Core Losses in Step-down Transformer Supplying Non-linear Loads

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Abstract **– The paper discusses the operation of a three-phase distribution transformer with a non-linear load connected. The influence of the harmonics, caused by the non-linear load, on the transformer's magnetic flux, magnetizing power and losses has been investigated. Analytical expressions for their calculation have been derived.**

Keywords: Harmonics, distribution transformer, core losses

I. Introduction

The wide usage of nonlinear electric power consumers (pulse-supplying sources with different purpose and power, supplying sources for different technological purpose, controlled motor drivers, etc) causes the electrical power quality aggravation. Due to its operation mode peculiarities this type of consumers deform the three-phase current waveforms. Therefore they become higher harmonics sources. The higher harmonics, which are distributed into supply grid, confuse the normal operation of the other consumers. On the other hand this harmonics cause the losses increasing in the equipment for energy transfer and distribution.

The aim of this paper is to study the influences of the harmonics, caused by the non-linear loads, upon the distribution transformer core losses.

II. Non-symmetry magnitic fluxes in three-phase transformer with symetric supply

In the design and analysis procedures it is usually assumed that the magnetic flux is equal for all three cores of the three-phase transformer. The three-phase transformers with power less than 1600 kVA are usually of core design (three-leg core) and their magnetic current is non-symmetrical. It is preferred practice today since it is lighter, smaller, cheaper and slightly more efficient [4].

Assume that the primary winding is supplied with symmetrical three-phase voltage. The line voltages are equals to:

$$\dot{U}_{AB} = \dot{U}\,; \quad \dot{U}_{BC} = a^2.\dot{U}\,; \quad \dot{U}_{CA} = a.\dot{U}\,.$$

where $a = \exp\left(j\dfrac{2\pi}{3}\right)$ and $j = \sqrt{-1}$.

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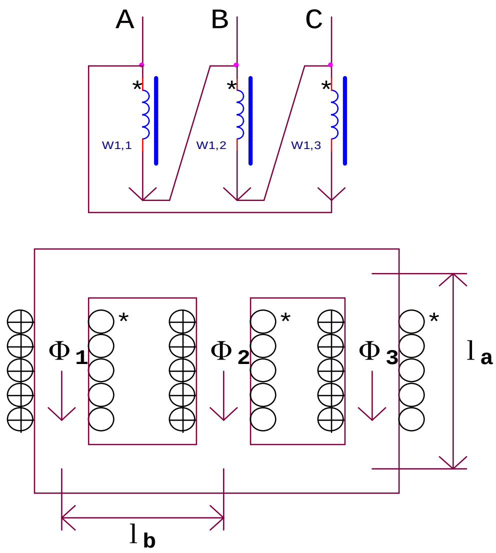

Fig.1.The three-phase core type transformer

Composing the three-phase core type transformer's equivalent circuit it is assumed that the leakage fluxes are negligible (fig.1). The useful magnetic fluxes in three cores are equals:

$$\begin{aligned}
\dot{\Phi}_1 &= \left(\frac{1}{1+2r_\mu} - a^2\right)\Phi_0 \\
\dot{\Phi}_2 &= \left(a^2 - a\right)\Phi_0 = \left(-j.\sqrt{3}\right)\Phi_0 \\
\dot{\Phi}_3 &= \left(a - \frac{1}{1+2r_\mu}\right)\Phi_0
\end{aligned} \qquad (1)$$

where Φ_0 - the provisional magnetic flux, is equal to:

$$\Phi_0 = \frac{3w_1 I_0}{\dfrac{1}{\Lambda_a} + \dfrac{2}{\Lambda_b}}$$

I_0 - mean value of the no-load current;
w_1 - number of the primary winding turns;
r_μ - ratio between core and yoke permeances (fig.1):

$$r_\mu = \frac{\Lambda_a}{\Lambda_b} = \frac{\mu.S_a}{l_a}.\frac{l_b}{\mu.S_b} = \frac{l_b}{l_a}.\frac{S_a}{S_b}$$

Λ_a, Λ_b -core and yoke permeances respectively equals to:

$$\Lambda_a = \frac{\mu.S_a}{l_a} \text{ и } \Lambda_b = \frac{\mu.S_b}{l_b}\,;$$

S_a, S_b - core and yoke cross-section;
l_a, l_b - core and yoke length respectively.

