

# Mathematical Model of a Current-Frequency Galvanomagnetic Converter

Anatoliy T. Aleksandrov

**Abstract** – A current-frequency converter has been synthesized on the basis of a digital magnetosensitive integrated circuit (IC). The paper studies the complex influence of the number of winding  $W$ , current  $I$ , voltage  $U_{DD}$  and resistance  $R$  over frequency  $f$  of the converter developed and it presents the respective theoretical-experimental models obtained, which reflect their influence. The dependencies obtained form the basis of the practical application of the current-frequency converter and set prerequisites for choosing an optimum mode of operation.

**Keywords** – magnetic field galvanomagnetic sensor, magneto-sensitive IC, current-frequency conversion system.

## I. INTRODUCTION:

On the basis of a digital magnetosensitive IC a galvanomagnetic current-frequency converter has been developed [1]. It exhibits a stable conversion characteristic, high noise immunity and low power consumption.

With a view to the practical use of this device and in order to obtain a complex and reliable assessment of its operation, it is necessary to investigate the complex influence of the number of windings  $W$ , current  $I$ , voltage  $U_{DD}$  and resistance  $R$  over frequency  $f$  and to obtain the respective theoretical-experimental models reflecting this influence.

## II THEORETICAL-EXPERIMENTAL MODEL:

In order to obtain the dependence  $f=f(W, I, U_{DD}, R)$  complete factor experiments of the type  $N=2^4$  have been conducted, each experiment having been repeated ( $n=4$ ), with simultaneous variation of the variable factors  $W$ ,  $I$ ,  $U_{DD}$ ,  $R$ , according to a predetermined plan of the experiment. The planning of the experiments and the processing of the results obtained have been carried out following the methodology presented in [2]. The type and levels of varying the factors are shown in Table 1. The areas of change in the parameters  $W$  and  $I$ , indirectly reflecting the effect of the magnetic field  $B$  on the converter operation, as well as in voltage  $U_{DD}$  and resistance  $R$ , have been determined after conducting preliminary single-factor experiments and they guarantee the normal operation of the current-frequency converter.

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The experimental investigations have been carried out at a constant ambient temperature  $T = 25^\circ\text{C}$ . The magnetic circuit has been solved experimentally and it exhibits constant values of magnetic permeability  $\mu=400$ , air gap  $8\div 10$  mm and magnetic circuit area  $S = 1600$  mm<sup>2</sup>.

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The relation between the natural and code values of the factors is set by the equation [2]:

$$x_i = \frac{2 \cdot (\ln z_i - \ln z_{iup})}{\ln z_{iup} - \ln z_{ilw}} + 1, \quad (1)$$

where:  $x_i$  - code value of the factor;

$z_i$  - natural value of the factor.

The plan of the experiment, as well as the results of measuring  $f$  of the current-frequency converter and the values of the target function  $y_1 = \ln f$  have been pointed in Table 2.

The presented values of the measured frequency logarithm are the average of the four observations carried out at each point:

$$\ln f = \frac{\ln f_1 + \ln f_2 + \ln f_3 + \ln f_4}{4}. \quad \text{The experimental setting}$$

presented in [1] has been used to measure  $f$ .

The dependence between frequency  $f$  of the converter and the parameters  $W$ ,  $I$ ,  $U_{DD}$ ,  $R$  is power function of the type:

$$f = c \cdot W^p \cdot I^q \cdot U_{DD}^s \cdot R^t. \quad (2)$$

In order to simplify the mathematical processing this power equation is linearized (converted into a first power polynomial) by taking logarithms:

$$\ln f = \ln c + p \cdot \ln W + q \cdot \ln I + s \cdot \ln U_{DD} + t \cdot \ln R \quad (3)$$

There are correlational links between the parameters  $W$ ,  $I$ ,  $U_{DD}$ ,  $R$ , as a result of which the equation reflecting the dependence  $f=f(W, I, U_{DD}, R)$  is presented in a general form by means of the polynomial:

$$y_1 = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{14} x_1 x_4 + b_{23} x_2 x_3 + b_{24} x_2 x_4 + b_{34} x_3 x_4 + b_{123} x_1 x_2 x_3 + b_{124} x_1 x_2 x_4 + b_{234} x_2 x_3 x_4 + b_{134} x_1 x_3 x_4 + b_{1234} x_1 x_2 x_3 x_4, \quad (4)$$

To check the reliability of the experimental results obtained, the dispersion of reproductivity (the experimental error) is determined  $S_y = 5,7 \cdot 10^{-3}$ . It has been calculated after determining the dispersions of all experiments consisting of  $n=4$  repeated observations and exhibiting the deviation of the repeated experiments significance from the arithmetic mean value of the output parameter, the sum of all dispersions

Table 1. Values of the Factors

Level	Natural values				Coded values			
	W	I	U <sub>DD</sub>	R	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>
	wind.	mA	V	kΩ				
Upper	450	2200	12	5,5	+1	+1	+1	+1
Zero	350	1450	10	3	0	0	0	0
Lower	250	700	8	0,5	-1	-1	-1	-1

