Two Stages Piece-Wise Linearization Method for Inteligent Transducers

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Abstract – **This paper presents the intelligent transducers suitable linearization methods, considering practical implementation. New method is proposed. By one look-up table, it transforms x-axis first, and then by other look-up table performs classic piece-wise linearization. Based on the example of inverse NTC characteristic, new method is compared with polynomial approximation and standard piece-wise linearization.**

Keywords – **Piece-wise, sensors, approximation, linearization.**

I. Introduction

Non-linear transfer characteristic of the sensor is possible to compensate with many software methods in intelligent transducers: segment linearization (”piece-wise”) [1] and [2], polynomial approximation on the one or more segments, or approximation by the rational functions [3]. This enables the use of strong non-linear sensor with stable characteristic. Speed and simplicity of the implementation are very important during the obtaining of the response of the linearization methods, till the linearization tables and needed coefficients are determined by the help of PC, with use of much more resources.

In the intelligent transducers processor power and memory for placing program, linearization table and temporal variables are limited [4]. This gives advantage to the methods which are appropriate to implement in mathematics with fixed point, i.e. piece-wise linear approximation versus polynomial.

In the estimation of the quality of the linearization method the following parameters should be considered:

- achieved accuracy of the linearization, like least mean squares deviation, or maximal deviation of the whole linearization range.
- time needed for response calculation in the device which does the linearization.
- used memory space as in EPROM for the linearization table or coefficients, so as in RAM for the saving of the temporal calculation results.
- the use of the program memory for the implementation of the linearization procedures.

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II. Standard Piece-Wise Linearization

All segmental linearizations and all graphic generation with appropriate error, presented in this paper, have done with specific purpose developed program for PC. Standard Piece-wise linearization is carried out in such a way that the range of input variable is divided into desired number of segments and the optimal line found for each segment by the method of least mean squares deviation. In order to obtain the continual transmition function some of the linear segments are connected in such a way that for the ordinate value on the border of segments the mean value of the ordinates adjacent lines are taken. It can be noticed that for the line determination the criteria of least mean squares deviation on segment is chosen, and the methods of the linearization are compared on the bases of maximal deviation. As for the all considered examples that are gained the maximal error is a little bit bigger than that which is optimally possible. On the other side, when the points of calibration are gained by real measuring, which itself considers the existence of less or bigger uncertain of the measuring results, the method of the minimal square is very appropriate in practice.

Piece-wise linearization gives very good results for almost all transferable characteristics and it is also used when the output values depend on two variables, for example for the temperature compensation as influential value [2].

As the example of the transferable function with distinctively non-linearity, this paper considers inverse characteristic of the NTC (negative temperature coefficient) resistant sensor. One way of temperature measuring by NTC is termistor voltage measuring with constant current (Fig. 1). Result of the A/D conversion is proportioned to resistance which is

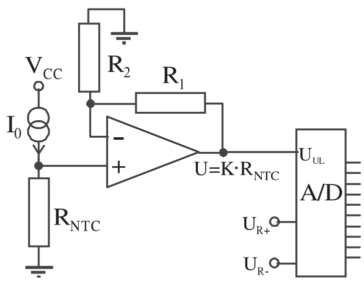

Fig. 1. NTC temperature sensor measuring circuit

the function of temperature. In order to calculate the temperature for the bases of the conversion results it is needed to carry out the linearization function, which is inverted termistor NTC characteristic, i.e. function $T = f_T(R)$. In the paper temperature range from 0° C up to 100° C is considered. In order to use universal procedures in mathematics with fixed point, the input and output range of all functions are normalized to lie from 0 up to 65535. The R_N variable is proportioned to the A/D conversion result, Eq. 1:

$$R_N = k_2 \cdot U_{A/D} = k_2 \cdot k_1 \cdot R_{NTC} \,. \tag{1}$$

The selection of the amplification, the range of the A/D converter, and conversion results averaging, in order to reject the noise, it can be adjusted that the R_N belongs to that range without additional scaling. After the linearization, final result of the temperature measurement is obtained by scaling according to formula $T_m[°C] = aT + b$, which enables two points recalibration.

In the part of small resistances inverse characteristic of the NTC has very large slope and alteration of the slope. Classic linearization by 16 segments gives maximum approximation error about 10% (Fig. 2). It can be seen that for the linearization of the distinctively non-linear characteristics, the small error of linearization (for example ¡ 1%) demands classic segment linearization with lot of segments. Strong characteristic non-linearity is often present in the small part of the input range, and it can be reduced if the input range is divided on unequal segments. The input data for the application of the Piece-wise linearization is the current number of segments, i.e. the ordinal number of the point in the table and the rest of the input value in the segment itself. For equal segment size these data are easy and quickly found by integer divide of the input variable with the segmental size. But, the difference of the segment size causes slow linearization method response, i.e. the defining of the current segment must be done in the loop.

