Analysis of the Chaotic Signals, Produced from Some Radiocommunication Circuits

Stoitscho V. Manev[1] and Vladimir Iv. Georgiev[2]

Abstract - **In this paper analysis of the structure of some circuits, designed to generate chaotic signals, has been made. The main purpose of the paper is to investigate the influence of the nonlinearity in the analyzed circuits upon the properties of the generated signals. An algorithm, based on the algorithm of Eckmann, Kamphorst, Ruelle and Ciliberto for determination of the Ljapunov's exponents of chaotic signals, has been applied. After the determination of the Ljapunov's exponents, conclusions have been made.**

Keywords - **chaotic circuits, chaotic attractors, chaotic signals Ljapunov exponents**

I. Introduction

The implementation of the chaotic signals in radiocommunications is of great interest. This paper can be considered as a continuation of the investigations, made in [5]. Different circuits, designed to generate chaotic signals, have been suggested and discussed in the publications. One of these circuits, most often used for this purpose, is the circuit, known as the Chua's circuit. In the presented paper investigations upon signals, produced by different parameter sets of the Chua's circuit, have been made. These variants of parameter sets have been discussed in [5]. An analysis, based on the Shilnikov's theorems, has been made there. Using the results, obtained in [5], in the presented paper a mathematical model of the system of differential equations, discussed there, has been made.

The chaotic signals, generated by means of the constructed mathematical model, have been investigated. By the computation of the Ljapunov's exponents, an algorithm, based on the well known Eckmann, Kamphorst, Ruelle and Ciliberto (EKRC) algorithm, has been used. This algorithm has been applied by the investigations, carried out upon different parameter sets of the Chua's circuit. From the values of the Ljapunov's exponents, obtained by the investigations, conclusions about the behavior of the circuit by respective parameter sets have been made. Based on the obtained results, an estimate about the ability of the investigated parameter sets of the Chua's circuit to generate different types of chaotic signals can be made.

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II. Experimental results

The dynamic behaviour of the observed circuits has been analyzed in 3-dimensional phase space. The phase space has been formed after the proposal, given in an example in [1].

The obtained digital values have been used as basis for further investigations. The time domain presentation and the autocorrelation function (ACF) of the signal have been shown in each considered case.

Following the methodology, proposed in [1], the value for the time delay (τ) in each case has been determined. An algorithm, based on the EKRC algorithm, has been implemented for determination of the Ljapunov's exponents and the Ljapunov's dimension.

For a fixed set of values of the parameters a chaotic signal has been produced. The initial conditions for the first parameter set of the Chua's circuit, which has been analyzed, are as follows:Ec=1V;E=8.3V;the conductance of the nonlinear element in the middle region Ga= -0.9167mS; the conductance of the nonlinear element in the outer regions Gb=-0.4091mS;the inductor L=23mH;the capacitor C1=15nF; the capacitor C2=100nF; the conductance G=0.625mS; u1=1e-12V;u2=2e-12V; i3=1e-12A.

Because of the restricted resolution by all graphical presentations of the investigated signals, only a highly reduced number of points have been presented. In fact, in the subsequent mathematical analysis much more points have been considered.

The generated signal has been shown in time domain, as follows bellow in Fig.1.

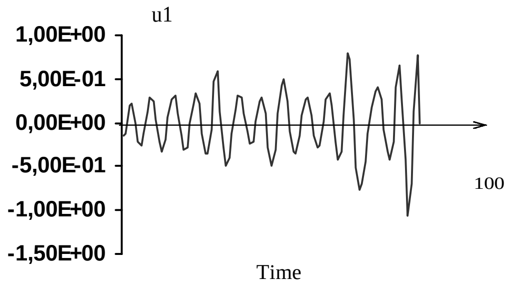


Fig.1

The accurate determination of the Ljapunov's exponents is from great importance for the study of the dynamical behaviour of the analyzed circuit. This requires analysis, based on the autocorrelation functions of the obtained signals. The precise calculation of the time value (τ), at which the autocorrelation function for the first time becomes equal to zero, is a necessary condition for the further computation of the Ljapunov's exponents. This is the reason, for every obtained signal this value to be determined by means of the corresponding ACF [1-2].

The corresponding autocorrelation function of the signal, displayed in time domain in Fig.1, has been presented in Fig.2.

