Analysis of the Processes in Some Radiocommunication Circuits, Designed to Generate Chaotic Signals

Stoitscho V. Manev[1] and Vladimir Iv. Georgiev[2]

Abstract - **In this paper analysis of the processes in some radiocommunication circuits, designed to produce chaotic signals, is presented. Using mathematical approach it has been shown, that the proposed circuits are appropriate for developing of chaotic generators. Some experimental results are shown. The obtained results have been discussed.**

Keywords – **Chaotic circuits, Chua's circuit**

I. Introduction

The generation of chaotic signals is from great interest for communication purposes. The problem is discussed in many papers[1-5].

Devices, designed to produce chaotic signals, can be used in communications to obtain signals with certain properties. A lot of the devices, designed for this purpose, can be attached to the class of circuits, based on the well known Chua's circuit, presented in Fig.1[1-5]. In purpose to make the mathematical analysis easier, piecewise - linear approximation of the volt - current characteristic of the nonlinear element has been made. The obtained characteristic can be divided in two regions with negative slopes, as presented in Fig.2 [1-4].

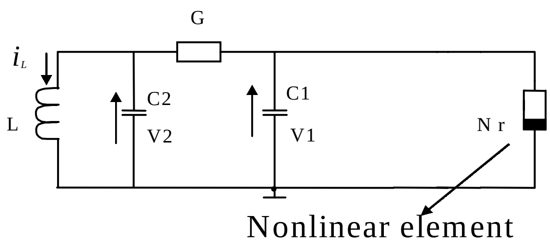

Fig.1.

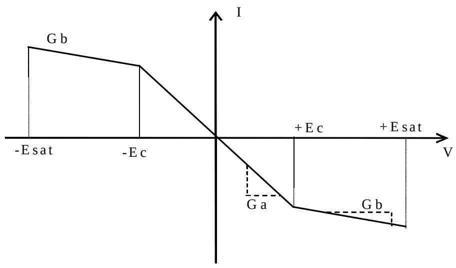

Fig.2

[1]Stoitscho Velizarov Manev - assistant in Dept. of Radiotechnic in Faculty of Communication and Communication Technologies in TU-Sofia, Bulgaria

[2]Vladimir Ivanov Georgiev - professor DSc. in Dept. of Theoretical Electrotechnic in TU-Sofia, Bulgaria

The processes in the Chua's circuit, can be described by means of the following equations [1]:

$$\frac{dv_1}{dt}=\frac{1}{C_1}.\left[G.(v_2-v_1)-\left(G_b.v_1+\frac{1}{2}(G_a-G_b).\left(\,|\,v_1+1|-|\,v_1-1|\right)\right)\right]$$

$$\frac{dv_2}{dt}=\frac{1}{C_2}.\left[G.(v_1-v_2)+i_L\right] \qquad (1)$$

$$\frac{di_L}{dt}=-\frac{v_2}{L}$$

In the equations, presented above, the admittances G_a and G_b are correlated to the graphical representation of the volt - current characteristic of the nonlinear element (Fig.2).

II. Approach, based on Shilnikov's theorems

The trajectories, obtained from the output signal, are located in 3-dimensional space. In order to make the mathematical analysis easier, this space can be separated in 3 regions.

The processes in the middle region can be described, by means of the value of the parameter G_a. In the outer regions the parameter G_b is used.

After the piecewise-linear approximation the processes can be examined in each region. It can be described analytically as a linear one.

In the middle region the following system of differential equations is valid [3]:

$$\frac{dI_3}{dt}=-\frac{1}{L}.V_2$$

$$\frac{dV_2}{dt}=\frac{1}{C_2}.I_3-\frac{G}{C_2}.(V_2-V_1) \qquad (2)$$

$$\frac{dV_1}{dt}=\frac{G}{C_1}.V_2-\frac{G_a{}'}{C_1}.V_1$$

where $G_a{}'=G+G_a$;