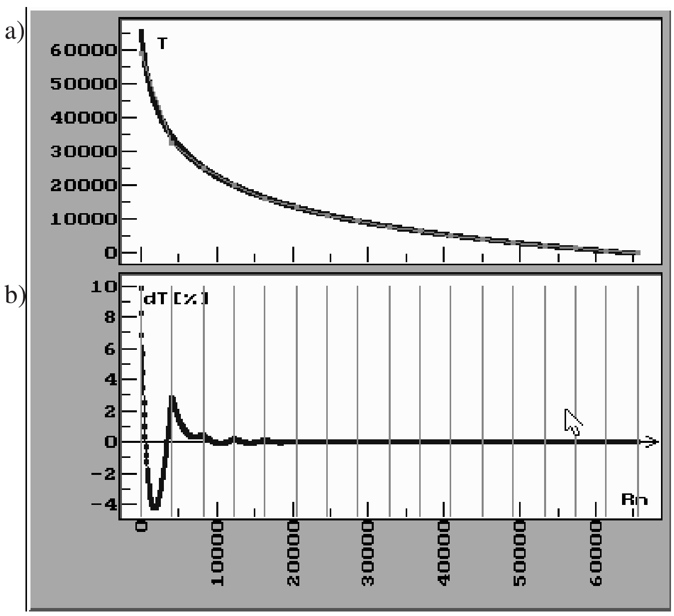

Fig. 2. Standard segment linearization

III. The Two Stage Piece-Wise Linearization Method

The method presented in this paper keeps simplicity and universality of the equal linear segment methods, and decreases total needed number of segments by linearization in two steps.

On the assumption that polynomial approximation is also implemented in the mathematics with fixed point, the price of this approach is the doubling of the calculation time. It can be compared with the second degree polynomial approximation on several segments. Concerning the memory space occupation, if we need 16 segments for the universal segment approximation, the accuracy will be compared with two-stage method of 8+8 segments.

The idea of the method is that the transformation of the x-axis is done before standard linear segment method application, so that the parts of the input range with strong nonlinearity are stretched, on account of the rest. The linear segment table also does this transformation. New characteristic in the function of transformed input variable is obtained, thus linear segment approximation can be performed with minor error than starting characteristic.

On the Figs. 3, 4 and 5 the example of the method by two times 8 segments is given. On the Fig. 3a function $f_2(z)$ is shown, i.e. temperature dependence of resistant with transformed abscissa, so that the starting part of the curve is stretched. After that, this function is approximated by 8 linear segments. The maximal error of this approximation of the whole range is 2,5% (Fig. 3b).

Input value is R. After the first transformation by linear segment table we get $z = f_1(R)$ (Fig. 4), and after the second transformation of z value, temperature is obtained as Eq. 2:

$$T = f_2(z) = f_2(f_1(R_N)) = f_T(R_N) \tag{2}$$

Let's define f_1, first. The input range of f_T is divided to desired number of segments (n=8), and for each segment

