Input-output based discrete-time disturbance estimator using sliding mode approach

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***Abstract* – This paper proposes input-output based discrete-time disturbance estimator structure, in which conventional passive digital filters are replaced with an active digital sliding mode controlled subsystem. In ideal sliding mode, complete disturbance rejection occurs and plant output follows nominal system. Estimator robustness is actively gained by providing sliding mode existence conditions. Simulation results show effectiveness of the proposed disturbance estimator.**

***Keywords* – Disturbance estimator, external disturbances, model uncertainties, sliding mode, robustness.**

I. Introduction

In almost every control environment the presence of external disturbances and model uncertainties is inevitable, which has significant impact on the performance of a controller. Hence, the performance of a control system is evaluated through its disturbance rejection ability and robustness to model uncertainties.

One approach in handling external disturbances and model uncertainties, which is usually regarded as an equivalent or generalized disturbance, is an employment of disturbance observers. The concept of disturbance observer is that the disturbance action can be efficiently compensated by feedback of the observed disturbance. Thus, disturbance observer enables real plant to behave like the nominal plant.

Generally, there are two methods in a disturbance observer design. The first is the design of a state space disturbance observer, which requires disturbance model to be employed in an augmented state observer. This approach is suitable for simple disturbances, such as bias and periodic disturbances. It is hardly used for arbitrary disturbances. However, it is proposed in [1] a design of discrete-time robust control system, containing a state space disturbance observer, in which modeling of disturbances is not required.

The other method of a disturbance observer design is based on transfer function approach. Conventionally, the disturbance observer is designed using inverse dynamic of a plant, [2]. Disturbance observer efficiency is dependent on the design of the so-called Q filter, which is essentially utilized for the causality of the observer. Q filter determines robustness and disturbance rejection performance, which is proved to be contradictory requirements, [3].

This approach may be viewed from the aspect of internal model concept. The IMPACT (Internal Model Principle And Control Together) structure incorporates both internal models of nominal plant and disturbances, in order to obtain rejection of known class of disturbances, robustness to parameter perturbations and desired dynamic response. Compensation of an arbitrary external disturbance may be obtained by on-line adaptation of disturbance internal model, [4]. A simplified IMPACT controlling structure [5] may be treated as a disturbance estimator.

As a nonlinear robust control strategy, which is theoretically insensitive to model uncertainties and external disturbances, variable structure systems (VSS) with their associated sliding mode behavior are very attractive for perturbed system control. Sliding mode control (SMC) essentially utilizes a switching control law to drive the state of the concerned system to a predefined sliding surface in the state space and to maintain the sliding motion along the surface into the equilibrium point [6]. Due to digital realization of VSC algorithms, analysis of discrete-time sliding mode control (DSMC) systems shows that in general only quasi-sliding modes can be achieved, i.e., the system trajectories are in a small bounded vicinity of the sliding surface.

Among many versatile applications, SMC has found its role in a system state observation [6], and thereby in disturbance observers. Discontinuous disturbance estimators, where VSS equivalent control theory is employed in disturbance estimation, are proposed in [7,8].

This paper proposes sliding mode controlled input-output based discrete-time disturbance estimator. Conventional passive digital filters responsible for estimator robustness and disturbances rejection are replaced with an active DSMC structure, due to its emphasized robustness property. If an ideal sliding mode is established, complete disturbance rejection occurs and plant output follows nominal system. Estimator robustness (stability) against model uncertainties is actively gained by providing sliding mode existence conditions. Thus, robustness and good external disturbances rejection property are no longer contradictory requests.

II. DSMC Based Disturbance Estimator

Consider a single-input single-output continuous time dynamic system described by the state space model

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$$\dot{\mathbf{x}}(t)=\mathbf{Ax}(t)+\mathbf{b}u(t)+\mathbf{j}v(t),\ y(t)=\mathbf{cx}(t), \quad (1)$$

where the state $x\in R^n$, the control $u\in R^1$, the external disturbance $v\in R^1$ and the output $y\in R^1$; $\mathbf{A}$, $\mathbf{b}$, $\mathbf{j}$, $\mathbf{c}$ are the constant matrix and vectors of appropriate dimensions. The discrete time representation of the dynamic system (1) is obtained by applying u through a zero-order-hold,

$$\mathbf{x}(k+1)=\mathbf{Ex}(k)+\mathbf{f}u(k)+\mathbf{w}(k),\ y(k)=\mathbf{cx}(k), \quad (2)$$

where T is a sampling period and

$$\mathbf{E}=e^{\mathbf{A}T},\ \mathbf{f}=\int_0^T e^{\mathbf{A}t}\mathrm{d}t\mathbf{b},\ \mathbf{w}(k)=\int_0^T e^{\mathbf{A}t}\mathbf{j}v(kT+T-t)\mathrm{d}t. \quad (3)$$