In this region the Jacobian matrix and the characteristic polynomial are [3]:

$$J_{F_a} = \begin{bmatrix} 0 & -\frac{1}{L} & 0 \\ \frac{1}{C_2} & -\frac{G}{C_2} & \frac{G}{C_2} \\ 0 & \frac{G}{C_1} & -\frac{G_a'}{C_1} \end{bmatrix} \qquad (3)$$

$$\lambda^3 + \left(\frac{G}{C_2} + \frac{G_a'}{C_1}\right).\lambda^2 + \left(\frac{1}{L.C_2} + \frac{G.G_a}{C_1.C_2}\right).\lambda + \frac{G_a'}{L.C_1.C_2}.$$

In order to analyse the behaviour of the Chua's circuit in the middle region, the eigenvalues of the Jacobian matrix have to be found. The processes in the outer regions can be described by the following system of differential equations [3]:

$$\frac{dI_3}{dt} = -\frac{1}{L}.V_2$$

$$\frac{dV_2}{dt} = \frac{1}{C_2}.I_3 - \frac{G}{C_2}.(V_2 - V_1) \qquad (4)$$

$$\frac{dV_1}{dt} = \frac{G}{C_1}.V_2 - \frac{G_b'}{C_1}.V_1 - \frac{I'}{C_1}$$

where:

$$G_b' = G + G_b,$$
$$I' = (G_b - G_a).E \quad if \quad V_1 < -E,$$
$$I' = (G_a - G_b).E \quad if \quad V_1 > E.$$

In the outer regions the Jacobian matrix and the characteristic polynomial are, as follows [3]:

$$J_{F_b} = \begin{bmatrix} 0 & -\frac{1}{L} & 0 \\ \frac{1}{C_2} & -\frac{G}{C_2} & \frac{G}{C_2} \\ 0 & \frac{G}{C_1} & -\frac{G_b'}{C_1} \end{bmatrix} \qquad (5)$$

$$\lambda^3 + \left(\frac{G}{C_2} + \frac{G_b'}{C_1}\right).\lambda^2 + \left(\frac{1}{L.C_2} + \frac{G.G_b}{C_1.C_2}\right).\lambda + \frac{G_b'}{L.C_1.C_2}.$$

The mathematical analysis, made in the outer regions, is similar to the analysis in the middle region.

In both cases after the determination of the eigenvalues, using the Jacobian matrices, the Shilnikov's method can be applied to prove the presence of chaotic behaviour .

Next, a general case of third order autonomous dynamical system is considered.

The equations, describing the processes in this system are, as follows [5]:

$$\frac{dX}{dt} = \zeta\,(X), \quad t \in R, \quad x \in R^3, \qquad (6)$$

where ζ is vector field.

Let $X_e \in R^3$. X_e is defined as an equilibrium point for (6), if $\zeta\,(X_e) = 0$.

Let the matrix $D\zeta\,(X_e)$ be:

1) real,
2) 3×3 dimensional,
3) the Jacobian derivative of ζ at X_e.

The equilibrium point X_e is defined as *hyperbolic saddle focus*, if the eigenvalues of $D\zeta\,(X_e)$ are of the form: γ, $\sigma \pm j\omega$, $\gamma.\sigma < 0$, $\omega \neq 0$, where: γ, σ, ω are real [5].

The two theorems, presented bellow, give the background for the chaotic behaviour of the Chua's circuit in the sense of Shilnikov [5]:

<u>Theorem1</u> (Homoclinic Shilnikov Method)

Given the third-order autonomous system in (6), where ξ is a C^2 vector field on R^3. Let X_e be an equilibrium point for (6). Suppose the following:

1) The equilibrium point is a saddle focus, whose characteristic eigenvalues satisfy the Shilnikov's inequality: $|\gamma| > |\sigma| > 0$

2) There exists a homoclinic orbit H based at X_e.

Then: 1) The Shilnikov map, defined in a neighbourhood of H, possesses a countable number of Smale horseshoes in its discrete dynamics.

2) For any sufficiently small C^1 - perturbation ζ of ξ^2, the perturbed system:

$$\frac{dX}{dt} = \zeta\,(X), \; x \in R^3 \qquad (7)$$

has at least a finite number of Smale horseshoes in the discrete dynamics of the Shilnikov's map, defined near H.

