-Sofia, Bulgaria, e-mail: dvstoyan@ie.bas.bg

The constant k^* is a positive integer chosen by the designer in order to achieve good tradeoff between the fast convergence rate of the system and the magnitude of the control u required to achieve this convergence rate.

Then a control law $u(k)$ is proposed which drives the system in such a way that the variable $s(k)$ actually changes according to the specification:

$$u(k) = -(\mathbf{c}^T\mathbf{b})^{-1}\Big\{\mathbf{c}^T\mathbf{A}\mathbf{x}(k) + d_0 - s_d(k+1) + \sum_{i=0}^{k}[s(i) - s_d(i)]\Big\} \quad (7)$$

This control design procedure is referred to as the reaching law approach.[2]

As it is shown in [3], the following holds:

$$|s(k) - s_d(k)| = |d(k) - d(k-1)| \le \triangle_d \overset{\text{Eq. (6)}}{\Longrightarrow} |s(k)| \le \triangle_d, \ \ k > k^* \quad (8)$$

$\triangle_d$ denotes the disturbance-rate and the parameter- change-rate limit.

III. Algorithm 2

In [1] the sliding mode motion design is proposed generating a continuous control input, thus eliminating chattering. Neither the explicit calculation of the equivalent control nor high gain inside the boundary layer is used. The algorithm is performed by means of the Lyapunov theory and is applied to a nonlinear system shown in the regular form:

$$\frac{\mathrm{d}\mathbf{x}_1}{\mathrm{d}t} = \mathbf{f}_1(\mathbf{x}_1,\mathbf{x}_2), \frac{\mathrm{d}\mathbf{x}_2}{\mathrm{d}t} = \mathbf{f}_2(\mathbf{x}_1,\mathbf{x}_2) + \mathbf{B}_2(\mathbf{x})\mathbf{u} + \mathbf{B}_2(\mathbf{x})\mathbf{d}(t) ; \\ \mathbf{x}_1 \in \Re^{n-m}, \mathbf{x}_2 \in \Re^m, \mathbf{u} \in \Re^m, \mathbf{f} \in \mathbf{F}^m, \quad (9) \\ rang\,[\mathbf{B}_2(\mathbf{x})] = m .$$

The components of control input and of the vector $(\mathrm{d}\mathbf{x}_2/\mathrm{d}t)$ are assumed bounded:

$$u_i \in [u_{i_{min}} u_{i_{max}}] ; \\ (\mathrm{d}\mathbf{x}_2/\mathrm{d}t) \in [\alpha_{min}, \alpha_{max}], \ (i = 1, ..., m) . \quad (10)$$

The motion of the system is restricted to belong to the manifold S

$$S = \{\mathbf{x} : \varphi(t) - \sigma_a(\mathbf{x}) = \sigma(\mathbf{x}, t) = 0\} ; \\ \sigma_a^T = [\sigma_{a1}, \sigma_{a2}, ..., \sigma_{am}] \in \mathbf{F}^m , \quad (11) \\ \varphi_a^T = [\varphi_{a1}, \varphi_{a2}, ..., \varphi_{am}] \in \mathbf{F}^m .$$

$\sigma_{ai}(t)$ and $\varphi_{ai}(t)$, $(i = 1, ..., m)$ are continuous functions. $\varphi_{ai}(t)$ and their first time-derivatives are bounded. These functions can be interpreted as the references to be traced by selected combinations $\varphi_{ai}(\mathbf{x})$ of the system's states.

For the system described by Eqs. (9), (10), and (11), the following design procedure is adopted:

- select a Lyapunov function candidate $\nu(\sigma)$, such that, if the Lyapunov stability criteria are satisfied, the solution $\varphi(t) - \sigma_a(\mathbf{x}) = 0$ is stable on the trajectories of the system described by Eqs. (9), (10), and (11);
- select a form which the time-derivative of the Lyapunov function should satisfy, and find control u such that selected form is achieved on the trajectories of the system described by Eqs. (9), (10), and (11);
- find the equations of motion on the selected manifold with designed control.

