# Analysis of an Asymmetric Inverter in Phase Space 

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#### Abstract

A model of an asymmetric inverter in the Phase space (PSM) have been build-up. By the means of the respective theoretical basis the main connections of the circuit parameters have been represented, with the mode parameters in PSM. The main dynamic conditions have been specified as in inverters definition. The common differential structure of the inverters is offered to your attention.


Keywords - inverter, model, phase space

## I. INTRODUCTION

The asymmetric inverter (AI) is a spreaded frequency converter and it is found in many variants. The electromagnetic processes in it are defined by differential equations similar to those of the serial inverter [1]. They are also a model for some more complex circuits at definite conditions [2]. The base circuit of an asymmetric inverter is shown in Fig. 1.


Fig 1 Base circuit of AI
It's load chain is a serial resonant CLR circuit. The equivalent parameters of the load are included in it.The load is usually an induction heating system, it could be also a ultrasonic converter or other consumer. The load circuit is connected to a DC source - E by driven and single-pole switch and a diode $-\mathrm{S}_{\mathrm{a}} \mathrm{D}_{\mathrm{a}}$, called "active". The second couple of switch and diode $-S_{p} D_{p}$, close the load chain directly and they are called "passive". The switches are electronic devices, for instance transistors. They are driven by a device, called "timer" for short. It implements an inverse control of the switches - when the first is active (closed), the other is not active (opened). The change of these states is made by tacts, by equal intervals called one-half periods $-\mathrm{T}_{\mathrm{T}} / 2$.

The goal of this report is to define a phase-space model of the inverter and also to structure the major connections between the elements of the model, respectively of the inverter. Making active the switch $\mathrm{S}_{\mathrm{a}}$ leads to current in the load chain and voltage across the capacitor. A differential
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[^0]dependence exists between the capacitor voltage and the current which determines the space of their interaction as in the PS is called Phase Model (PM). The model includes Phase Vector (PV) and it's specific kinematics. The vertex of Phase space (PS). It is a plane marked by a Cartesian coordinate system. The interpretation of the inverter processes the PV traces the Phase Trajectory (PT). It is a closed curve consisting of two spiral arcs - segments - active and passive. The general view of the Phase Model is presented in Fig. 2. It is related to inverters with Q -factor $\mathrm{q}=2$. The one-half


Fig 2 Phase model of AI period development angle is set by the timer: $\theta_{\mathrm{s}}=3,4$ radians which is about $195^{\circ}$. The upper arc is the active segment of the PT. It's start is the commutation point with coordinates $\mathrm{u}_{0}$ $=0,79$ and $\mathrm{i}_{0}=0,32$. The commutation point will be an object of special attention.

## II. THEORETICAL BASIS

The generalization and universalization of the particular circuit parameters is made using well-known procedure. The x -axis shows the voltage across the capacitor divided by the constant E . The source voltage - E [V] with it's dimension (volts) is treated as a scale for the x -axis. Thus the x -axis markers are relative and universal values released from the dimension of the particular voltage. The $y$-axis presents the current across the inductunce again in universal relative units. As a dimensional scale a current $-\mathrm{E} / \rho[\mathrm{A}]$ is applied, where $\rho$ $=\sqrt{\mathrm{L} / \mathrm{C}}$ is the wave resistance of the load circuit.

The following equations (Eqs. 1) present the voltage and current in the PS:

$$
\begin{align*}
& \mathrm{u}(\theta)=1-\mathrm{u}_{\mathrm{s}} \exp (-\varepsilon \theta) \cos \theta  \tag{1}\\
& \mathrm{i}(\theta)=\mathrm{i}_{\mathrm{s}} \exp (-\varepsilon \theta) \sin \theta
\end{align*}
$$

The angle $\theta=\omega \mathrm{t}$ is the phase argument limited by the frames of the one-half period $0 \leq \theta \leq \theta_{\mathrm{s}}$, where it's boundary value "development angle" is $\theta_{s}=\left(\mathrm{T}_{\tau} / \mathrm{T}_{0}\right) \pi$.