By assuming zero initial conditions, Z-transform of (2) yields

$$Y(z)=\mathbf{c}(z\mathbf{I}-\mathbf{E})^{-1}\mathbf{f}U(z)+\mathbf{c}(z\mathbf{I}-\mathbf{E})^{-1}\mathbf{W}(z). \quad (4)$$

Using Eq. (4), the system output y in terms of the control u and the external disturbances d may be expressed as

$$y(k)=G(z)u(k)+d(k), \quad (5)$$

where

$$G(z)=\frac{z^{-1}B(z^{-1})}{A(z^{-1})},\ d(k)=\frac{z^{-1}\mathbf{H}(z^{-1})}{A(z^{-1})}\mathbf{w}(k),$$
$$A(z^{-1})=z^{-n}\det(z\mathbf{I}-\mathbf{E}),\ B(z^{-1})=z^{-n+1}\mathbf{c}\,\mathrm{adj}(z\mathbf{I}-\mathbf{E})\mathbf{f}, \quad (6)$$
$$\mathbf{H}(z^{-1})=z^{-n+1}\mathbf{c}\,\mathrm{adj}(z\mathbf{I}-\mathbf{E})=\left[H_1(z^{-1})\quad H_2(z^{-1})\right].$$

The polynomials $A(z^{-1})$ and $B(z^{-1})$ are assumed to be stable and relatively prime, where z^{-1} denotes unit-delay operator.

The main concept is to compensate action of the equivalent disturbance, consisting of model uncertainties and external disturbance, by feedback of the observed equivalent disturbance, and thereby to obtain nominal model behavior. Consider the control structure proposed in Fig. 1., which is composed of the real plant (5) and the disturbance estimator in the local loop. The extraction of the equivalent disturbance q in the disturbance estimator is obtained using discrete transfer function of the nominal model $G_n(z)$. The mismatch between the real plant and nominal model inevitably exists due to uncertainties of the plant parameters. The real plant may be reliably described by

$$G(z)=G_n(z)(1+\delta G(z)), \quad (7)$$

where its perturbation is limited by the multiplicative bound of uncertainties $\left|\delta G(e^{j\omega T})\right|\le\gamma(\omega),\ \omega\in[0,\pi/T]$. According to Eq. (7) and Fig. 1., the extracted equivalent disturbance is

$$q(k)=d(k)+G_n(z)\delta G(z)u_k(k). \quad (8)$$

In order to improve disturbance estimator robustness, an active controlling structure is employed instead of conventionally used passive digital filter. A reasonable choice is a DVSC system due to its emphasized robustness property. Signal $\hat{q}$ is an estimation of the compensated equivalent disturbance portion. If DSMC ensures $\hat{q}=q$ (an ideal sliding mode occurs), the control signal may be described as $u_{sm}(k)=G_n^{-1}(z)q(k)$. It can be easily proved that the system output behaves as a nominal model $y(k)=G_n(z)u(k)$. Since DSMC systems enables only quasi-sliding mode, certain but small bounded error between q and $\hat{q}$ will exist, whose value depends on the employed control algorithm.

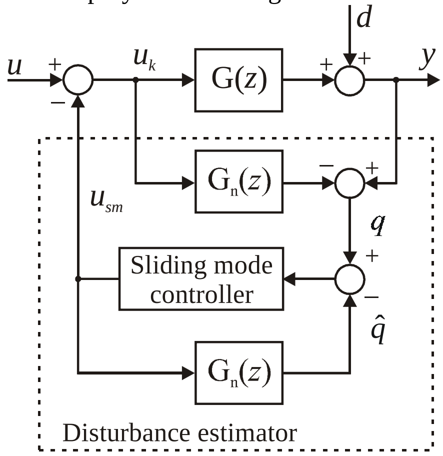

Fig. 1. Structure of DSMC based disturbance estimator

Robustness against model uncertainties is actively gained by providing sliding mode existence conditions. Thus, robustness and good external disturbances rejection property are no longer contradictory requests. From the control design aspect, the proposed method of equivalent disturbance compensation may be treated as a discrete-time tracking control problem with measurable but unknown in advance reference signal $q(k)$. Since digital controller steers a nominal model not a real plant, all state variables are available. This enables variety of DSMC algorithms to be utilized, which successfully handle this control task.