3) Both the original system (equation (6)) and the perturbed system (equation (7)) exhibit horseshoe chaos (homoclinic chaos)[5].

<u>Theorem2</u> (Heteroclinic Shilnikov Method)

Given the third - order autonomous system in (6), where ξ is a C^2 vector field on R^3. Let X_{e1} and X_{e2} be two distinct equilibrium points for (6). Suppose the following:

1) Both X_{e1} and X_{e2} are saddle foci that satisfy the Shilnikov's inequality: $|\gamma_i| > |\sigma_i| > 0 \quad (i = 1, 2)$

With the further constraint:

$$\sigma_1.\sigma_2 > 0 \quad or \quad \gamma_1.\gamma_2 > 0$$

2) There is a heteroclinic loop H_l, joining X_{e1} to X_{e2}, that is made up of two heteroclinic orbits $\mathrm{H}_i \quad (i = 1, 2)$.

Then: 1) The Shilnikov map, defined in a neighbourhood of H_l, possesses a countable number of Smale horseshoes in it's discrete dynamics.

2) For any sufficiently small C^1 - perturbation ζ of ξ^2, the perturbed system:

$$\frac{dx}{dt} = \zeta\,(x), \qquad x \in R^3 \qquad (8)$$

has at least a finite number of Smale horseshoes in the discrete dynamics of the Shilnikov's map, defined near H_l.

3) Both the original system equation (6) and the perturbed system equation (8) exhibit heteroclinic chaos [5].

The aim of the analysis, made in the presented paper, is to examine the influence of the Shilnikovs inequalities (see above) on the character of the signal, produced from the Chua's circuit.

In order to achieve this, in the presented study the following algorithm has been developed:

1) For the initial conditions the Jacobian matrix (3) in the middle region has been constructed.

2) By means of the values of the parameters, given in the initial conditions, the Jacobian matrices (5) for the outer regions have been constructed.

3) The eigenvalues, respectively for the matrices (3) and (5), have been determined.

4) The real parts: γ and σ from the eigenvalues have been determined.

5) Using the Shilnikov's inequalities a procedure has been developed. The essence of this procedure is: such values of the parameters, taking part by the construction of the Jacobian matrices, to be determined, so as the Shilnikov's inequalities to be satisfied. To achieve this, the following steps are necessary:

* The eigenvalues should be checked up, if they satisfy the requirements, defined by the Shilnikov's inequalities.

* If these requirements are not fulfilled, another procedure starts. It includes search of such set of values of parameters, which is appropriate to satisfy the Shilnikov's inequalities.

III. Experimental results

As starting point for the investigations a circuit, analogous to this, presented in [3], has been used. In Fig.1 the structure of the analyzed circuit is presented. It consists of: an inductor L, two capacitors C1 and C2, a linear resistor G and a nonlinear element.

The values of the parameters are: the inductor L=23mH;the capacitor C1=15nF; the capacitor C2=100nF; the conductance: G=0.625m S;

In Fig.2 the volt-current characteristic is given. The negative signs indicate, that the slopes of the volt - current characteristic in the examined regions are both negative. In order to obtain characteristic in such a form (Fig.2), the absolute value of Ga must be greater then the absolute value of Gb. This requirement is included in the proposed algorithm. A search for values of the parameters Ga and Gb has been made, in order to satisfy the Shilnikov's inequalities.

The experimental results illustrate the above procedure. They are based on the following initial conditions:

* a conductance of the nonlinear element in the middle region Ga= - 0.9167 mS.

* a conductance of the nonlinear element in the outer regions Gb= - 0.4091 mS.

For this case for γ and σ, concerning the dynamics of the system in the middle region, the values:

$\gamma = 2.4805e+004$; $\sigma = -5.8051e+003$ have been obtained.

The eigenvectors for the middle region are the columns of the matrix, presented bellow:

$$\begin{bmatrix} 0.0011 + 0.0008i & 0.0011 - 0.0008i & -0.0002 \\ 0.4877 - 0.3385i & 0.4877 + 0.3385i & 0.1276 \\ -0.8047 & -0.8047 & 0.9918 \end{bmatrix}$$

The values, obtained for γ and σ by the investigations in the outer regions, are as shown bellow:

$\gamma = -2.1732e+004$; $\sigma = 544.0226$.