Table 2. Plan of the Experiment

Coded values of the factors					Values of the Output Parameters					$\hat{y}_1$
x <sub>0</sub>	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	f <sub>1</sub>	f <sub>2</sub>	f <sub>3</sub>	f <sub>4</sub>	y <sub>1</sub> =ln f	
+1	-1	-1	-1	-1	445	446	445	446	6,0986	6,0999
+1	+1	-1	-1	-1	487	488	488	488	6,1898	6,1885
+1	-1	+1	-1	-1	3205	3203	3190	3196	8,0704	8,0717
+1	+1	+1	-1	-1	2389	2401	2408	2425	7,7856	7,7843
+1	-1	-1	+1	-1	285	284	283	281	5,6463	5,6476
+1	+1	-1	+1	-1	384	382	380	380	5,9441	5,9428
+1	-1	+1	+1	-1	2477	2461	2478	2480	7,8136	7,8149
+1	+1	+1	+1	-1	1938	1952	1957	1982	7,5793	7,5780
+1	-1	-1	-1	+1	633	633	636	637	6,4532	6,4545
+1	+1	-1	-1	+1	719	720	720	722	6,5796	6,5783
+1	-1	+1	-1	+1	3762	3779	3818	3822	8,2415	8,2428
+1	+1	+1	-1	+1	2987	2996	3018	3022	8,0083	8,0070
+1	-1	-1	+1	+1	266	265	262	261	5,5740	5,5753
+1	+1	-1	+1	+1	428	426	426	427	6,0562	6,0549
+1	-1	+1	+1	+1	2832	2835	2838	2844	7,9541	7,9554
+1	+1	+1	+1	+1	2251	2265	2289	2292	7,7294	7,7281

$\sum_{u=1}^{16} S_u^2 = 5.10^{-4}$ , the maximum dispersion  $S_{u\max}^2 = 10^{-4}$  and a check of dispersions uniformity according to Kohren's criterion ( $G_R=0,1686 < G_{R\text{table}}=0,2674$ ) at a significance level  $\alpha=0,05$ .

The coefficients in the mathematical model (4) have been determined in a coded form:

$$b_i = \frac{\sum_{u=1}^N y_{1u}}{\sum_{u=1}^N x_{iu}^2}; \quad b_{ij} = \frac{\sum_{u=1}^N y_{1u} \cdot x_{iu} \cdot x_{ju}}{N};$$

$$b_{ijg} = \frac{\sum_{u=1}^N y_{1u} \cdot x_{iu} \cdot x_{ju} \cdot x_{gu}}{N};$$

$$b_{ijgk} = \frac{\sum_{u=1}^N y_{1u} \cdot x_{iu} \cdot x_{ju} \cdot x_{gu} \cdot x_{ku}}{N}, \quad (5)$$

where: n = 1 - N; N - number of the experiment;  
i, j, g, k - 1, 2, 3, 4 - number of the factor.

The significance of the coefficients in the regression equation (4) has been determined according to Student's criterion. The confidence interval of the regression coefficients  $\Delta b_i = 1,4 \cdot 10^{-3}$  at a level of significance  $\alpha = 0,05$ .

After removing the insignificant coefficients of the equation the following theoretical model is obtained:

$$\hat{y}_1 = 6,9828 + 0,915x_2 - 0,1956x_3 - 0,0918x_4 - 0,1234x_1x_2 + 0,0388x_1x_3 + 0,0176x_1x_4 + 0,0689x_2x_3 - 0,0062x_2x_4 - 0,0505x_3x_4 - 0,0315x_1x_2x_3 - 0,0099x_1x_2x_4 - 0,0067x_2x_3x_4 +$$

$$+ 0,0376x_1x_3x_4 - 0,0119x_1x_2x_3x_4 \quad (6)$$

The theoretical model obtained satisfies the requirements of the adequacy check according to Fisher's criterion at a significance level  $\alpha=0,05$ :  $F_R = 3,2086 < F_{Rtable} = 4,051$ . This check shows that the dispersion of the experimental results obtained compared to the theoretical dependence does not exceed the present value. The dispersion of inadequacy characterizing the model accuracy is  $S_{AD}=0,0102$ .

The process quality of description is good and it is assessed using the multiple correlation coefficient  $r=0,9998$ . The significance of the multiple correlation coefficient is determined according to Fisher's criterion ( $F_R=30660 > F_{Rtable}=1,991$ ).