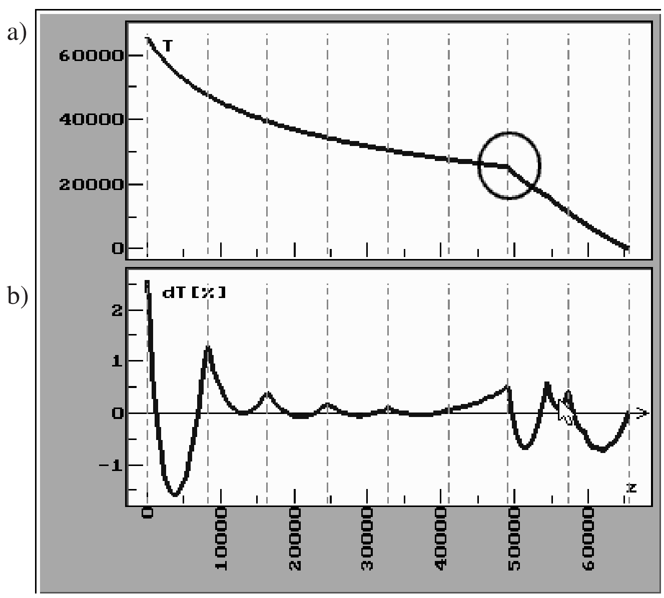

Fig. 3. Error of transformed curve piece-wise approximation

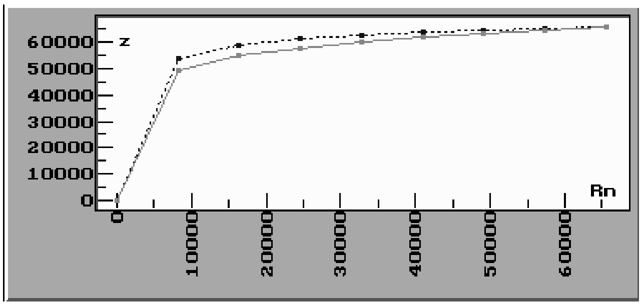

Fig. 4. The function which transforms R – axis

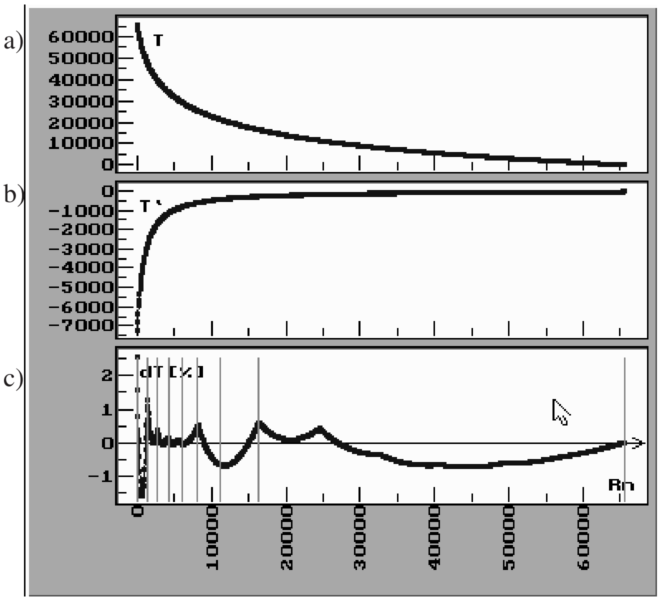

Fig. 5. Obtained error of presented method

apart defines maximal and minimal value of the derivation, and their difference, i.e. $\triangle T_i' = |T_{i\max}' - T_{i\min}'|$. Numerical obtained derivation of the $f_T(R_N)$ can be seen on Fig. 5b. Hence, the value F is determined by $F = \sum_{i=1}^{N} \triangle T_i'$. Knots of the first function $z = f_1(R)$ has ordinate according to Eq. 3:

$$z_0 = 0,\ z_i = z_{i-1} + \left[\frac{\triangle T_i'}{F} \cdot 65536\right] \tag{3}$$

The values on the R-axis are equidistant arranged as on Eq. 4 and Fig. 4 - a doted line.

$$R_i = i \cdot \left[\frac{65536}{n}\right] \tag{4}$$

Function f_1 is monotonous increased, which enables that one-valued of $f_2(z)$ is kept. It is achieved that the parts of the characteristic with the large second degree derivate stretch on the count of the rest of the part, which has been the aim of this transformation. The first Piece-wise linearization is stretched 6 times. The transformation of the input variable from the range of 0 up to 65535 to the same range is also obtained, which enables the use of the universal linearization procedure in intelligent transducer.

The Fig. 5a shows the function which is linearized (it's equal with the function on the Fig. 2a, the Fig. 5b represents the derivation of the function, and the Fig. 5c shows the linearization error. The error is obtained by the linearization function shown on the Fig. 3, but the same error is shown with linear change of the input value R_N. On the Fig. 3b the equidistant vertical lines represent the segment boundaries which respond to those on the Fig. 5c. The interspace between those segmental boundaries has been changed caused by transformation of the R_N - axis.

On the Fig. 3a it can be seen that after the first transformation smooth function $T = f_T(R)$ gets ridge points (circled area) so on them, caused by sudden change of the slope, valuable error of linearization can be expected. As to avoid this, the following procedure is applied: the ordinate values of the first table, which have large transformation in comparison with previous one, are rounded to values which correspond to abscissas of the second linearization table, to amount of $i \cdot \left[\frac{65536}{n}\right]$. In this way ridge points of the function $f_2(z)$ will coincide with the knots of the second linearization function, so they will not take in addicted error. Function $z = f_1(R)$ transformed by exposed method is shown in Fig. 4 – full line.

IV. Comparison to Classical Methods

Independent from the linearization methods, it is needed to consider the way of obtaining the input data, i.e. the way of the transfer function defining:

- by measuring of the pair of points in the calibration process, which is a long-lasting procedure for the great number of points and high accuracy.
- on the bases of the previous knowledge of the physics of sensor, known functional dependence is used, thus for the each concrete sensor several constants (at least the offset and the slope) has to be defined by several measurements.

