Realization of Variable IIR Digital Filters as a Cascade of Third or Higher Order Identical Sub-filters

Georgi Stoyanov[1], Ivan Uzunov[2] and Masayuki Kawamata[3]

Abstract – **A new approach to design high tuning accuracy variable IIR filters as a cascade of *N* identical filters of order higher than second is proposed in this paper. The identical sub-filters are realized as parallel allpass structures and their sensitivities are minimized to obtain higher accuracy and wider range of tuning compared to other known variable filters. All theoretical results derived are verified experimentally.**

Keywords – **IIR Digital filters, Variable/tunable digital filters, Sensitivity, Tuning accuracy, Cascade realization**.

I. INTRODUCTION

Variable IIR filters are usually designed by employing the spectral (allpass) transformations of Constantinides (TC) [1], [2]. But when the prototypes are IIR filters, delay-free loops appear after the TC. Due to the attempts to eliminate these delay-free loops, no precise, without limitations, real-time tuning of IIR filters is known until now – all methods are approximate and valid only in a narrow range of values of the tuned parameter and over some limited frequency range. Most of the known methods are based on truncated Taylor series expansions, applied on real or complex coefficient parallel-allpass-structure [3] (called MNR-method after the names of the authors Mitra, Neuvo and Roivainen) or on cascade [4] and wave [5] realizations. The MNR-method is considered as the best known, but we have shown in Ref. [6] that the magnitude characteristics are degrading even when the LP/HP (lowpass/highpass) filter cutoff frequency or the bandpass and bandstop (BP/BS) bandwidth (BW) are tuned over a very limited frequency range. We have increased considerably the range and the accuracy of tuning by introducing a sensitivity minimization as an additional design step [6]. We have developed also a new approach [7], [8], based on a cascaded connection of several identical sub-filters. It permits an easy tuning of the cutoff frequency of the LP filter without having to use TC and truncated Taylor series expansions when using sub-filters of first or second order. We have developed and investigated [8],[9] such tunable sub-filter structures (of first and second order) with a very high tuning accuracy for narrow-band realizations.

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In this work we propose to realize the sub-filters of higher than second order as parallel allpass structures and investigate the applicability and the merits of such an approach.

II. APPROXIMATION USING A PRODUCT OF SEVERAL EQUAL TRANSFER FUNCTIONS

Our new method of design and realization of variable digital filters (VDF) is based on the usage of several cascaded identical filter blocks, each of them providing a very simple tuning of a given frequency parameter by varying a single multiplier coefficient. We are concerned with development of approximation procedures meeting only lowpass filter specifications. Variable BP/BS filters are obtained then by applying the constrained TC [1][2] on the variable LP filter. This transformation provides also an independent tuning of the central frequency, while the tuning of the cutoff frequency of the prototype LP filter is varying the BW of these BP/BS filters.

The magnitude specifications of the desired LP variable filter are: pass-band (PB) from 0 to ω_p (for digital filters) or Ω_p (for analog), stop-band (SB) from ω_s or Ω_s to infinity, maximum variation of the PB attenuation A_p, dB and minimum SB attenuation A_s, dB. And we have to find a total transfer function (TF) $H(z)$ (digital) or $T(s)$ (analog) represented as a product of N equal individual TFs $H_i(z)$ or $T_i(s)$, each of them of order n:

$$H(z) = H_i^N(z), \qquad T(s) = T_i^N(s), \tag{1}$$

$$H_i(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots\ a_n z^{-n}}{1 + b_1 z^{-1} + b_2 z^{-2} + \dots\ b_n z^{-n}}, \tag{2}$$

$$T_i(s) = \frac{a_0 + a_1 s + a_2 s^2 + \dots\ a_m s^m}{b_0 + b_1 s + b_2 s^2 + \dots\ b_n s^n},\ m \le n. \tag{3}$$

These TFs might be of Butterworth, Chebyshev or elliptic type and in the process of design we have to determine the minimal number N of the individual TFs $H_i(z)$ or $T_i(s)$, necessary to meet the specifications with given (selected) type (maximally flat, equiripple or other) and order n. A step-by-step design procedure for this is given in [7][8], it is performed in the analog domain and is using the popular in the classical filter theory Characteristic function $k(\Omega)$.