The selection of Lyapunov function should be as simple as possible; hence, the first choice is a quadratic form

$$\nu = \frac{\sigma^T\sigma}{2} . \quad (12)$$

According to the Lyapunov theory, the solution $\sigma(\mathbf{x}, t) = 0$ will be stable if the time-derivative of the Lyapunov function can be expressed as

$$\frac{\mathrm{d}\nu}{\mathrm{d}t} = -\sigma^T\mathbf{D}\sigma, \ \mathbf{D} > 0 . \quad (13)$$

It is shown in [1] that control can be calculated as

$$u = sat\left[u_{eq} + (\mathbf{B}_2)^{-1}\mathbf{D}\sigma\right] \quad (14)$$

and, using equality $\mathbf{B}_2 u_{eq} = \mathbf{B}_2 u + \mathrm{d}\sigma/\mathrm{d}t$, control u is finally

$$u(t) = sat\left[u(t^-) + (\mathbf{B}_2)^{-1}\left(\mathbf{D}\sigma + \frac{\mathrm{d}\sigma}{\mathrm{d}t}\right)\right] , \quad (15) \\ t = t^- + \triangle, \triangle \to 0 .$$

Here $\triangle$ denotes the time-delay necessary for the calculations.

For the system given in the regular form, the following model holds during the sliding mode:

$$\frac{\mathrm{d}\mathbf{x}_1}{\mathrm{d}t} = \mathbf{f}_1(\mathbf{x}_1,\mathbf{x}_2) \ \wedge \ \frac{\mathrm{d}\sigma}{\mathrm{d}t} + \mathbf{D}\sigma = 0 \quad (16)$$

Discrete-time versions of Eqs. (15) and (16) can be written as

$$u(kT) = sat\Big[u(kT - T) + (\mathbf{B}_2)^{-1}\Big(\mathbf{D}\sigma(kT) + \frac{\mathrm{d}\sigma(kT)}{\mathrm{d}t}\Big)\Big] \quad (17)$$

$$\sigma(kT) = (\mathbf{I} - T\mathbf{D})\sigma(kT - T) \quad (18)$$

T is the sampling interval, $\mathbf{I}$ is the identity matrix. If matrix $\mathbf{D}$ is selected diagonal with $d_{ii} = 1/T$ then Eq. (18) equals zero and sliding mode will occur after finite number of sampling intervals. Further simplifications can be introduced by substituting $\mathrm{d}\sigma(kT)/\mathrm{d}t$ by its first order approximation

$$u(kT) = sat\Big[u(kT - T) + (\mathbf{B}_2 T)^{-1}\Big((\mathbf{I} + T\mathbf{D})\sigma(kT) - \sigma(kT - T)\Big)\Big] . \quad (19)$$

IV. Algorithm 3

In [4] a linear time-invariant system is considered:

$$\dot{\mathbf{x}} = \mathbf{A}_c\mathbf{x}(t) + \mathbf{b}_c u(t) \quad (20)$$

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$$u(t) = u(kT), \ kT \le t < (k+1)T, \\ k \in N^0 = \{0, 1, 2, ...\}, \ T > 0 . \quad (21)$$

An equivalent discrete-time representation is then for the perturbed system

$$\delta\mathbf{x}(kT) = A_\delta(T)\mathbf{x}(kT) + A_\delta(T)\mathbf{x}(kT) + \mathbf{b}_\delta(T)u(kT) + \mathbf{d}_\delta(T)\mathbf{f}(kT) \quad (22)$$

$$\dot{\mathbf{x}} \approx \delta\mathbf{x}(kT) = \big[\mathbf{x}((k+1)T) - x(kT)\big]/T \quad (23)$$

and $\triangle\mathbf{A}_\delta(T) \in \Re^{n\times n}$ is a matrix of uncertainties, $\mathbf{d}_\delta(T) \in \Re^{n\times 1}$, $\mathbf{f}(kT)$ is a bounded external disturbance with

$$|\mathbf{f}(kT)| \le \mu,\ \forall k \in N^0 \ . \quad (24)$$

The matching conditions are assumed, and therefore

$$\mathbf{d}_\delta(T) = \mathbf{b}_\delta(T)\ . \quad (25)$$

The goal is to impose $s = 0$ as the sliding mode hyperplane

$$s = \mathbf{c}_\delta(T)\mathbf{x},\ \mathbf{c}_\delta(T) \in \Re^{1\times n}\ . \quad (26)$$