III. DSMC Design

Robust chattering-free DSMC algorithm [9] based on state-space approach is chosen for the digital sliding mode controller. Controller design is demonstrated on the second order system. Consider a discrete-time nominal model.

$$\mathbf{x}(k+1)=\mathbf{Ex}(k)+\mathbf{f}u_{sm}(k),\ y(k)=\mathbf{cx}(k), \quad (9.a)$$

$$\mathbf{E}=e^{\mathbf{A}T}=\begin{bmatrix}e_{11} & e_{12}\\ e_{21} & e_{22}\end{bmatrix},\ \mathbf{f}=\int_0^T e^{\mathbf{A}t}\mathrm{d}t\mathbf{b}=\begin{bmatrix}f_1\\ f_2\end{bmatrix},\ \mathbf{c}=\begin{bmatrix}1 & 0\end{bmatrix}. \quad (9.b)$$

It is more convenient to consider tracking error dynamics, which may be obtained using coordinate transformation. A new state space vector $z(k)=[z_1\quad z_2]^{\mathrm{T}}$ is introduced, defined as $z_1(k)=q(k)-x_1(k)$, $z_2(k)=z_1(k)-z_1(k-1)$, $\hat{q}(k)=x_1(k)$, which has to be driven to zero by the control force $u_{sm}(k)$. The discrete-time model (9) is transformed to

$$\mathbf{z}(k+1)=\mathbf{E}_x\mathbf{z}(k)+\mathbf{f}_x u_{sm}(k)+\mathbf{h}x_2(k)+\mathbf{g}q(k+1)+\mathbf{p}q(k),$$
$$\mathbf{E}_x=\begin{bmatrix}e_{11} & 0\\ e_{11}-1 & 0\end{bmatrix},\mathbf{f}_x=\begin{bmatrix}-f_1\\ -f_1\end{bmatrix},\mathbf{h}=\begin{bmatrix}-e_{12}\\ -e_{12}\end{bmatrix}, \quad (10)$$
$$\mathbf{g}=\begin{bmatrix}1\\ 1\end{bmatrix},\mathbf{p}=\begin{bmatrix}-e_{11}\\ -e_{11}\end{bmatrix}.$$

The basic sliding mode control philosophy comprises global stabilization of the control system by steering system states onto predefined sliding surface, and maintaining subsequent motion along that surface into the state space origin. Let the switching function be

$$s(k) = \boldsymbol{\sigma}\mathbf{z}(k),\ \boldsymbol{\sigma} = [\sigma \quad 1] \tag{11}$$

Control law $u_{sm}(k)$ should be determined which provides the desired motion constrained into the quasi-sliding domain, defined as small bounded vicinity of the sliding line $s(k) = 0$. Sliding line parameter σ defines sliding mode dynamics, that is, compensation dynamics of the equivalent disturbance. An adequate adoption of the slope σ should provide stable system eigenvalues with desired dynamics.

According to Eqs. (11) and (10), system motion toward the sliding line is presented by

$$\begin{aligned} s(k+1) = s(k) + \boldsymbol{\sigma}[(\mathbf{E}_x - \mathbf{I})\mathbf{z}(k) + \mathbf{f}_x u_{sm}(k) \\ + \mathbf{h}x_2(k) + \mathbf{g}q(k+1) + \mathbf{p}q(k)]. \end{aligned} \tag{12}$$

Let the control signal be in the form

$$\begin{aligned} u_{sm}(k) = \frac{-1}{\boldsymbol{\sigma}\mathbf{f}_x}\{\boldsymbol{\sigma}[(\mathbf{E}_x - \mathbf{I})\mathbf{z}(k) + \mathbf{h}x_2(k) + (\mathbf{p} + 2\mathbf{g})q(k) \\ - \mathbf{g}q(k-1)] + \operatorname{sgn}(s(k))\min(|s(k)|, \alpha T)\}, \end{aligned} \tag{13}$$

under constraint $\boldsymbol{\sigma}\mathbf{f}_x \neq 0$. Control signal has two modes. The first mode, nonlinear with respect to $s(k)$ and active outside the region $|s(k)| < \alpha T$, ensures reaching the region in a finite number of steps; afterwards, the second linear mode provides reaching the quasi-sliding domain in one step. Thus, a discrete-time sliding mode is achieved employing smooth control signal, [9].

To prove that the proposed control law (13) ensure above described motion, and to determine the related switching gain as well as the width of the quasi-sliding domain, the following two supportive lemmas are given.