In order to increase the accuracy in the real conditions, disregarding whether the transfer function is defined by the set of points or known mathematical formula, it is needed to carry out more measurements than the minimal needed for the coefficients defining. If the polynomial approximation or the approximation according to beforehand defined curve is continued, by the use of the mathematical programs of the general purpose (MCAD, ORIGIN...) the needed coefficients can be defined by the iterative methods, concerning as a criteria the minimization of the square error on the whole range of the input variable.

For the given instance of the NTC temperature sensor, direct dependency of the resistance from the temperature is known as Eq. 5:

$$R_{NTC} = R_{25} \cdot e^{\beta\left(\frac{1}{273.16+T} - \frac{1}{298.16}\right)} \tag{5}$$

On concrete sample of the sensor in the temperature chamber, by four wired measurement of the resistance by the use of multimeter HP3478A, the 20 pairs of calibration points have been obtained, on which base the determined coefficients R_{25} and β have been defined by the use of the ORIGIN program. Afterwards, the inversion function is mathematically defined, and the input and the output ranges are linearly scaled to the range from 0 up to 65535. Based on such defined dependence, by the use of the ORIGIN program, the 1000 points of the function $T = f_3(R_N)$ has been generated. Those points are the input data for all linear and polynomial approximations which have been carried out, and whose accuracy is presented in this paper.

Standard polynomial approximation on the whole range of the input variable, even of the 9$^{\text{th}}$ degree, gives the error larger than 2%, for the given example of the inverse NTC characteristic. In the Table 1 maximal error of the piece-wise polynomial approximation of the second and the third degree is given. As distinctively non-linearity is on the small part of the range, even second and the third degree polynomial, on the small number of the segments, does not give significantly better results.

Table 1. The maximal errors of the polynomial approximation

Number of segments	4	8	16	32
Second degree	16%	8%	3,5%	1,1%
Third degree	9%	4%	1,3%	0,3%

In the Table 2 results achieved by the standard segmental linearization are given. The results achieved by the suggested method (by the two tables with equal number of the segments) are also given in Table 2. By analyzing the required memory for the tables, or coefficients of polynomial approximation, classical segment linearization with 16 segments, should be compared with two-stage method with 8+8 segments, or approximation by the second-degree polynomial on 8 segments.

Table 2. The maximal errors of the standard segment approximation and suggested method

Number of segments	8	16	32	64
Classic segment	17,5%	10%	4,5%	1,75%
Number of segments	4+4	8+8	16+16	32+32
Two-stage method	12,5%	2,5%	0,7%	0,12%

For the given example of the distinctively non-linear function, the suggested method gives significantly better results than standard segment linearization, especially if the higher accuracy of linearization is required. It also gives better results than polynomial approximation, even the third degree, with easier implementation. The response of the linearization is obtain by doubled application of the same procedures, thus negligible larger program memory is required.

V. Conclusion

It is on disposal the whole range of software methods for the linearization of the transferable characteristics in intelligent transducers. However, for the linearization of the extremely non-linear characteristics, the accuracy in order of 1% is hardly achieved by the polynomial approximations and by the standard piece-wise linearization also. We should have in mind that the linearization has to be carried out by integer mathematics, or in fixed point, and that in the intelligent transducers the program memory, as well as the memory for the linearization tables saving, is very often limited.

Suggested two-stage method is easy to realize even with the limited processor recourses, and it gives less maximal linearization error in relation to the polynomial approximation and standard Piece-wise linearization. Program memory occupancy needed for the implementation of the linearization procedures is only insignificantly bigger than standard Piece-wise linearization.

References

[1] G. Horn, J. Huijising, *Integrated smart sensors, design and calibration, doctoral diss.*, Delft, Kluwer Academic Publisher, Netherlands, 1998.

[2] A. Flammini., D. Marioli, & A. Taroni, Application of an optimal look-up table to sensor data processing, *IEEE Trans. Instrum. Meas.*, 48 (4), 1999, 813-816.

[3] W. T. Bolk, A general digital linearising method for transducers, *J.Phys.E: Sci. Instrum.*, 18, 1985, 61-64.

[4] L. Barford, & Q. Li, Choosing designs of calibration transducer electronic data sheets, *HP Labs Technical Reports,* 1998, HPL-98-166, http://www.hpl.hp.com/techreports/98/HPL-98-166.pdf

[5] D. Živanović, M. Arsić ”Universal Intelligent Sensor Module”, *ETRAN, Conference Proceedings*, Vol. I, pp.121-123., Zlatibor, Serbia, 1995.

[6] D. Živanović, G. Djordjević, M. Arsić, ”A Method Of Measurement Results Processing Definition For Intelligent Sensor Module”, *IT’96, Conference Proceedings*, pp.468-471, Žabljak, Serbia,1996