Respectively the eigenvectors, concerning the processes in the outer regions, are the columns of the matrix:

$$\begin{bmatrix} -0.0009 + 0.0008i & -0.0009 - 0.0008i & -0.0003 \\ 0.3151 + 0.3578i & 0.3151 - 0.3578i & -0.1734 \\ 0.8790 & 0.8790 & 0.9848 \end{bmatrix}$$

It is obvious, that the obtained values for γ and σ in the middle and in the outer regions fulfil the requirements from the Shilnikov's inequalities and their polarity are in accordance with the expected polarity, resulting from the analysis in [3].

Alternatively, another set of initial conditions:

Ga= - 0.9215 mS; Gb= - 0.3991 mS produce the following values for γ and σ in the middle region:

$\gamma = 2.5117e+004$; $\sigma = -5.7999e+003$

The eigenvectors are the columns of the matrix

$$\begin{bmatrix} 0.0011 + 0.0008i & 0.0011 - 0.0008i & -0.0002 \\ 0.4922 - 0.3381i & 0.4922 + 0.3381i & 0.1274 \\ -0.8021 & -0.8021 & 0.9919 \end{bmatrix}$$

For the outer regions $\gamma = -2.2384e+004$, $\sigma = 536.8802$ have been obtained .The eigenvectors are presented as columns of the matrix

$$\begin{bmatrix} -0.0009 + 0.0008i & -0.0009 - 0.0008i & -0.0003 \\ 0.3272 + 0.3587i & 0.3272 - 0.3587i & -0.1731 \\ 0.8742 & 0.8742 & 0.9849 \end{bmatrix}$$

The inequalities in the Shilnikov's theorems are satisfied.

Another set of initial conditions: L=18mH; C1=10nF; C2=100nF;G=(1/1700)S;G= 0.58824 mS; Ga=-0.9167mS; Gb=-0.4091 mS produce:

$\gamma = 3.8713e+004$; $\sigma = -5.8762e+003$

in the middle region. The eigenvectors in this region are presented as columns of the matrix:

$$\begin{bmatrix} 0.0011+0.0011i & 0.0011-0.0011i & -0.0001 \\ 0.5271-0.2845i & 0.5271+0.2845i & 0.0993 \\ -0.8008 & -0.8008 & 0.9951 \end{bmatrix}$$

For the outer regions the following values have been obtained: $\gamma = -2.6246e+004$; $\sigma = 1.2246e+003$. The corresponding eigenvectors are the columns of the matrix, shown bellow:

$$\begin{bmatrix} 0.0009-0.0008i & 0.0009+0.0008i & -0.0003 \\ -0.2952-0.2997i & -0.2952+0.2997i & -0.1402 \\ -0.9072 & -0.9072 & 0.9901 \end{bmatrix}$$

The change in the values of Ga and Gb, as follows:

Ga= -0.9215 mS; Gb= -0.3991mS;

leads to a respective change in the eigenvalues of the Jacobian matrices in the middle and in the outer regions:

In the middle region: $\gamma = 3.9168e+004$,

$\sigma = -5.8620e+003$.

The eigenvectors are the columns of the matrix

$$\begin{bmatrix} 0.0011+0.0011i & 0.0011-0.0011i & -0.0001 \\ 0.5316-0.2840i & 0.5316+0.2840i & 0.0988 \\ -0.7980 & -0.7980 & 0.9951 \end{bmatrix}$$

In the outer regions : $\gamma = -2.7203e+004$,

$\sigma = 1.2035e+003$.

The eigenvectors in the outer regions are displayed as columns of the matrix

$$\begin{bmatrix} 0.0009-0.0008i & 0.0009+0.0008i & -0.0003 \\ -0.3086-0.3009i & -0.3086+0.3009i & -0.1395 \\ -0.9023 & -0.9023 & 0.9902 \end{bmatrix}$$

It is obvious, that the inequalities in the Shilnikov's theorems are satisfied.