The frequency is $\bar{\omega}=\sqrt{(\mathrm{CL})^{-1}-\delta^{2}}$;the fade $\delta=\mathrm{R} / 2 \mathrm{~L}$; the relative fade $\varepsilon=\delta / \omega$ or approximately: $\varepsilon=1 / 2 \mathrm{q}$, where the Qfactor: $\mathrm{q}=\sqrt{\mathrm{L} / \mathrm{C}} / \mathrm{R} . \mathrm{T}_{\tau}$ is the period set by the timer; $\mathrm{T}_{0}=$ $2 \pi / \omega$ is the period of the free oscillations. It is important to notice that the inequality: $\mathrm{T} \tau>\mathrm{T}_{0}$ is always true. The inverter always works at development angle $\theta_{\mathrm{s}}>\pi$.

## III. Phase Model Elements

The voltage is marked on the real axis and the current on the imaginary axis when treating the phase plane as complex. This gives the PV suitable complex form (Eq. 2):

$$
\begin{equation*}
\mathrm{F}(\theta)=-1+(1+\mathrm{Ka}) * \exp ((-\varepsilon+\mathrm{j}) * \theta) \tag{2}
\end{equation*}
$$

The complex function (Eq. 2) is a spiral segment. The spiral start point $\mathrm{k}_{\mathrm{a}}$ by it's coordinates $\mathrm{k}_{\mathrm{a}}\left(\mathrm{u}_{0}, \mathrm{i}_{0}\right)$ determines the commutation mode of the inverter. The start and the commutation processes are typical for the zero value of the argument $\theta=0$. At the consequent rising of the argument the vertex of the $\mathrm{PV}-\mathrm{F}(\theta)$ traces the active segment of the Phase trajectory. The center of this spiral arc is on the x-axis at the point -1 . The end of the segment is at the borderline value of the argument $\theta=\theta_{\mathrm{s}}$. This is the other commutation point passive $\mathrm{k}_{\mathrm{p}}$ where the connection between the active and passive arc of the PT is realized
The typical for the inverters change of the PV centering is made at the commutation point. This transferring of the PV at the beginning of the coordinate system where the center of the passive arc is, is made discrete at the moment when it's vertex is at the commutation point.
The energy interpretation of this phenomenon gives an opportunity for a more different approach toward the inverters. The transferring of the vector beginning at the new center determines the new analytic definition of the PV valid for the passive one-half period - Eq. 3:

$$
\begin{equation*}
\mathrm{F}\left(\theta_{\mathrm{s}}+\theta\right)=К р * \exp ((-\varepsilon+\mathrm{j}) * \theta) \tag{3}
\end{equation*}
$$

The spiral of the passive segment starts from the point $k_{p}$, develops according Eq. 3 and ends at the active commutation point $\mathrm{k}_{\mathrm{a}}\left(\mathrm{u}_{0}, \mathrm{i}_{0}\right)$. The center transferring of the PV is made at the both commutation points.

Another type of commutation is implemented at the cross points of every spiral arc with the x -axis. The current is transferred from the appropriate switch to it's diode at these points. This is an apparatus-device commutation and it is not of special energy importance. The functions of the inverter stay fluent and unchanged.
The components of the PM can be treated also as a conventional functions of the time. The inverter current imaginary component of the PV is developed in Fig. 3. It is related to inverters with Q -factor $\mathrm{q}=2$ and one-half period angle $\theta_{\mathrm{s}}=4,4$ radians or $252^{\circ}$.

The commutation point where the active sub-circuit diode current is "transferred" to the passive sub-circuit transistor is marked as broken arc. At the end of the function of the current the another commutation is made with which the active transistor starts the next period "accepting" the current. At the "zeros" of the current function where it crosses the x -axis the diode commutations are made. The current is "transferred" from the transistor to it's corresponding diode there.


