# Neural network for polar image processing 

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#### Abstract

The wide application of neural network in image processing is less developed in polar image direction. The polar image transformations give the specific advantages for some classes of images, which are composed of objects with circular performances. It is the goal of this article to propose a suitable neural network structure for polar image processing. Keywords: Polar Images, Neural network, Image processing.


## I. Introduction

The polar images are very important for some applications in which it is necessary to select the objects with circular performance [1]. The classical methods for image segmentation can do this selection, bur it is not possible to use the specific features of circular objects in images. The other direction of polar image processing are the active image processing, are the active image processing systems [2], in which it is used the eye point of view as a origin of image sampling and processing. These systems are similar and a simple model of some animal eyes system [3]. The application of these methods of polar image processing is very useful in robotic systems.

The goal of this article is to propose a suitable and efficient structure of a neural network for polar image processing.

## II. THE POLAR IMAGES REPRESENTATION

Most of the polar images processing systems used a nonuniform sampling as a model of an animal eye. This method give an image representation with densest sample points in the centre and regions of interest, whilst maintaining a wide field of view, but it requires camera movements to allow such regions to be selected. This disadvantages of this is the necessarily of special cameras. The non-uniform method of sampling, which is used very often, is log-polar sampling. In log-polar sampling, pixels are indexed by ring number $R$ and wedge number $W$, related to ordinary $x, y$ image coordinates by the mapping:

$$
r=\left[\left(x-x_{c}\right)^{2}+\left(y-y_{c}\right)^{2}\right]^{1 / 2}
$$

$$
\begin{equation*}
\theta=\tan ^{-1}\left(\frac{y-y_{c}}{x-x_{c}}\right) \tag{1}
\end{equation*}
$$

(2)

[^0]$$
R=\frac{\left(n_{r}-1\right) \log \left(r / r_{\min }\right)}{\log \left(r_{\max } / r_{\min }\right)}
$$
(3)
$$
W=\frac{n_{w} \theta}{2 \pi}
$$
(4)
where
$r, \theta$ are polar coordinates;
$x_{c}, y_{c}$ - position of the centre of the log-polar sampling pattern;
$n_{r}$ and $n_{w}$ - the numbers of rings and wedges, respectively;
$r_{\text {min }}$ and $r_{\text {max }}$ - radii of the smallest and largest rings of samples.

A log-polar sampling image is one whose samples are centered on points mapping to integral $R$ and $W, R \in\left\{0, \ldots, n_{r}-1\right\}, W \in\left\{0, \ldots, n_{w}-1\right\}$. this separation. The separation between sample points is proportional to distance from the sample centre. This arrangement appears to be approximated by the ganglion cells of the primate retina and the visual cortex. In this representation, image expansion and rotations about $\left(x_{c}, y_{c}\right)$ become shifts in $R$ and $W$, but image transaction has a more complex effect.

In order to keep a pixel's nearest neighbors in orthogonal directions at approximately equal distance from it, the following constraint is needed:

$$
\begin{equation*}
r_{\min }=r_{\max } \ell^{-2 \pi\left(n_{r}-1\right) / n_{w}} \tag{5}
\end{equation*}
$$

Log-polar images are displayed on orthogonal $(R, W)$ axes, but this is misleading since it leads them to be regarded as "distorted" representations.

It is possible to define:

$$
\begin{equation*}
\rho=\log r=-\log \cos \theta \tag{6}
\end{equation*}
$$

The graphical relation between $\rho$ and $\theta$ is shown in Fig.1.


