A new algorithm for Doppler ambiguity resolution

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Abstract – A new algorithm for velocity ambiguity resolution in coherent pulsed Doppler radar is proposed, based on the spectrum analysis of the nonuniformly sampled data. The highest resolved Doppler ambiguity rate is estimated and simulations results are represented to show the algorithm effectiveness.

Keywords – Doppler ambiguity, single target

Introduction - In radar systems the periodically nonuniform sampling is used to avoid the blind speed effect. For a given set of PRFs, Chinese Remainder Theorem (CRT) has been established to resolve ambiguity [1]-[3]. The problem with the CRT approach is that a small range error on a single PRF (pulse repetition frequency) can course a large error in the resolved range and there is no indication that this has happened. To avoid this problem, a clustering algorithm is also suggested with the minimum square error criterion [4]. It has good anti-error ability and expensive computational throughout. An algorithm based on the choice of particular values for the PRFs is provided for velocity ambiguity resolution, where a quasi-maximum likelihood criterion is maximized for ambiguity order estimation [5].

Considering the blind area of both in time and frequency, this algorithm is so limited by particular PRFs that it is not fit for other combination of PRFs to resolve the velocity ambiguity. In view of the shown disadvantages of the described methods, a new algorithm is proposed to resolve the velocity ambiguity using the spectrum analysis of the nonuniformly sampled data.

Mathematical background – As is well known, the spectrum of a signal sampled uniformly in the time domain is periodic in the frequency domain. In contrast, the spectrum of a nonuniformly sampled signal is not periodic, which enables unambiguity frequency analysis at the frequencies above half the virtual sampling frequency, defined by the mean harmonious of the selected pulse repetition frequencies [6]. The proposed algorithm is based on this aperiodicity according to the spectrum estimation equation [6] using two consecutively changed *PRFs*:

$$X(m+pN/2)=X_{2k}(m)+e^{j\frac{2\pi}{N}(m+pN/2)(1-\rho)}X_{2k+1}(m), \quad (1)$$

where $\rho=\dfrac{PRF_1-PRF_2}{PRF_1+PRF_2}$ - nonuniformly sampling constant; N - number of sampling points; $m\in\left[0;N/2-1\right]$ - frequency bin number; p - integer number; X_{2k} and X_{2k+1} - *DFT* of even and odd samples respectively

The corresponding power spectrum is calculated by taking the modulus-squared spectral coefficients:

$$\left|X(m+pN/2)\right|^2=\left|X_{2k}(m)\right|^2+\left|X_{2k+1}(m)\right|^2+ \\ +\operatorname{Re}\hat{X}\cos\theta-\operatorname{Im}\hat{X}\sin\theta \quad (2)$$

where $\theta=2\pi m/N-\pi p\rho$ and $\hat{X}=X_{2k}(m)\overset{*}{X}_{2k+1}(m)$

The Doppler ambiguity resolution is made by spectrum peak detection at two stages. The first stage defines the bin number of the spectrum peak below half the sampling frequency and the second stage locates the peak of the dominant alias frequencies envelope according to the equation:

$$\frac{\partial\left\{\left|X(m'+pN)\right|^2\right\}}{\partial p}=0 \quad (3)$$

By substituting the equation (2) into (3), the resolution equation is described as follows:

$$tg\left(\frac{2\pi}{N}m'-2\pi p\rho\right)+\frac{\operatorname{Im}\hat{X}}{\operatorname{Re}\hat{X}}=0, \quad (4)$$

From the known trigonometric formula $tg(\alpha-\beta)=\dfrac{tg\alpha-tg\beta}{1+tg\alpha tg\beta}$, the equation (4) is modified to the next equation:

$$tg(2\pi p\rho)=\frac{tg\left(\frac{2\pi}{N}m'\right)+\frac{\operatorname{Im}\hat{X}}{\operatorname{Re}\hat{X}}}{1-\frac{\operatorname{Im}\hat{X}}{\operatorname{Re}\hat{X}}tg\left(\frac{2\pi}{N}m'\right)}=Y \quad (5)$$

Since the trigonometric function $tg(x)$ is defined in the range $-\pi/2\le x\le\pi/2$, the Doppler ambiguity rate can be uniquely established when the following expression is implemented:

$$|p|\le\left(4|\rho|\right)^{-1} \quad (6)$$

In this case the Doppler ambiguity rate is calculated from the next equation:

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$$p = \mathrm{int}\left(\frac{1}{2\pi\rho} arctgY\right) \qquad (7)$$

Therefore, the obtained equation (7) resolves the Doppler ambiguity of a single target in the limitation of equation (6) using spectrum analysis of the nonuniformly sampled data. It is clear that the highest resolved Doppler ambiguity rate is inversely proportional of the nonuniformly sampling constant, but the detailed analysis shows that in this case the proposed algorithm is more sensitive to the additive noise. So there is a settlement by compromise of this parameter.