The following assumption ensures that the relative degree of variable s, seen as an output, with the respect to the control signal u is one

$$\mathbf{c}_\delta(T)\mathbf{b}_\delta(T) = 1\ . \quad (27)$$

The reaching law is defined as follows

$$\delta s(k) = -\Phi(s(k), \mathbf{X}(k))\ , \quad \delta s = \frac{s(k+1)-s(k)}{T} = \mathbf{c}_\delta(T)\delta\mathbf{x}(k)\ , \quad (28)$$

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}(k) \\ \hat{\mathbf{x}}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}(k-1) \end{bmatrix}\ , \quad \text{by definition } \hat{\mathbf{x}}(0) = \mathbf{x}(0)\ , \quad (29)$$

The control u is then

$$u(k) = -\mathbf{c}_\delta(T)\mathbf{A}_\delta(T)\mathbf{x}(k) - \Phi(s(k), \mathbf{X}(k))\ . \quad (30)$$

According to the theorem from [4], it is sufficient that the following conditions for Φ are met:

$$\begin{gathered} \Phi(s,\mathbf{X}) = s/T,\ \mathbf{X} \in \mathbf{S}(T)\ , \\ \gamma T(d_m||\mathbf{X}||_1 + \mu)/|s| < T\Phi(s,\mathbf{X})/s < 1\ , \\ \mathbf{X} \notin \mathbf{S}(T),\ \gamma > 1,\ \varepsilon > \mu,\ \eta_2 > d_m\ . \end{gathered} \quad (31)$$

The following function satisfying these conditions is proposed in [4]

$$\Phi(s,\mathbf{X}) = \min\left(\frac{|s|}{T}, \sigma + q|s| + r||\mathbf{X}||_1\right)\operatorname{sgn}(s)\ , \quad 0 \le qT < 1,\ r \ge d_m\gamma,\ \sigma > \gamma\mu\ . \quad (32)$$

The vicinity of the hyperplane is defined by

$$\mathbf{S}(T) = \Big\{\mathbf{X} \in \Re^{2n} : s = |\mathbf{c}_\delta(T)\mathbf{x}| < \frac{\sigma T + rT||\mathbf{x}||_1 + rT||\hat{\mathbf{x}}||_1}{1-qT}\Big\}\ , \quad (33)$$

$$\sigma = 9,\ q = 0,\ r = 0.011,\ (\gamma = 1.1).$$

V. Comparison of Algorithms

In order to compare the proposed algorithms, each of them is applied to the model of a DC motor with neglected electric-time constant, also used in [4] for the verification purposes

$$\dot{x_1} = x_2,\ \dot{x_2} = -16x_2 - 680u\ , \quad (34)$$

where $x_1 = \theta_d - \theta$ (θ is the angular position of the rotor shaft), $x_2 = -\omega$ (ω is the rotor velocity), and u is the control signal. Corresponding to the matrix representation of the Eq. (20), it can be written

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 \\ 0 & -16 \end{bmatrix},\ \mathbf{b}_c = \begin{bmatrix} 0 \\ -b_c \end{bmatrix} = \begin{bmatrix} 0 \\ -680 \end{bmatrix}\ . \quad (35)$$

The Eq. (34) with external disturbance included, according to Eqs. (22), (25), satisfying Eq. (24), is as follows

$$\dot{x_1} = x_2,\ \dot{x_2} = -16x_2 - 680(u - f)\ , \quad (36)$$

$$f = \mu\big(|2-t| - 1\big)\ . \quad (37)$$

The adopted external disturbance waveform f of Eq. (37) with $\mu = 0.7$ (the same as in [4]) is shown in Fig. 1.