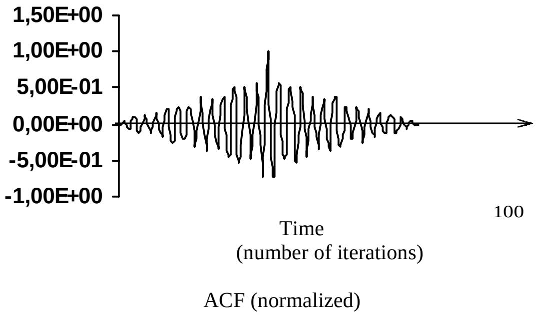


Fig.2

The value of τ for the discussed signal has been computed after calculations, based on the ACF , using the full number of iterations of the obtained signal, in contrast to the small fraction of points, presented in Fig.2, because of the limited resolution by the graphical representation of the results.

For determination of the Ljapunov's exponents and the Ljapunov's dimension an algorithm, based on the EKRC algorithm, has been implemented. The following results have been obtained:

$\lambda_1 = 0.1630$, $\lambda_2 = -0.0600$, $\lambda_3 = -0.3119$,

$D_{Ljap} = 3.7153$,

the sum of the positive Ljap exponents = 0.1630,

the sum of the negative Ljap exponents = - 0.3719.

The presence of: (1) Ljapunov's exponents with positive value,

(2) the satisfied inequality between the sum of the negative Ljapunov's exponents, dominated against the sum of the positive Ljapunov's exponents ,

(3) the small value of the second Ljapunov exponent,

are evidence for chaotic behaviour of the generated signal.

Let consider an another set of initial conditions: the conductance of the nonlinear element in the middle region Ga= - 0.9215 mS , the conductance of the nonlinear element in the outer regions Gb= - 0.3991 mS.

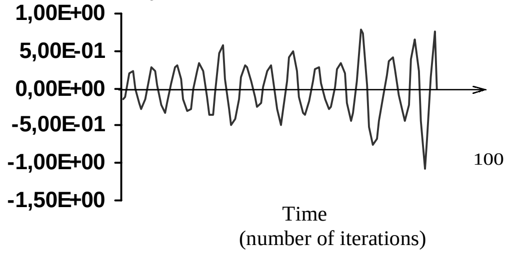


Fig.3.

By these initial conditions another chaotic signal has been obtained. The normalized characteristic in time domain has been presented respectively in Fig.3.

The corresponding autocorrelation function of the signal has been presented in Fig.4.

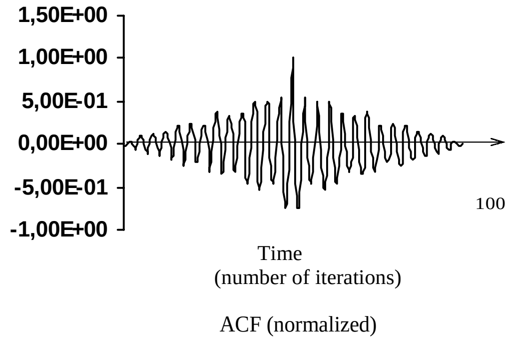


Fig.4.

The further computations of the Ljapunov's exponents lead to the following results:

$\lambda_1 = 0.1666$, $\lambda_2 = -0.0511$, $\lambda_3 = -0.3383$,

$D_{Ljap} = 4.2592$,

the sum of the positive Ljap exponents = 0.1666,

the sum of the negative Ljap exponents = - 0.3895.

The signal , displayed in Fig.4, is treated by the analysis, concerning the Ljapunov's exponents for the chaotic signal, explained above. By means of the Ljapunov's exponents chaotic behaviour in the presented investigated signal has been established. The Ljapunov's dimension is similar to the dimension, obtained in the previous case.

By a change of the initial conditions, as follows:
Ec=1V; E=8.3V; L=18mH; C1=10nF; C2=100nF;
G=(1/1700)S; Ga=(-55/60)mS ; Gb=(-9/22)mS,
another chaotic signal has been generated.

The normalized chaotic signal in time domain has been shown in Fig.5

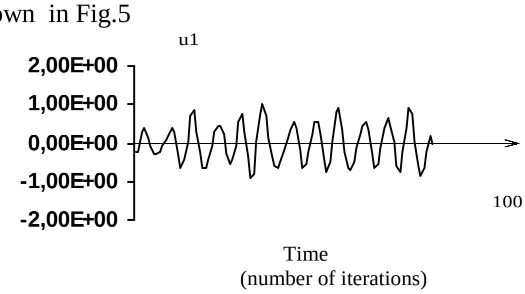


Fig.5

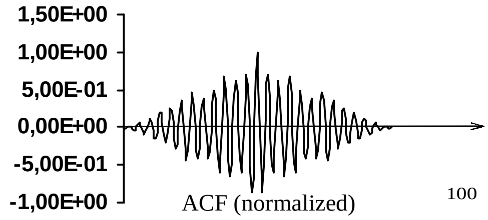


Fig.6

The corresponding normalized autocorrelation function of the obtained signal has been presented above in Fig.6.