Lemma 1, [10]**:** *System trajectories described by*

$$s(k+1) = s(k) + f(k+1) - \alpha T \operatorname{sgn}(s(k)) \tag{14}$$

where $f(k)$ *is some bounded function and* $\alpha > |f(k+1)|/T$, $\forall k \in N$, *will reach region* $|s(k)| < \alpha T$ *from any initial state* $s(0)$ *in finite number of steps.*

Lemma 2, [10]**:** *If* $q(t)$ *is an arbitrary smooth function with bounded time derivative* $|\ddot{q}(t)| \leq R$, *then the following discrete time inequality holds* $|q(kT+T) - 2q(kT) + q(kT-T)| \leq 2T^2R$.

The designed discrete time variable structure controller is summarized by the following theorem.

Theorem: *Consider the discrete time system (10) under the action of the control signal (13), where the switching function is chosen as (11). If the switching gain satisfies*

$$\alpha > 2TR|\boldsymbol{\sigma}\mathbf{g}|, \tag{15}$$

the discrete-time quasi-sliding mode will arise from any initial state in a finite number of steps in the domain defined by

$$|s(k+1)| \leq 2T^2R|\boldsymbol{\sigma}\mathbf{g}|. \tag{16}$$

Proof: Since the system motion starts outside the region $|s(k)| < \alpha T$, by virtue of Eq. (13), Eq. (12) becomes

$$s(k+1) = s(k) - \alpha T \operatorname{sgn}(s(k)) + \boldsymbol{\sigma}\mathbf{g}[q(k+1) - 2q(k) + q(k-1)]. \tag{17}$$

Under condition (15), according to lemmas 1 and 2, system trajectories (17) will reach the region in a finite number of steps. Governed further by the linear phase of the control (13), system motion is described by

$$s(k+1) = \boldsymbol{\sigma}\mathbf{g}[q(k+1) - 2q(k) + q(k-1)], \tag{18}$$

indicating that the quasi-sliding domain is entered in the next step, whose width (16) is proved by (18) and lemma 2. □

It is evident from (16) that $s(k+1) = 0$ if $\ddot{q}(t) = 0$, implying that the proposed sliding mode based disturbance estimator completely rejects step and ramp like equivalent disturbances. In all other cases, when $R \neq 0$, it provides $O(T^2)$ accuracy.

VI. Simulation Results

The effectiveness of the proposed DSMC based disturbance estimator has been investigated by simulation. The plant is a DC motor, whose continuous-time model (1) is defined with

$$\mathbf{x} = \begin{bmatrix} \omega \\ i_r \end{bmatrix}, \mathbf{A} = \begin{bmatrix} 0 & k/J \\ \frac{-k}{L_r} & \frac{-R_r}{L_r} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ \frac{1}{L_r} \end{bmatrix}, \mathbf{j} = \begin{bmatrix} \frac{-1}{J} \\ 0 \end{bmatrix}, \mathbf{c}^{\mathrm{T}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{19}$$

The nominal plant (model) parameters are: $R_{rn} = 1\Omega$, $B_n = 0$ $L_{rn} = 2.5 \cdot 10^{-3} H$, $J_n = 3.267 \cdot 10^{-3} kgm^2$, $k_n = 0.33$. Using the discrete-time model (2) with the sampling time $T = 10^{-3} s$ the following nominal discrete transfer function is obtained according to Eqn. (6)

$$G_n(z) = \frac{z^{-1}(0.0177384 + 0.0155257z^{-1})}{1 - 1.65934z^{-1} + 0.67032z^{-2}}. \tag{20}$$

The plant is subjected to parameter and external disturbances. Load torque, shown in Fig. 2., acts as an external disturbance. Model uncertainty, i.e., the mismatch between the real plant and the nominal model, is defined by the following values: $R_r = R_{rn} \cdot 1.3$, $L_r = L_{rn} \cdot 1.2$, $J = J_n \cdot 2$, $k = k_n \cdot 1.1$. Consequently, the real plant discrete transfer function is

$$G(z) = \frac{z^{-1}(0.00805062 + 0.00696901z^{-1})}{1 - 1.64289z^{-1} + 0.648344z^{-2}}. \tag{21}$$

Parameters of the digital sliding mode controller have been selected as: $\alpha = 10$, $\sigma = 10$. The main control loop contains linear digital controller $G_r(z) = (1 - 0.985z^{-1})/(1 - z^{-1})$, which involves an integral action. The controlled DC motor is subjected to a step angular velocity reference.