Fig.1. Graphical relation between $\rho$ and $\theta$.
The objects representation in log-polar images is very important, because it give the possibility to decide, which objects are easy to select and separate in log-polar images. As an example of an object representation, in log-polar images here is a representation of straight line:

$$
\begin{equation*}
\rho=\rho_{l}-\log \cos \left(\theta-\theta_{l}\right) . \tag{7}
\end{equation*}
$$

The peak of the convolution output will be at $\left(\rho_{l}, \theta_{l}\right)$. The cases for straight line $x=1$ in orthogonal Cartesian axes give from expression (7):

$$
\rho=-\log \cos \theta
$$

(8)
and is shown in Fig.1.
It is possible to perform an efficient convolution by multiplication in the Fourier domain. It is possible to find a closed - form expression for the Fourier transform of the straight line in log-polar space.

To find the Fourier transform, it is possible to taka a path integral along the line in log-polar space. Let $S$ be the standard line $\rho=-\log \cos \theta$ from the equation (8) with element $d \rho$ in $(\rho, \theta)$ space, then the integral is:

$$
\begin{equation*}
F\left(k_{\rho}, k_{\theta}\right)=\int_{s} \ell^{-i k_{\rho} \rho} \ell^{j k_{\theta} \theta} w(\rho, \theta) d s, \tag{9}
\end{equation*}
$$

where
$w(\rho, \theta)$ is the wiegnt factor to allow convergence:

$$
w(\rho, \theta)=(\cos \theta)^{1-\alpha} 0<\alpha<1
$$

(10)

The larger values of $\alpha$ make $F$ more localized to the minimum of $\rho$. An suitable value of $\alpha$ is 0.2 .

If substitute $d \theta / d s=\cos \theta$, the integral becomes:

$$
\begin{equation*}
F\left(k_{\rho}, k_{\theta}\right)=\int_{-\pi / 2}^{+\pi / 2}(\cos \theta)^{i k_{\rho}-\alpha} \ell^{-i k_{\theta} \theta} d \theta \tag{11}
\end{equation*}
$$

The general methods for polar images processing is based on using local logical on arithmetic operators for object feature selection (lines, edges, bars etc.). It is possible to propose a suitable structure of a neural network and to use the ability of the neural network to be learning for the object feature extraction in log-polar images. The structure of this proposed neural network is shown in Fig.2.

As an input image it is possible to use a conventional Cartesian image following by a log-polar transformation or an active camera input image which is directly represented in log-polar coordinates. The structure of polar neural network can be chosen from traditional types of neural networks: perception Hopfield, Kehonen etc. with appropriate layers, inputs and layers numbers. The neural network input can be represented as a two vectors:

$$
\begin{aligned}
& I M_{R}=\left[\begin{array}{l}
i r_{0} \\
i r_{1} \\
\cdot \\
\cdot \\
i r_{n_{r}-1}
\end{array}\right] R \in\left\{0, \ldots, n_{r}-1\right\} \\
& I M_{W}=\left[\begin{array}{l}
i w_{0} \\
i w_{1} \\
\cdot \\
\vdots \\
i w_{n_{w}-1}
\end{array}\right] W \in\left\{0, \ldots, n_{w}-1\right\}
\end{aligned}
$$

(13)
in accordance with equations (1), (2), (3) and (4). The learning process can be performed with the following rules: forward, back propagation, associative etc. and is used to collect the objects features as edges, lines etc. At the fig. 2 it is shown a last general block named feature, processing as an example to using the extracted the proposed neural network features. This feature processing can be made for a concrete application and output of this block can be used for a following post processing. But as a general necessary application of the output information it is shown at Fig. 2 with dashed line the definition and updating of origin $\left(x_{c}, y_{c}\right)$ of current input image if this is a real complete log-polar image processing system with an active controlled camera.

## IV. Test and conclusion

A Matlab simulation is prepared for the proposed neural network structure for log-polar images processing. Some of practical results of learning and classification with this neural network for some typical features in polar images edges, bars etc. are shown in Fig.3.


Fig.3. Practical results of neural network testing.
These practical results are only for a confirming the possibility of using such neural network structure and to continue its investigation, extension and real practical applications.

## REFERENCE:

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Fig.2. Structure of proposed neural network


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