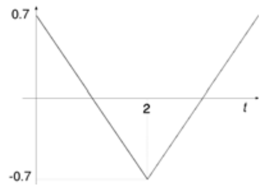

Fig. 1. The external disturbance waveform f

A relation between two discrete representations (given by the Eqs. (22) and (1)) should be established to evaluate the external disturbance in the Eq. (1) in the form $\mathbf{f}(kT) = [0\ \ f_1]^T$. It can be performed easily after rearranging the terms in the Eq. (22) and dividing by T(sampling period) both sides of the equation; hence, the following holds

$$\mathbf{A}_\delta = \frac{\mathbf{A}-\mathbf{I}}{T},\ \mathbf{b}_\delta = \frac{\mathbf{b}}{T},\ \triangle\mathbf{A}_\delta = \frac{\triangle\mathbf{A}}{T}\ , \quad (38)$$

$$b_\delta f = \frac{f_1}{T}\ . \quad (39)$$

The discrete-time representation parameters expressed in the form of Eq. (1) for the system described by Eq. (36), are [5]

$$\begin{gathered} \mathbf{A} = \exp(A_cT) = \begin{bmatrix} 1 & \frac{1}{16}(1-\exp(-16T)) \\ 0 & \exp(-16T) \end{bmatrix}, \\ \mathbf{b} = \int_0^T \exp(\mathbf{A}_c\tau)\mathbf{b}_c \mathrm{d}\tau = \\ = \begin{bmatrix} \frac{85}{32}(1-16T-\exp(-16T)) \\ \frac{85}{2}(1-\exp(-16T)) \end{bmatrix}. \end{gathered} \quad (40)$$

The reference value θ_d is 100 rad. The desired system response is related to the degree of exponential stability, being $\exp(-\alpha T)$, α=15 s^{-1}. According to the procedure described

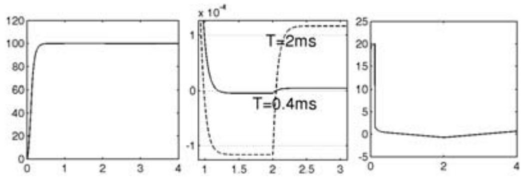

Fig. 2. Algorithm 1: time response, steady-state error and control signal respectively

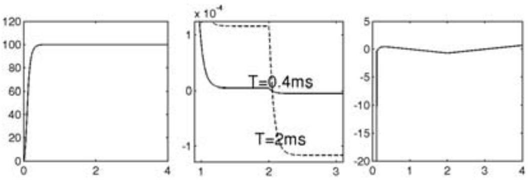

Fig. 3. Algorithm 2: time response, steady-state error and control signal respectively

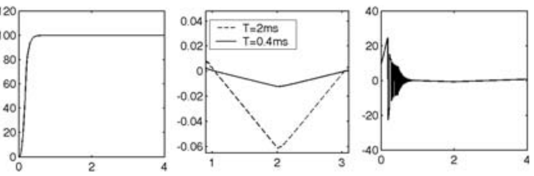

Fig. 4. Algorithm 3: time response, steady-state error and control signal respectively

and applied in [4], the sliding hyperplane parameters $\mathbf{c}_\delta$ are as follows

$$\begin{array}{ll} T = 0.4ms: & \mathbf{c}_\delta(T) = [-0.0220632 - 0.00147088], \\ & \mathbf{c}_\delta(T)\mathbf{A}_\delta(T) = [0, 0.00146618]\,; \\ T = 2ms: & \mathbf{c}_\delta(T) = [-0.0220808 - 0.00147204], \\ & \mathbf{c}_\delta(T)\mathbf{A}_\delta(T) = [0, 0.00144863]\,; \\ T = 10ms: & \mathbf{c}_\delta(T) = [-0.0221655 - 0.00147758], \\ & \mathbf{c}_\delta(T)\mathbf{A}_\delta(T) = [0, 0.00136288]\,. \end{array} \quad (41)$$