Usually, the core and yoke cross-sections are equal. For the TMA type transformers type (manufactured by the Elprom

Trafo CH AG - Kjustendil, Bulgaria) the ratio l_b/l_a is about 0.55. Hence, the magnetic flux in inner core is almost 30% bigger than the fluxes in outside cores. Usually in the design procedures for three-phase transformer is used the magnetic fluxes mean value [5,6].

III. Electromagnetic Processes in Transformer Supllying Non-Linear Load

Nonlinear loads, connected to the low voltage supplying grid, cause an energy flow formed by higher harmonics with different numbers [7]. This energy flow is directed from the non-liner load to the step-down transformer and further to the medium voltage power-supplying grid. Hereafter it will be referred "Secondary Power Flow" (SPF) and will be designated with $W_{(k)}$, to be differentiated from the "Primary Power Flow" designated with $W_{(1)}$ comprising only the basic harmonic (fig.2) [5].

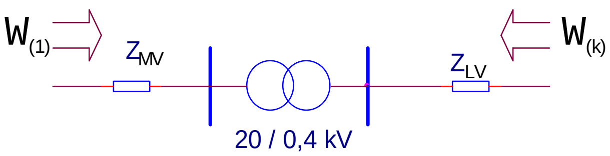

Fig.2. Scheme of the energy flow

It is typical of the discussed distribution transformer mode, that:

(a) The magnetic flux with frequency k^n, created in the magnetic core of the transformer by the secondary power flow and the main magnetic flux with grid frequency, generated by the primary winding and by the primary power flow, superimpose in the transformer magnetic core. The total magnetic flux of the transformer is a result of this superimposition and is non-sinusoidal.

(b) In terms of secondary power flow, the transformer is in a close to the short-circuit mode, as its primary winding is connected to middle voltage power supplying grid, with very low resistance for a harmonic with k^n frequency;

(c) While the primary winding is fed by a sufficiently powerful source (theoretically with unlimited power), regarding to the secondary power flow, the transformer feeding device has a limited power [1].

(d) The principle of superposition is not applicable in the analyses of the discussed mode. Therefore, we have to use the magnetization curves *B(H)* and take the momentary induction values for each of the harmonics. The position of the working point on the magnetization curve is determined by both magnetic fluxes - the basic frequency magnetic flux and the superimposed partial k^n frequency magnetic flux;

(e) Owing to the non-linear characteristics of the transformer and the presence of currents with different frequencies, it is necessary to transfer provisionally some of the parameters to one and the same basic frequency - the grid frequency. The following condition is accepted as the basis of this transferring – the core losses, caused by the k^n harmonic of the secondary winding current and the losses, transferred to the basic frequency, should be equal, that is:

$$P_{c(k)} = \hat{P}_{c(k)} \qquad (2)$$

It is accepted hereafter to use the symbol ˆ to denote quantities, caused by the secondary current harmonics, which have been transferred to the basic frequency.

From all that, it can be concluded that the operation of the nonlinear load, low voltage supplying grid, the step-down distribution transformer and middle voltage supplying grid should be analyzed as interrelated.

In the analysis of the electromagnetic processes, it is accepted that the partial voltage of the secondary winding has sinusoidal waveform. Consequently the magnetic flux $\Phi_{(k)}$ has also sinusoidal waveform and is added to the basic frequency magnetic flux $\Phi_{(1)}$.