An approximation using N equal terms is far from optimal and there are many limitations that have to be clearly defined. These limitations depend on the type of the individual TF and on the selected approximation parameters. It might be even impossible to meet some difficult filter specifications doesn't matter how high the number N is taken. These limitations are investigated in details in [7][8]. Here we need a more general evaluations taking into account the order n of the individual TFs and some simple parameter describing how difficult the specifications are. We choose to use the "rectangularity coefficient" r (calculated in s- or in z-domain) as such parameter:

$$r = \frac{\Omega_s}{\Omega_p} \quad \text{or} \quad r = \frac{\tan \frac{\omega_s \tau}{2}}{\tan \frac{\omega_p \tau}{2}}, \tag{4}$$

where τ is the sampling interval and r is taking values within the limits $1 \le r < \infty$ with r=1 for an ideal LP filter.

If $A_{s_{\max}}$ is the highest value that A_s can achieve at its points of minimum (in the stop-band) for a given approximation and values of n while $N \to \infty$, we can derive very handy formulae connecting only A_p, r and $A_{s_{\max}}$ and permitting the most general possible way of comparing different approximations.

For Butterworth-type of individual TF we get (starting from the results in [7][8])

$$A_{s_{\max}} = A_p r^{2n}, \tag{5}$$

which is used to calculate the curves shown in Fig. 1.

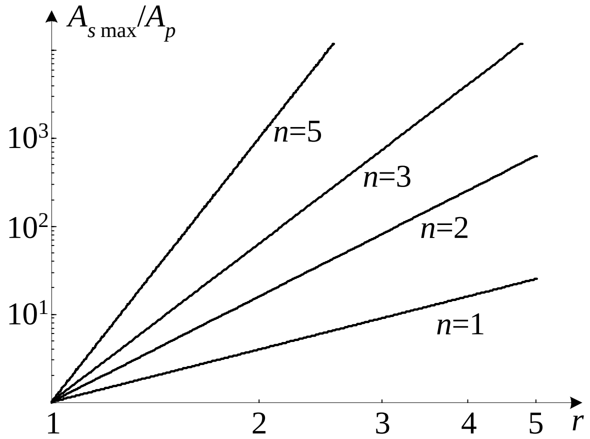

Fig. 1 Upper limits of the SB attenuation for different order n of the individual Butterworth type transfer functions

It is clear from Fig. 1 that the ratio $A_{s_{\max}} / A_p$ cannot exceed the value of 100 even for very easy filter specifications (r=3, for example) if we use an unlimited number of Butterworth-type second-order sections as individual sub-filters.

If the individual TFs are of Chebyshev type, we obtain the following very general expression

$$A_{s_{\max}} = A_p \,\mathrm{ch}^2(n \,\mathrm{Arch}\, r). \tag{6}$$

A family of curves, obtained by using Eq. (6), is shown in Fig. 2. It is seen once again that in order to obtain a filter with quite a high selectivity (r<2), it will be necessary to use individual sub-filters of order higher than second even when these sub-filters are with Chebyshev type of TF. These filters are, on the other hand, much more capable, compared to the filters with Butterworth individual TF (which is very easy to anticipate).

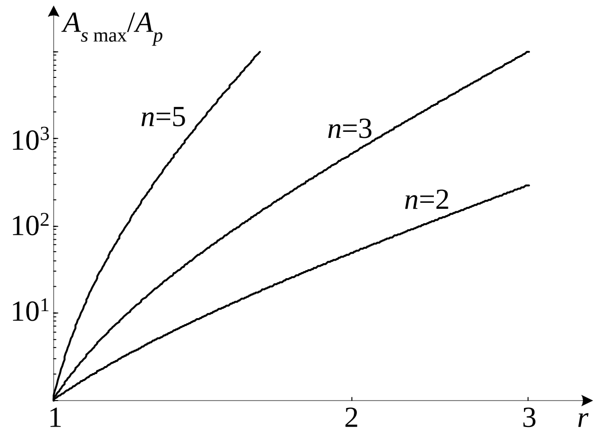

Fig. 2 Upper limits of the SB attenuation for different order n of the individual Chebyshev type transfer functions