To apply the Algorithm 1, Eq. (40) is used, along with

$$\mathbf{b}_\delta = \frac{\mathbf{b}}{T} \Rightarrow (\mathbf{c}_\delta \mathbf{b}_\delta T)^{-1} = (\mathbf{1} \cdot T)^{-1} = \frac{1}{T} \quad (42)$$

and a certain approximation, arising from Eqs. (38) and (41)

$$\mathbf{c}_\delta \mathbf{A}_\delta T = \mathbf{c}_\delta(\mathbf{A} - \mathbf{I}) \Rightarrow \mathbf{c}_\delta \mathbf{A} = (\mathbf{c}_\delta \mathbf{A}_\delta)T + \mathbf{c}_\delta \approx \mathbf{c}_\delta\,. \quad (43)$$

Finally, the control signal is generated by

$$u(k) = -\frac{1}{T}[s(k) - s_d(k+1) + Z(k)]\,, \quad (44)$$

$$Z(k) = Z(k-1) + s(k) - s_d(k)\,. \quad (45)$$

The choice of k^* is governed by the desired sliding-surface reaching time, being $t_r = 120$ ms. Hence

$$k^* = \frac{t_r}{T}\,. \quad (46)$$

The simulation results for the Algorithm 1, applied to the perturbed system of Eq. (36), are given in the Fig. 2. The system response waveform is in the Fig. 2a, and steady-state error in Fig. 2b, but for three different sampling periods, the same as in Eq. (41). The control signal is presented in the Fig. 2c.

The steady-state error of the controlled variable x_1 can be evaluated by means of Eq. (8). $\triangle_d$ can be expressed as (Eqs. (3), (39) and (37))

$$\begin{aligned} \triangle_d &\overset{\text{Eq. (3)}}{=} c_2 \triangle f_1 \overset{\text{Eq. (39)}}{=} c_2 b_\delta T \triangle f \\ &= c_2 b_\delta T \left(f(t)|_{t=kT} - f(t)|_{t=(k-1)T} \right) \end{aligned} \quad (47)$$

$$\triangle_d \overset{\text{Eq. (37)}}{=} c_2 b_\delta T \cdot \mu T = c_2 b_\delta \mu T^2\,.$$

Since in steady state $x_2 \to 0$ and therefore $|s(k)| \approx c_1 x_1$

$$x_1 \approx \frac{c_2}{c_1} b_\delta \mu T^2\,. \quad (48)$$

The simulation values from Fig. 2b match the values from Eq. (48). Thus the simulation scheme and the Algorithm 1 itself are verified.

Control design according to the Algorithm 2 is not based on the discrete-time representation like the other two algorithms. The continuous-time system (36) is already in the regular form [6]. That is why c_2 is chosen to be $c_2 = 1$, and to keep the same system response features, c_1 takes the value of c_2/c_1 from the Eq. (41). It is also chosen that $d_{ii} = 1/T \Rightarrow T\mathbf{D} = \mathbf{I}$, and according to the Eq. (19)

$$u(kT) = sat[u(kT-T) + (b_\delta T)^{-1}(2s(kT) - s(kT-T))]. \quad (49)$$

The simulation results shown in Fig. 3 are as expected.

The Algorithm 3 is designed according to the Eqs. (30), (32) and (33) and simulation results shown in Fig. 4 verify the simulation scheme and the algorithm itself.