What is characteristic of the discussed mode of operation is that the magnetization currents of the primary winding with basic frequency and the magnetizing currents of secondary winding with higher frequency, form independent of one another magnetic fluxes, which superimposed. Therefore we cannot define general magnetization current in the common sense of this term. We can only introduce and use it so long as it helps us determine the position of the working point on the magnetization curve and afterwards, to determine the value of the partial magnetizing current.

IV. Core Losses Due to the Secondary Power Flow

The core losses can be determined in the following way:

$$P_c = \sum_{j=1}^{n} p_{\frac{1,0}{50}} . B_{m,j}^2 \left(\frac{f}{50}\right)^{1,3}$$

where $p_{\frac{1,0}{50}}$ is the value of the relative core losses at peak flux density of 1T and frequency f=50*Hz*.

$B_{m,j}$ - the peak value of the flux density for the j[th] element of the magnetic circuit.

Assuming that all elements of the magnetic circuit have equal cross-sections $S_j = Const$, it can be concluded that $B_{m,j} = Const$, and the transferring condition acquires the following expression:

$$\sum_{j=1}^{n} p_{\frac{1,0}{50}} . \hat{B}_{m,(k),j}^2 \left(\frac{f}{50}\right)^{1,3} = \sum_{j=1}^{n} p_{\frac{1,0}{50}} . B_{m,(k),j}^2 \left(\frac{k.f}{50}\right) \qquad (3)$$

After applying

$$k_{B(k)} = \sqrt{k^{1,3}} = k^{0,65} \qquad (4)$$

for the peak of the flux density transferred to basic frequency the result is:

$$\hat{B}_{m(k)} = k_{B(k)} . B_{m(k)} \qquad (5)$$

Due to the presence of two power flows through the transformer, the magnetic flux created by the k^n harmonic of the secondary winding current superimposes to the basic