Experiments with another set of initial conditions: L=16mH; C1=15nF; C2=100nF; G=(1/1500)S; Ga = - 0.9167 mS; Gb= - 0.4091 mS, have been carried out. Respectively: $\gamma = 2.1856e+004$ and $\sigma = -5.9270e+003$ in the middle region have been obtained.

The eigenvectors in this region are the columns of the matrix

$$\begin{bmatrix} -0.0014-0.0008i & -0.0014+0.0008i & -0.0003 \\ -0.4176+0.3884i & -0.4176-0.3884i & 0.1159 \\ 0.8214 & 0.8214 & 0.9933 \end{bmatrix}$$

For the outer regions

$\gamma = -2.4002e+004$; $\sigma = 82.2005$

have been obtained. The eigenvectors in the outer regions can be obtained, using the columns of the matrix:

$$\begin{bmatrix} -0.0012+0.0010i & -0.0012-0.0010i & -0.0004 \\ 0.3308+0.4054i & 0.3308-0.4054i & -0.1519 \\ 0.8522 & 0.8522 & 0.9884 \end{bmatrix}$$

Experiments with another set of initial conditions, have been carried out:L=33mH; C1=22nF; C2=100nF; G=(1/1500)S; Ga= -0.9167 mS; Gb= - 0.4091 mS;

The following bellow values for γ and σ in the middle region have been obtained:

$\gamma = 1.6225e+004$; $\sigma = -5.7633e+003$.

The eigenvectors in this region are the columns of the matrix

$$\begin{bmatrix} -0.0011-0.0006i & -0.0011+0.0006i & -0.0003 \\ -0.4593+0.3588i & -0.4593-0.3588i & 0.1584 \\ 0.8126 & 0.8126 & 0.9874 \end{bmatrix}$$

For the outer regions the following values have been obtained: $\gamma = -1.8856e+004$

$\sigma = 240.9648$

Respectively the columns of the matrix:

$$\begin{bmatrix} -0.0009+0.0007i & -0.0009-0.0007i & -0.0004 \\ 0.3381+0.3880i & 0.3381-0.3880i & -0.2296 \\ 0.8574 & 0.8574 & 0.9733 \end{bmatrix}$$

are the eigenvectors in the outer regions.

From the results, obtained by the investigations of different sets of values of parameters of the elements of the Chua's circuit, becomes obvious, that the requirements of the inequalities in the Shilnikov's theorems are satisfied.

IV. Conclusions

* An algorithm to study chaotic signals by means of Chua's circuit, has been proposed.
* Some illustrative experiments have been carried out.
* The obtained results have been examined, if they are with accordance to the requirements, imposed by the Shilnikov's theorems.
* It has been shown, that by appropriate choice of the values of the components in the Chua's circuit, using the proposed algorithm, chaotic signals have been obtained.

References

[1] L.Chua, G.N.Lin, "Canonical realization of Chua's circuit family", *IEEE Trans. CAS,* Vol. 37, No.7, 1990.

[2] M.P.Kennedy, "Three steps to chaos-part I: Evolution", *IEEE Trans.CAS* , Vol.40, No.10, 1993.

[3] M.P.Kennedy, " Three steps to chaos-Part II: A Chua's circuit primer", *IEEE Trans.CAS,* Vol.40, No.10, 1993.

[4] L.Chua, Ch.W.Wu, An.Huang, G.-Q.Zhong, "A universal circuit for studying and generating chaos - Part I: Routes to chaos", *IEEE Trans.CAS* , Vol.40, No.10, 1993.

[5] Christopher P.Silva, "Shilnikov's Theorem - A Tutorial", *IEEE Trans.CAS,* Vol.40, No.10, 1993.

[6] St. Manev , Vl. Georgiev, "Chaotic signals, generated by some circuits - comparative study (correlation analysis)", ICEST 2003 - Sofia.

[7] St. Manev , Vl. Georgiev, "Chaotic signals, generated by some circuits - comparative study (Ljapunov exponents)", ICEST 2003 - Sofia.