Fig 3 Current of AI

## IV. Phase Model Recurrent Dependencies

The inverter processes development matching to the passive arc is nearly the same as for the active segment and it is due to the functional similarity between Eqs. (2) and (3). The imaginary part of the phase vector - current by modulus repeats the former one-half period. The real part of the PV voltage differs only with a constant of "one" which is the distance between the two spiral centers. There is a distinct symmetry between the passive and active sectors of the PM and this will be proved by the analysis of the stationary mode of the inverter.
Let us examine the inside of the PM. There is a rhomboid between the active and the passive sectors marked by the spiral centers C1 and C0 and commutation points $\mathrm{k}_{\mathrm{a}}$ and $\mathrm{k}_{\mathrm{p}}$. The outside angles of the rhomboid at the centers are the spiral one-half period angles $\theta_{\mathrm{s}}=\pi+\alpha$. The diagonal between the centers divides it to two identical triangles.

Each of the triangles is characterized with:

1. It's base is the single segment of the x -axis.
2. Opposite the single base of the triangles at the commutation points resides the complain component of the spiral angle $\alpha\left(\theta_{\mathrm{s}}=\pi+\alpha\right)$.
3. The other faces of the triangles are respectively the start and the final values of the PV. The reduction connection between the start and final modules of the PV is valid also for the lengths of the other two faces:

$$
\begin{equation*}
\text { СоКа }=\text { С1Ка } * \exp \left(-\varepsilon \theta_{\mathrm{s}}\right) \tag{4}
\end{equation*}
$$

That is enough to build-up the triangle which apex completely defines the whole stationary mode of the inverter. That is the direct solution of the stationary mode without using the connections with the former transitional process. The problem will be especially treated comprehensively.

## V. Phase Model Dynamics

The vertex of the PV traces the PT by rotation with permanent angle velocity. The permanent reduction of it's modulus expresses the realized or so called "active" energy. This is only the one side of the dialectical energy parity at he inverters. The discrete periodical transferring of the beginning of the PV from the first spiral center to the other is the typical for the inverters transferring to another subsystem with increased energy potential. The PM represents accurately the rise of the potential energy with the increased modulus
immediately after the commutation. Thus the energy realization which the PM expresses as a permanent modulus reaction of the PV corresponds to the discrete postcommutation modulus rise. This gives an ability for an alternative approach by the means of which the inverter processes and parameters can be predicted and designed in a untraditional way.

## VI. Phase Model Accents

The mentioned spiral centers are the steady solutions images of the differential equations which are valid during the various one-half periods of the inverter. Their positioning in the Phase Space presents a specific classification of the inverter types. They reside on the x -axis of the PS for all inverters which load circuit includes a serial capacitor as it is at the examined object.
The described single-round distance between the spiral centers stay also at the so called "half-bridge" inverter and in the various pseudo symmetrical circuits too. At them the PM and the inverter processes have the form shown in Fig 2, with the difference that the $y$-axis is in a symmetrical position expressing a symmetry at the voltage too. The spiral centers are positioned respectively at the points $+0,5$ and $-0,5$ on the real axis.

The PT at the bridge inverters is not much different too. It's centers are only at a double distance on the x -axis points $\pm 1$. This makes possible and stable the operation of the bridge inverters also at an increased fading in the load circuit.

Could be said that he presented Phase Model is valid not only at the asymmetric inverter but it includes the processes in a wilder class of inverters.

It excites interest a wider classification of the inverters made not only by circuit parameters but especially by the steady solutions of their differential systems which at the present PM figure as centers of the spiral arcs. Such position is typical for all inverters which have serial capacitor in the load circuit. However many inverters exit where these centers reside at other - alternative position. Such type of inverter is for instance the known Parallel inverter. Another alternative is the "Inverter with inductive commutation" [3].

## VII. CONCLUSION

The Phase Model presents the dynamic connection between the circuit and mode parameters of the inverters. The voltage, the current, the time and the energy of the inverter are scaled in one drawing. This gives opportunities for:
$\checkmark \square$ General classification of the inverters by the steady solutions of their differential equations;
$\checkmark \square$ Structuring the dynamic model and thus defining the noun "inverter" determining it from the other powerful generators;
$\checkmark \square$ Implementing the element connections at the PM for application aims which was only demonstrated by the direct "incoming" in a stationary mode.

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