Numerical example - The proposed algorithm is tested using Matlab® routine. To illustrate the significance of the algorithm, we consider a single sinusoidal waveform f=13kHz with added white Gaussian noise ($SNR=5dB$), sampled with two consequently changed PRFs – PRF_1=1818Hz and PRF_2=1723Hz in N=128 points. The chosen initial values specify the values of the nonuniformly sampling constant and the highest resolved Doppler ambiguity rate to 0,026829 and 9 respectively. The spectrum power of the signal is calculated below and above half the sampling frequency according to equation (1), and the simulation results are shown at fig. 1 and fig. 2 respectively. The figure 1 defines two spectrum peaks – one real spectrum peak at 622,4Hz and second parasitic one at 1493,3Hz. The parasitic spectrum peak is descended from the nonuniform sampling nature, described at our previous work [6]. The real resolved frequency is equal to 13,002kHz, which is the nearest point to the real simulated waveform. The second resolved frequency, defined by the parasitic spectrum peak, is equal to 15,658kHz, which is recognized as parasitic Doppler frequency due to extended spectrum analysis, shown at fig.2. It shows that the real Doppler frequency is located at the peak of the dominant alias frequencies envelope according to equation (3).

The figure 1 and figure 2 demonstrate the first and second stage of the Doppler ambiguity resolution respectively as they define the bin number of the spectrum peak and the value of the Doppler ambiguity rate, which is set to 7 according to equation (7).

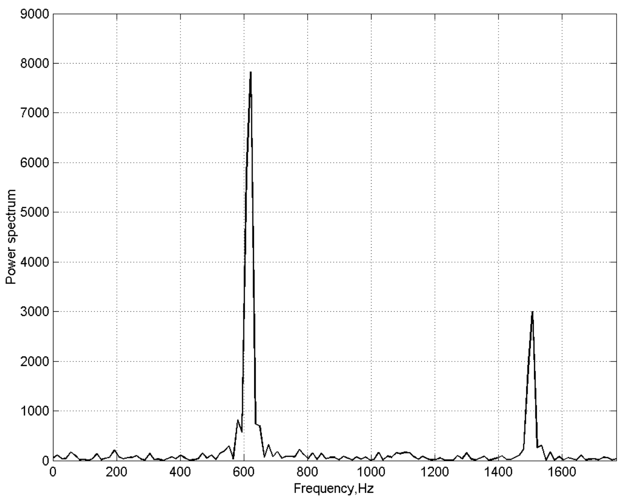

Figure 1 – Spectrum analysis at the frequencies below half the virtual sampling frequency

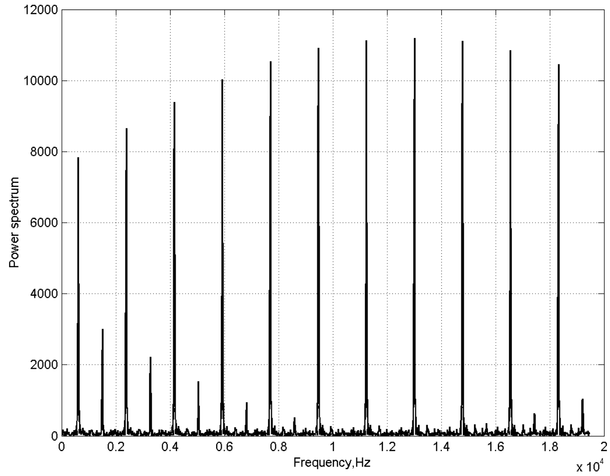

Figure 2 – Extended spectrum analysis at the frequencies above half the virtual sampling frequency

So the proposed algorithm resolves successfully the Doppler ambiguity in the low SNR environments using simple estimation technique as described in §2. The fast implementation of the unambiguous estimation procedure is based on the fast Fourier transform (*FFT*) and spectrum coefficients computation only at selected frequencies. The main disadvantage of the proposed algorithm consists in the inability to resolve the Doppler frequencies in the multiple target situations, when the difference of the target radial velocities is divisible by the virtual sampling frequency.

Conclusion - A new approach to PRF ambiguity resolving has been described. The presented algorithm allows resolving the Doppler ambiguity of a single target according to the obtained equation (7) and the highest resolved Doppler ambiguity rate is defined by equation (6). This algorithm provides good stability in low SNR environments and simple computation mechanism. It can find an application in advanced radar processors, included in low PRF search radars, or in other Doppler systems to determine the object radial velocity.

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