It is impossible to derive a single compact formula for the case of elliptic type of individual TFs. In [7][8] we have derived very general expressions (different for n even and n - odd), which are used to obtain the following more concrete formulae:

$$A_{s_{\max}} = \frac{A_p r^4}{\mathrm{sn}^8\left(\frac{K(1/r)}{2}\right)} = A_p r^4 \left(1 + \sqrt{1 - \frac{1}{r^2}}\right)^4 \quad \text{for } n = 2, \tag{7}$$

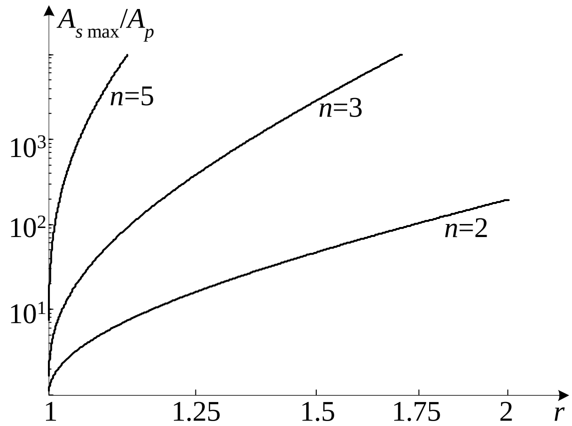

Fig. 3 Upper limits of the SB attenuation for different order n of the individual elliptic type transfer functions

$$A_{s_{\max}} = \frac{A_p r^8}{\mathrm{sn}^8\left(\frac{K(1/r)}{3}\right)} \quad \text{for } n = 3, \tag{8}$$

$$A_{s\,\max} = \frac{A_p r^{12}}{\mathrm{sn}^8\left(\frac{K(1/r)}{5}\right)\mathrm{sn}^8\left(\frac{3K(1/r)}{5}\right)} \quad \text{for } n = 5, \tag{9}$$

where sn is the Jacobi's "sinus elliptic" function and $K(1/r)$ – the function of Jacobi, calculated as a complete elliptic integral of first kind (see [8] for details).

It is seen from Fig. 3 that almost any kind of difficult filter specifications will be met if elliptic transfer functions with order higher than second are used to realize the individual sub-filters.

In Fig. 4 the upper limits of the stop-band attenuation, achievable with identical first- (IFOS) and second-order sections (ISOS), are given in order to have a base for comparisons. These limits are, in fact, much lower because instead of $N \to \infty$, we are using only several IFOS or ISOS.

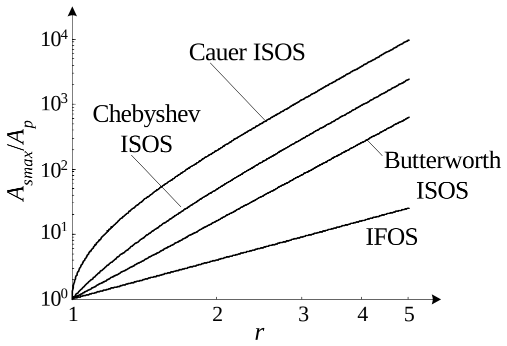

Fig. 4 Upper limits of the SB attenuation with IFOS or ISOS

III. REALIZATION OF THE HIGHER ORDER SUB-FILTERS

We have developed and investigated in [8],[9] excellent variable first and second order filter sections for IFOS and ISOS with truly independent high accuracy tuning without using any truncated Taylor series expansions. It appeared, however, to be impossible to synthesize such structures with higher than second order. No such structures have been found also in the literature. In order to solve the problem, we have to accept the MNR approach (to use truncated Taylor series expansions) [3] but only for the realization of the sub-filters. It will provide an easy tuning of the cutoff frequency of the LP individual filters by varying a single multiplier coefficient. But as our structure is a cascade of several low-order order sections (even though obtained as parallel-allpass-structures) , it will have much lower sensitivity in the stop-band, compared to that of the totally parallel allpass structure, which is behaving really badly, as shown in [6]. Next, we propose to use our approach advanced in [6], namely to introduce an additional step in the design of the sub-filers, consisting of a sensitivity minimization (within the frequency range of interest) of the first- and second-order allpass sections used to realize these sub-filters. There is another problem with the parallel-allpass-structure – it cannot realize even-order LP TFs with real coefficients. We could still use it to realize our even-order sub-filters with complex coefficients, but they will be quite impractical (complex structures with low tuning accuracy). Thus we shall concentrate mainly on the realization of third- and fifth-order sub-filters, putting, in fact, out of consideration only sub-filters of fourth-order (it is also impractical to use sub-filters of order higher than fifth, because we lose then all the merits of the cascade realization).