The Ljapunov components and the Ljapunov's dimension, computed for the case, considered here, have the following values:

$\lambda_1 = 0.1404$, $\lambda_2 = -0.0832$, $\lambda_3 = -0.3406$

$D_{Ljap} = 2.6880$,

the sum of the positive Ljapunov exponents = 0.1404,

the sum of the negative Ljapunov exponents = - 0.4237.

The computation of the sums, respectively of the positive and the negative Ljapunov's exponents is from great importance for the precise identification of the obtained signals and the correct determination of their characteristics.

The results show, that the examined signal exhibits chaotic behaviour. The value of the Ljapunov's dimension is a small one comparing with the analogous values, obtained in the previous cases, but it is sufficient to prove the presence of chaotic behaviour of the analyzed signal.

Let change the parameter set of the initial conditions in the following way: Ga=(- 0.9215)mS;
Gb=(- 0.3991)mS;

In result a new chaotic signal has been generated.

The time domain presentation of the normalized new chaotic signal is displayed in Fig.7.

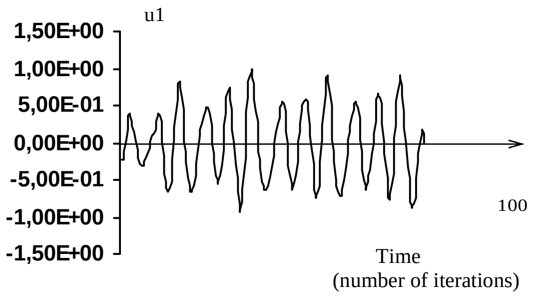
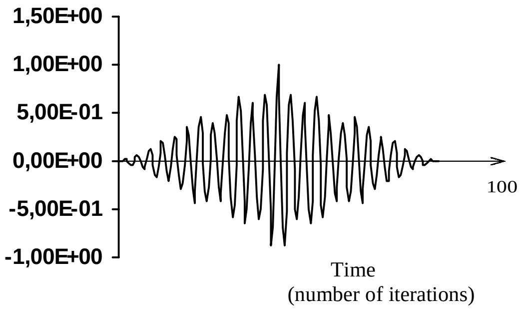


Fig.7

The normalized ACF, related to the signal, considered here, is shown bellow in Fig.8.



ACF (normalized)

Fig.8

After computations, using the full set of the discrete values for the produced signal, the following results for the Ljapunov's exponents and the Ljapunov's dimension have been obtained:

$\lambda_1 = 0.1648$, $\lambda_2 = -0.0833$, $\lambda_3 = -0.3289$,

$D_{Ljap} = 2.9777$,

the sum of the positive Ljap exponents is: 0.1648,

the sum of the negative Ljap exponents is: - 0.4122.

The value of the Ljapunov's dimension, calculated here, is bigger than the Ljapunov's dimension, obtained in the previous case.

Another initial conditions, as shown bellow:
L=16mH; C1=15nF; C2=100nF; G=(1/1500) S;
Ga= - 0.9167 mS; Gb= - 0.4091 mS are accepted.The obtained signal has been shown in time domain in Fig.9.

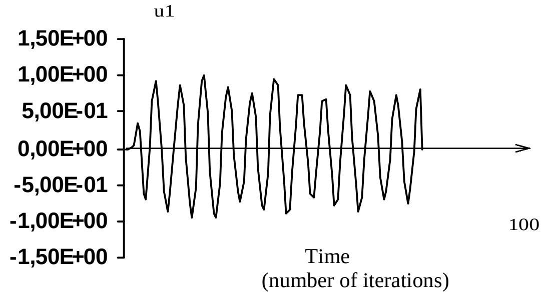


Fig.9

The corresponding ACF is presented in Fig.10.