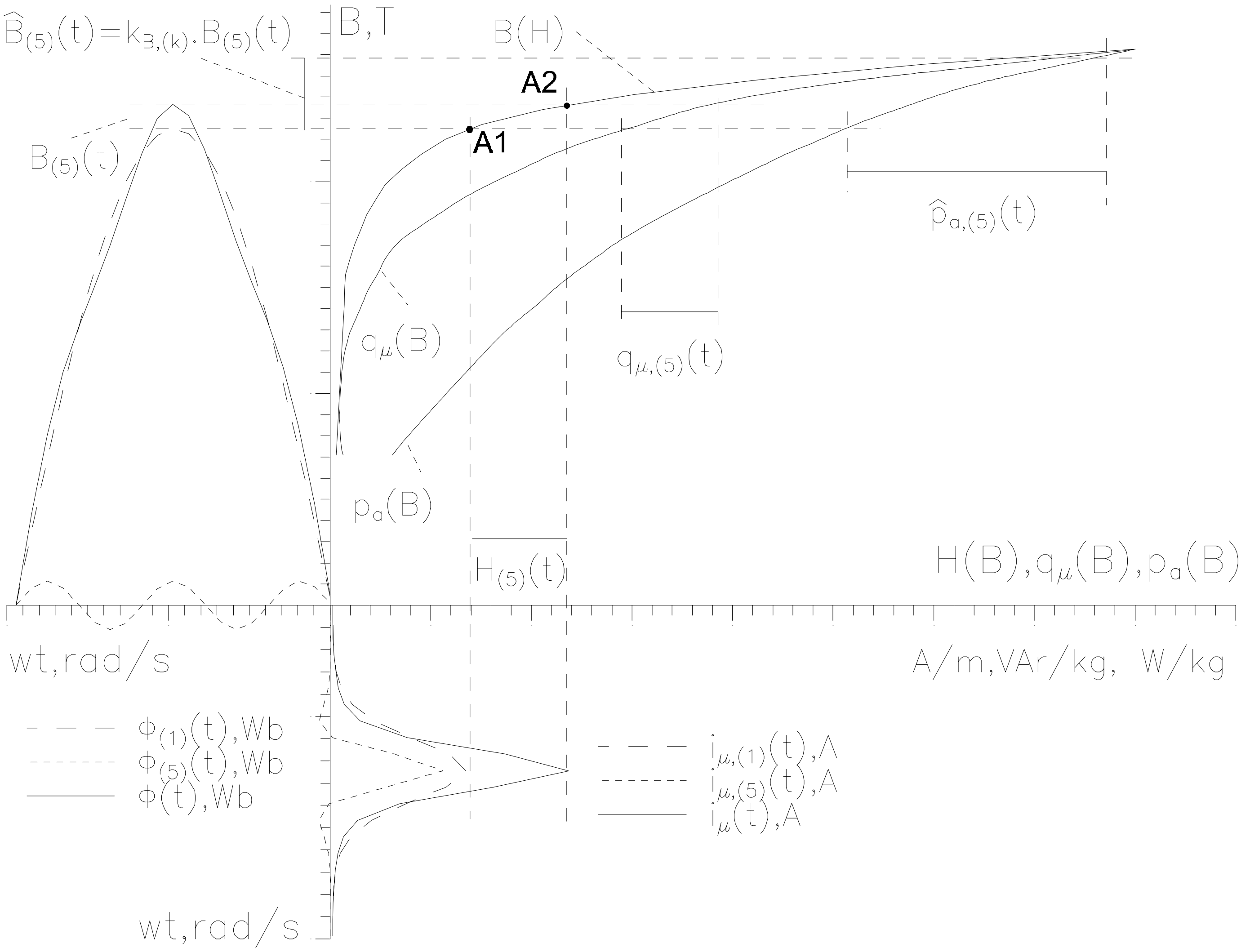

Fig.3. Determining the additional core losses of the transformer for the 5^n harmonic

magnetic flux with grid frequency The value of the k^{-n} harmonic magnetic flux can be presented as:

$$\Phi_{(k)}(\omega.t)=\frac{\mu.w_2}{l_m}.S_a.i_{\mu,(k)}(\omega.t) \quad (6)$$

where $i_{\mu,(k)}(k\omega.t)$ is the k^n harmonic magnetization current, of the secondary winding;

S_a - cross-section of the magnetic core;

l_m - the average length of the magnetic circuit.

The magnetization curve *B(H)*, the relative core losses curve $p_a(B)$, and the relative magnetizing power curve $q_\mu(B)$ for the steel are used to determine the parameters of the transformer for the k^n harmonic. For creating a precise model of a step-down transformer, it is necessary to take into account some other characteristics typical of its magnetic circuit, such as the magnetization power of the air gaps $q_{\mu,\delta}(B)$, the residual magnetic flux density B_r, etc. As the aim of this research is to investigate the influence of the harmonics to the transformer core losses they are not under consideration in this paper.

Fig 3 illustrates the core losses increasing due to the higher harmonics magnetic fluxes. It is assumed that only partial flux $\Phi_{(5)}$ superimposes to the basic magnetic flux $\Phi_{(1)}$. The basic magnetic flux $\Phi_{(1)}$ determines the basic working point on the magnetizing curve. With $\Phi_{(1)} = \Phi_{(1),max}$ the flux density $B_{(1)}$ reaches its peak value (designated with A1 in figure 3).

The sinusoidal voltage of the secondary winding creates a partial magnetic flux $\Phi_{(5)}$ with frequency 5 times the basic one. Supposing the magnitude of $\Phi_{(5)}$ is given. The flux $\Phi_{(5)}$ superimposed to the basic flux $\Phi_{(1)}$, causes partial hysteresis loop around the working point with frequency *kf=5f* and periodic oscillation of the magnetic induction $B_{(1)}$. In fig.3 with the given values, the magnetic induction *B* reaches the peak values, marked as A2 on the magnetization curve.