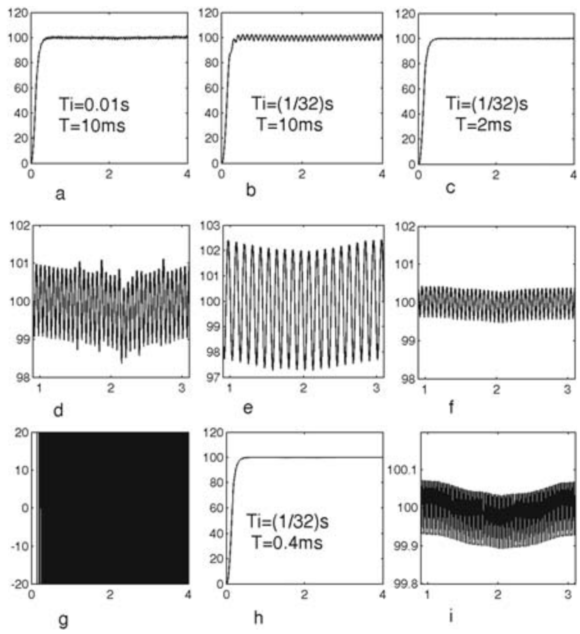

Fig. 5. The effects of unmodeled dynamics in the Algorithm 2:
– First two rows: time responses for respectively Ti=0.01s, T=10ms; Ti=(1/32)s, T=10ms and Ti=(1/32)s, T=2ms
– The third row: control signal u and time response for Ti=(1/32)s and T=0.4ms

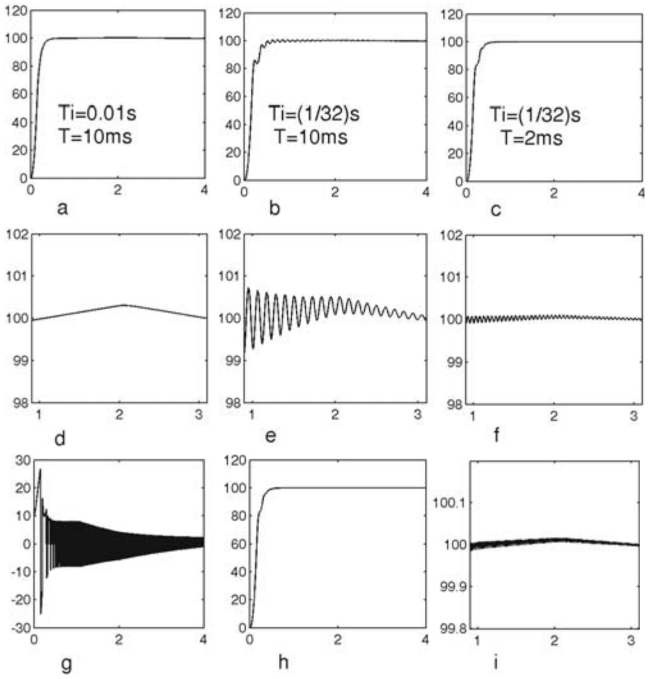

Fig. 6. . The effects of unmodeled dynamics to Algorithm 3:
– First two rows: time responses for respectively Ti=0.01s, T=10ms; Ti=(1/32)s, T=10ms and Ti=(1/32)s, T=2ms
– The third row: control signal u and time response for Ti=(1/32) s and T=0.4ms

Unmodelled dynamics is introduced in the system as an additional pole with time-constant T_i and the Coulomb friction. The simulation of the corresponding system with Algorithm 1 could not finish because going to infinity. If the Algorithm 2 is applied, the simulation results are those shown in Fig. 5.

The simulation results for Algorithm 3 are shown in Fig. 6.

Obviously, it can be seen from Figs. 5 and 6 that Algorithm 3 is better solution if chattering amplitude reduction is the most important task.

The simpler version of Algorithm 1 (without sums in the Eqs. (5) and (7)) provides the sliding surface steady-state error of $|s(k)| \leq \delta_d$. It should be emphasized that the system controlled by this version of the Algorithm 1 is stable with the chattering amplitude greater than in two other cases.

VI. Conclusion

Three possible control algorithms are compared regarding chattering reduction. Their robustness to parameter uncertainties and external disturbance presented in the original papers are verified. Their different behaviors regarding chattering reduction at presence of unmodelled dynamics are treated in this paper and the best performance algorithm is proposed.

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