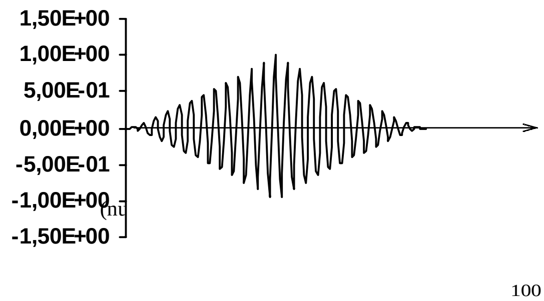


Fig.10

After computations, using the full set of the discrete values for the produced signal, the following results for the Ljapunov's exponents and the Ljapunov's dimension have been obtained:

$\lambda_1 = 0.2042$, $\lambda_2 = -0.1145$, $\lambda_3 = -0.3298$,

$D_{Ljap} = 2.7840$,

the sum of the positive Ljap exponents = 0.2042

the sum of the negative Ljap exponents = - 0.4443

A preliminary control of the values of the parameter set has been made, if the examined values satisfy the requirements of the Shilnikov's theorems. By means of the algorithm, proposed in [5], only these values have been selected, which are with accordance with the requirements, imposed by the theorems, considered in [5].

Changing the initial conditions, as shown bellow:
L=33mH; C1=22nF; C2=100nF; G=(1/1500)S;
Ga=(-55/60)mS; Gb=(-9/22)mS, an another chaotic signal has been obtained.

The examined signal has been presented in time domain respectively in Fig.11.

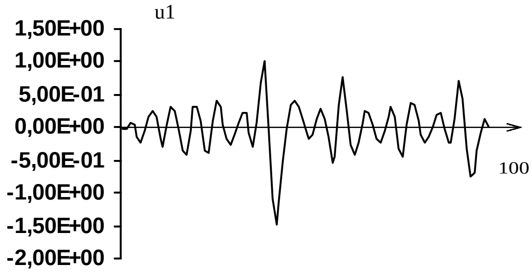
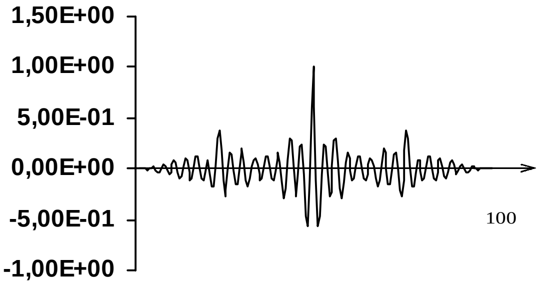


Fig.11

The autocorrelation function, related to the time domain characteristic, is presented bellow in Fig.12:



ACF (normalized)

Fig.12

After computations, using the full set of the discrete values for the produced signal, the following results for the Ljapunov's exponents and the Ljapunov's dimension have been obtained:

$\lambda_1 = 0.2155$, $\lambda_2 = 0.0037$, $\lambda_3 = -0.2933$

$D_{Ljap} = 2.7472$,

the sum of the positive Ljap exponents = 0.2192 ,

the sum of the negative Ljap exponents = - 0.2933

The comparison between the obtained values allows, conclusion for presence of signal with chaotic behaviour to be made.

The values, obtained for the Ljapunov's exponents, show a variation, depending on the set of the values of parameters. As expected, the magnitudes of the Ljapunov's dimension are significant.

The analysis of the dependence between the structure of the Chua's circuit, designed to generate chaotic signals and the characteristics of the generated signals is from great importance for the understanding the processes in this circuit. The appropriate choice of the type of the nonlinearity, used in the circuit, permits signals with chaotic behaviour to be produced.

III. Conclusions

* A circuit, designed to generate chaotic signals, has been investigated.
* Numerous sets of values of the parameters of the elements, included in the Chua's circuit, have been examined.
* The dependence between the change in the sets of the values of their parameters and the corresponding change in the form of the obtained chaotic signals has been found.
* The sets of values of the parameters of the elements in the Chua's circuit are chosen in accordance with the requirements of the Shilnikov's theorems.
* Signals, obtained from the discussed circuits, have been analyzed and presented in time domain.
* The corresponding autocorrelation functions have been shown.
* Based on the obtained autocorrelation functions, computations, necessary for the determination of the Ljapunov exponents, have been carried out.
* The Ljapunov exponents have been obtained.
* The Ljapunov's dimensions have been computed.
* The Ljapunov's exponents made possible, a qualitative analysis of the examined chaotic signals to be done. By means of the values, obtained for the Ljapunov's exponents, the relationship between these exponents and the Kolmogorov's entropy can be investigated.
* Comparison between some of the obtained Ljapunov's dimensions has been made.
* Conclusions about the ability of the examined sets of values of the parameters of the elements for producing of chaotic signals have been made.
* All of the observed signals satisfy the criteria for the presence of chaotic behaviour.

References

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