Owing to the strongly expressed non-linearity of the magnetization curve, the relative magnitude of the magnetic induction for the k^n harmonic is determined graphically directly by the curve of *B = f(H);*

$$H_{(k)}(t)=H(t)-H_{(1)}(t) \quad (7)$$

The magnetizing current $i_{\mu,(k)}$ of the k^n harmonic is also plotted in quadrant 4. It can be determined by subtracting the magnetizing current with basic frequency $i_{\mu,(1)}$, from the resultant magnetizing current i_μ, determined by the total magnetic flux $\Phi(t)=\Phi_{(1)}(t)+\Phi_{(5)}(t)$.

$$i_{\mu(k)}(t)=i_{\mu}(t)-i_{\mu(1)}(t) \qquad (8)$$

The curve $q_\mu=f(B)$, plotted in quadrant 1, allows to determine the magnitude of the relative magnetization power for the k^n harmonic.

$$q_{\mu(k)}(t)=q_{\mu}(t)-q_{\mu(1)}(t) \qquad (9)$$

Having the curve of the relative core losses at basic frequency $f=50Hz$ in quadrant 1, $p_a = f(B)$ the relative losses in the steel, caused by the k^n harmonic in the secondary current, can be determined.

For this purpose, the obtained value of $B_{(k)}$ has to be transferred to the basic frequency by multiplying to the coefficient $k_{B,(k)}$. By adding the transferred magnetic flux density $\hat{B}_{(k)}$ to the magnetic flux density $B_{(1)}$, the peak value of the resultant magnetic flux density is obtained. This value is used to determine the total core losses. The relative active losses, due to the magnetic flux with frequency higher than the basic one, are determined with the equation:

$$\hat{p}_{a(k)}(t)=p_a(t)-p_{a(1)}(t) \qquad (10)$$

The ration of the components of the power - $\hat{p}_{a(k)}$, $q_{a(k)}$, determines the loss angle for the k^n harmonic.

$$tg\left(\alpha_{loss(k)}\right)=\frac{\hat{p}_{a(k)}(t)}{q_{a(k)}(t)} \qquad (11)$$

The active component of the transformer current is:

$$\hat{i}_{a(k)}=i_{\mu(k)}.tg\left(\alpha_{loss(k)}\right) \qquad (12)$$

It is important to note that all discussed values are the momentary ones.

The average values of the magnetizing power and the core losses for the k^n harmonic are determined for one period of the basic harmonic. This is so, because it is the basic magnetic flux that determines the position of the working point, around which the partial hysteresis cycle with frequency kf is located.

Active power for the k^n harmonic

$$P_{a(k)}=\frac{G_a}{T}.\int_0^T \hat{p}_{a(k)}(t).dt \qquad (13)$$

where G_a is the core steel masse.

Reactive power for the k^n harmonic

$$Q_{\mu(k)}=\frac{G_a}{T}.\int_0^T q_{\mu(k)}(t).dt \qquad (14)$$

The core losses increasing is sum of losses causing by the harmonics with different frequency:

$$P_{a,SPF}=\sum_{k=1}^{\infty}P_{a(k)} \qquad (15)$$

and respectively, the generalized magnetizing power is:

$$Q_{\mu,SPF}=\sum_{k=1}^{\infty}Q_{\mu(k)} \qquad (16)$$

V. Conclusion

The nonlinear loads connected to the secondary winding cause higher harmonics power flow from the load to the transformer. It forms higher harmonics magnetic flux in the transformer magnetic core, which increases values of the flux density and moves the working point on the magnetization curve in saturation zone. As nowadays the transformer operate with a relatively high value of the magnetic flux density, this movement to the high saturation zone leads to sharp increase of the core losses. The additional core losses, due to the higher harmonics may have value commensurable with the losses due to the basic harmonic. On the other hand, due to the transformer core is non-symmetry, the magnetic flux, respectively magnetic flux density in the innes and the outside cores are different then the average values. Hence, the increase of the core losses is bigger for the inner core. The design engineers should take this fact into account when they design distribution transformer, supplying non-linear loads.

VI.References:

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