After passing from coded to natural values of the variable quantities the following empirical dependence is obtained, which reflects the complex influence of  $W, I, U_{DD}, R$  over the converter frequency  $f$ .

$$\begin{aligned} \ln f = & 56,6834 - 8,9485 \cdot \ln W - 5,5601 \cdot \ln I - 39,5986 \cdot \ln U_{DD} + \\ & + 31,5508 \cdot \ln R + 1,059 \cdot \ln W \cdot \ln I + 6,1218 \cdot \ln W \cdot \ln U_{DD} - \\ & - 4,5645 \cdot \ln W \cdot \ln R + 4,9442 \cdot \ln I \cdot \ln U_{DD} - 4,2177 \cdot \ln I \cdot \ln R - \\ & - 14,7804 \cdot \ln U_{DD} \cdot \ln R - 0,7745 \cdot \ln W \cdot \ln I \cdot \ln U_{DD} + \\ & + 0,6177 \cdot \ln W \cdot \ln I \cdot \ln R + 2,175 \cdot \ln W \cdot \ln U_{DD} \cdot \ln R + \\ & + 1,9691 \cdot \ln I \cdot \ln U_{DD} \cdot \ln R - 0,2922 \cdot \ln W \cdot \ln I \cdot \ln U_{DD} \cdot \ln R \end{aligned} \quad (7)$$

When  $U_{DD}=12V$  and  $R_2=5k\Omega$  the theoretical-experimental model has the form:

$$f = e^{-50,0402} \cdot W^{7,6247-1,0401 \cdot \ln I} \cdot I^{7,8127} \quad (8)$$

Since with  $W < 250$  the converter switches on at  $I > 1900mA$ , to obtain the dependence  $f' = f(W, I, U_{DD}, R)$ , with the change in  $W=100 \div 400$  windings, complete factor experiments of the type  $N=2^4$  have been conducted, each experiment having been repeated ( $n=4$ ), with simultaneous variation of the variable factors  $W, I, U_{DD}, R$ , according to a predetermined plan of the experiment. The areas of change in the parameters  $W, I, U_{DD}, R$ :  $W=(100 \div 400)$  windings,  $I=(1800 \div 2000)mA$ ,  $U_{DD}=(10 \div 12)V$ ,  $R=(0,5 \div 5,5)k\Omega$ , have been determined after conducting preliminary single-factor experiments and they guarantee the normal operation of the current-frequency converter.

The form of the theoretical-experimental model obtained, which reflects the complex influence of the parameters  $W, I, U_{DD}, R$  over frequency  $f'$  under these conditions, is:

$$\begin{aligned} \ln f' = & 188,5906 - 25,3304 \cdot \ln W - 22,0062 \cdot \ln I - 89,4842 \cdot \\ & * \ln U_{DD} + 324,2198 \cdot \ln R + 3,0857 \cdot \ln W \cdot \ln I + \\ & + 12,1223 \cdot \ln W \cdot \ln U_{DD} - 47,3797 \cdot \ln W \cdot \ln R + 11,1825 \cdot \ln I \cdot \\ & * \ln U_{DD} - 2,9998 \cdot \ln I \cdot \ln R - 134,4424 \cdot \ln U_{DD} \cdot \ln R - 1,5196 \cdot \\ & * \ln W \cdot \ln I \cdot \ln U_{DD} + 6,304 \cdot \ln W \cdot \ln I \cdot \ln R + 17,8401 \cdot \ln W \cdot \\ & * \ln U_{DD} \cdot \ln R + 19,7871 \cdot \ln I \cdot \ln U_{DD} \cdot \ln R - \\ & - 26335 \cdot \ln W \cdot \ln I \cdot \ln U_{DD} \cdot \ln R \end{aligned} \quad (9)$$

When  $U_{DD}=12V$  and  $R=5k\Omega$  the theoretical-experimental model has the form:

$$f' = e^{-49,6336} \cdot W^{7,6723-1,0767 \cdot \ln I} \cdot I^{7,868} \quad (10)$$

### III. ANALYSIS AND CONCLUSIONS

The theoretical-experimental models obtained describe the conversion characteristic of the current-frequency converter with a high degree of accuracy and enable the working out of an assessment on the degree of influence of the parameters  $W, I, U_{DD}, R$  over the frequency  $f$ .

The frequency  $f$  of the converter discussed depends to the greatest extent on the magnitude of the current  $I$ , the increase in current causing an increase in frequency, the conversion characteristic being linearized above a certain value of  $I$ . With the increase in the number of windings  $W$ , rise in  $R$  and decrease in voltage  $U_{DD}$ , the influence of current over the frequency  $f$  weakens.

The frequency of the converter depends on the number of windings  $W$  of the electromagnet, their increase causing the frequency to decrease following an exponential law. The influence of  $W$  is most significant when  $I > 1800$  mA. When  $I=1500$  mA the number of windings does not influence  $f$ . When  $I > 1500$  mA the frequency decreases with the increase in  $W$ , and when  $I < 1500$  mA the tendency is reversed.

The converter frequency depends on the supply voltage of the magnetosensitive IC, the increase in  $U_{DD}$  causing a decrease in frequency. The degree of influence of  $U_{DD}$  over  $f$  rises with the reduction in the number of windings  $W$ , the change being 1,6 times when  $W=100$  windings, and 1,3 times when  $W=400$  windings. This dependence of  $f$  on  $U_{DD}$  can be used for temperature stabilisation.

The converter frequency depends on the resistor resistance  $R$ , its increase causing the frequency to increase according to an exponential law. The degree of influence of  $R$  decreases with the decrease in the number of windings  $W$ .

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