Once it is decided, the design problem is reduced to development of convenient first- and second-order allpass sections with low sensitivities for the frequency range over which the filter will be tuned. We have shown in [6] that for the most common case – variable narrow-band LP filter (having all his TF poles near z=1) – our very low sensitivity allpass sections, shown in Fig. 5b and Fig.6b and named respectively ST and LS, are by far the best known.

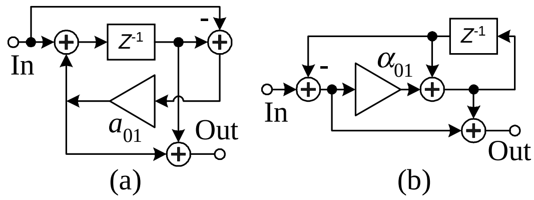

Fig. 5. First-order all-pass sections: (a) MH section, (b) ST section

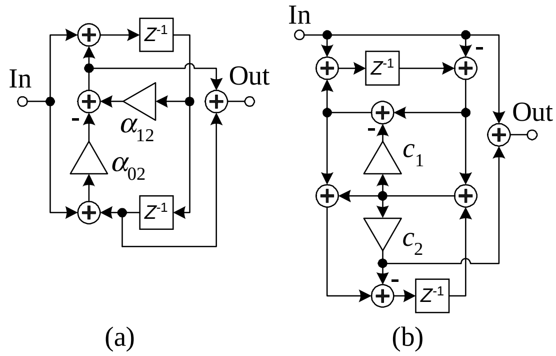

Fig. 6. Second-order all-pass sections: (a) MH2B, (b) LS

They realize the following TFs

$$H(z)_{ST} = \frac{z^{-1} - (1-\alpha_{01})}{1-(1-\alpha_{01})z^{-1}}, \tag{10}$$

$$H(z)_{LS} = \frac{1\text{-}c_2 - (2-2c_1-c_2)z^{-1} + z^{-2}}{1-(2-2c_1-c_2)z^{-1} + (1\text{-}c_2)z^{-2}}. \tag{11}$$

In Figs. 5a and 6a we have shown the allpass sections most often mentioned in the literature [3] – these of Mitra and Hirano, called respectively MH and MH2B.

The design of third and fifth-order sub-filters as parallel-allpass-structures (Fig. 7a) might be very simple and straightforward. Once $H_i(z)$ (2) is determined, its poles p_0 and $p_{1,2} = r_1 e^{j\theta_1}$ (for n=3) and p_0, $p_{1,2} = r_1 e^{j\theta_1}$, $p_{3,4} = r_3 e^{j\theta_3}$ (for n=5) are readily known. Then, for n=3, $A(z)$ (Fig. 7a) will be a second-order allpass TF with poles $p_{1,2}$, while $B(z)$ will be of first-order with a real pole p_0. The denominator polynomials of $A(z)$ and $B(z)$ are thus easily found and the corresponding nominators are their mirror image polynomials.

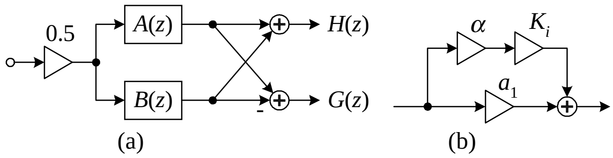

Fig. 7. Parallel-allpass-structure-based realization (a) and variable coefficient realization (b)

For n=5 $A(z)$ will be a third-order allpass TF with poles p_0 and $p_{3,4}$, while $B(z)$ will be of second-order with poles $p_{1,2}$ (supposing $\theta_3 > \theta_1$).