Step responses of the controlled systems with and without disturbance estimator are shown in Fig. 2, as well as the load torque. Due to the integral action of the main loop controller, the uncompensated system output $y_0(t)$ has zero error only in the section of step like disturbance action. In other sections, system error is significant. The compensated system output $y(t)$ demonstrates an excellent disturbance rejection performance.

According to the enlarged scale of the switching function plotted in Fig. 3., it is confirmed the complete rejection of step and ramp like disturbances by the proposed disturbance estimator. The deviation from the zero value in other sectors is significantly small.

Error signal of he compensated system is shown in Fig. 4. in the enlarged scale also. It may be noticed that the system has no error in the case of step, ramp and parabolic like disturbance action. This is a result of the combined action of the disturbance estimator and the main controller with integral action.

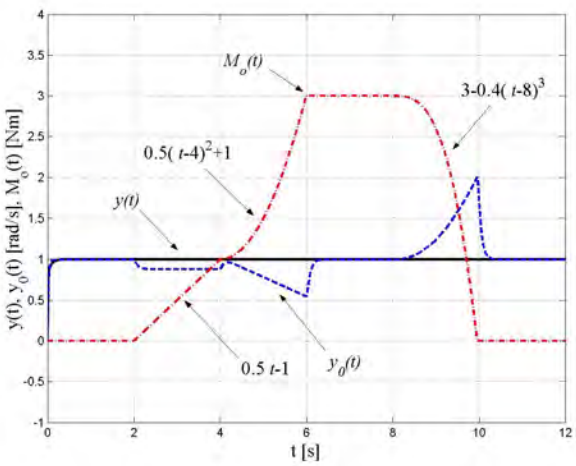

Fig. 2. Step responses of uncompensated system $y_0(t)$, compensated system $y(t)$; and external disturbance $M_o(t)$.

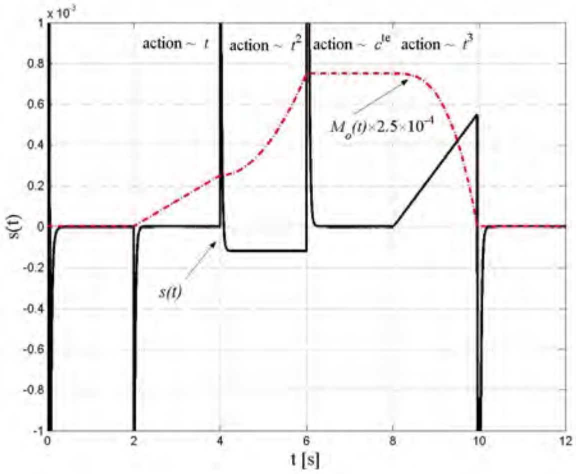

Fig. 3. Switching function $s(t)$.

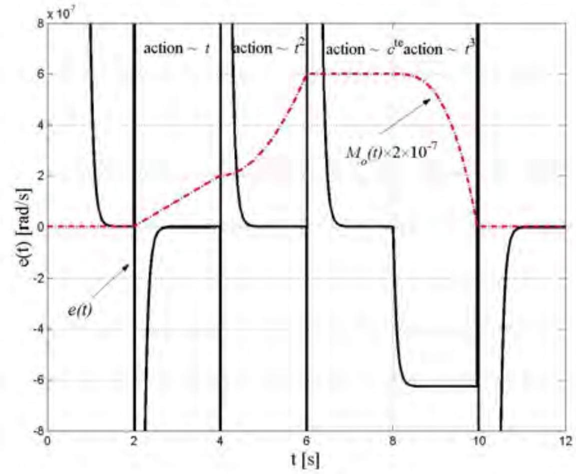

Fig. 4. Error signal $e(t)$ of the compensated system.

V. Conclusion

The proposed input-output based discrete-time disturbance estimator structure, where conventional passive digital filters are replaced with active DSMC subsystem, inherits recognized sliding mode properties with respect to internal and external perturbations. Estimator robustness against model uncertainties is actively gained by providing sliding mode existence conditions. Thus, robustness and good external disturbances rejection property are no longer contradictory requests.

From the control design aspect, the proposed equivalent disturbance compensation method may be viewed as a discrete-time tracking control problem with measurable but unknown in advance reference signal. Sliding mode digital controller governs a nominal model not a real plant, where all state variables are available, offering variety of DSMC algorithms to be utilized. The applied state-space DSMC algorithm enables complete rejection of step and ramp like equivalent disturbances. In all other cases it provides an $O(T^2)$ accuracy.

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