For n=3 and usage of the ST and LS sections it produces:

$$\alpha_{01} = 1 - p_0\,,\ c_1 = 0.5(1 - 2r_1\cos\theta_1 - r_1^2),\ c_2 = 1 - r_1^2. \quad (12)$$

Each multiplier then is made tunable as shown in Fig. 7b by adding a parallel branch αK_i, where α is the tuning factor and K_i is calculated as

$$K_\alpha = \alpha_{01}(\alpha_{01} - 2); \quad (13)$$

$$K_{c1} = c_1(2c_1 + c_2 - 4); \qquad K_{c2} = c_2(2c_1 + c_2 - 2). \quad (14)$$

Similar formulae are easily derived for the design of ST and LS for n=5.

IV. EXPERIMENTS

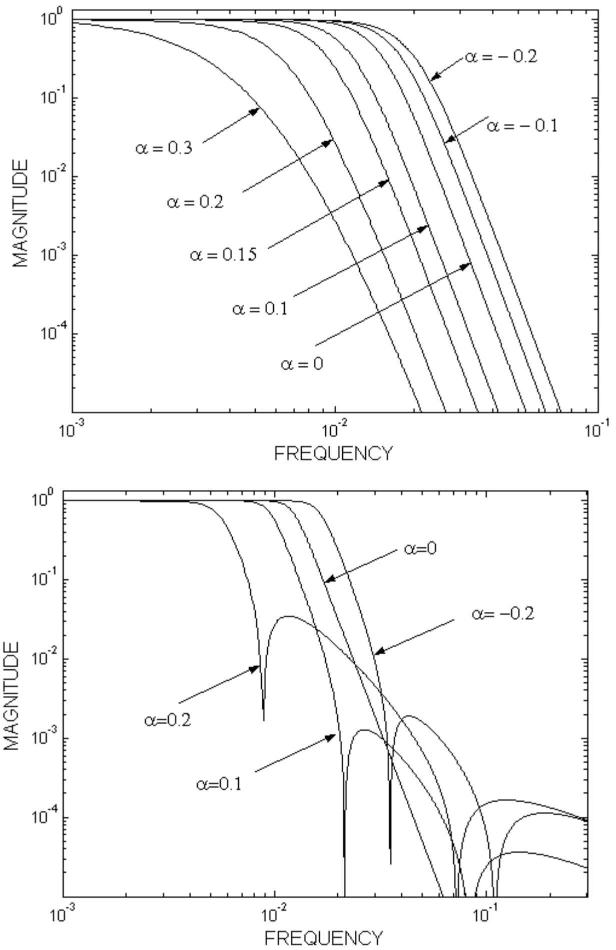

Fig. 8. Tuning of a Butterworth variable filter realized as a cascade of three 3rd order sub-filters (a) and as a MNR-filter (b)

In order to verify the proposed approach and the derived expressions, we have designed and simulated many variable filters based on cascades of identical third or fifth order sub-filters. In Fig. 8a, b the results are shown for filter meeting the following specifications: ω_p=0.01 (tunable from 0.005 to 0.015), ω_s=0.03, A_p=2 dB and A_s=55 dB and using a Butterworth approximation. Our method is producing a 9th order TF, realized as a cascade of 3 sub-filters of 3rd order while the MNR-filter is of 7th order and is realized with MH and MH2B sections. It is seen in Fig. 8 that our filter is easily tuned even over wider frequency range while the SB magnitude of the MNR-filter is destroyed and gone out of the specifications and some transmission zeros are appearing even when $\alpha < |0.1|$.

V. CONCLUSIONS

A new approach to design high tuning accuracy variable IIR filters as a cascade of N identical sub-filters of order higher than second is proposed in this paper. The identical sub-filters are realized as parallel all-pass structures and their sensitivities are minimized to obtain higher accuracy and wider range of tuning compared to other known variable filters. The limitations of the new approach are investigated and simple design procedures are proposed. All theoretical results and the superiority of our method are verified experimentally.

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