Analysis of two-dimensional LMS Error -Diffusion Adaptive Filter

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Abstract - **An analysis of error-diffusion adaptive filter for image halftoning is presented. The filter coefficients are adapted on the basis of 2D LMS algorithm. The received results shown that the filter leads to more uniform values distribution in the homogeneous areas and more precise edges reproduction in the output bi-level image.**

Keywords - image quantization, error diffusion, adaptive filtration, LMS adaptation.

I. Introduction

Digital image halftoning is used to transform gray scale images into bi-level ones, which give the appearance of containing various shades of gray. Most widespread are the following two basic transforming techniques - "error diffusion" and "ordered dither" [1], [2], [3]. The use of error diffusion methods results in a better edge reproduction, but also to appearance of artefacts in the homogeneous image areas. This disadvantage is due to the constant values of weights of the two-dimensional error filter, which deals with image pixels in a certain causal window.

In order to minimize the image distortions an adaptive error-diffusion filter is presented. The coefficients of error filter are adapted with the help of generalized 2D LMS Widrow algorithm [5], [6], [7] and the compare threshold on the base of the image histogram is calculated [7].

II. Mathematical Description

The input m-level halftone image and the output n-level ($2 \le n \le m/2$) image of dimensions *M x N* can be represented by the matrices:

$$\boldsymbol{C} = \{\mathbf{c}(k,l)\ /\ k = \overline{0,M-1};\ l = \overline{0,N-1}\},$$
$$\boldsymbol{D} = \{\mathbf{d}(k,l)\ /\ k = \overline{0,M-1};\ l = \overline{0,N-1}\}. \qquad (1)$$

Transformation of the image elements **c**(k,l) into **d**(k,l) is accomplished by the adaptive error diffusion quantiser (AEDQ) shown on Fig. 1. The quantiser operation is described by the following equation:

$$\mathbf{d}(k,l) = Q[\mathbf{c}_f(k,l)] = \begin{cases} q_{0,} \text{ if } \mathbf{c}_f(k,l) < T_0 \\ q_{p,} \text{ if } T_{p\text{-}1} < \mathbf{c}_f(k,l) < T_p \ (p = \overline{1, n-2}) \\ q_{n\text{-}1,} \text{ if } \mathbf{c}_f(k,l) < T_0 \end{cases} \qquad (2)$$

where $q_p \le q_{p+1} \le m$ ($p = \overline{0, n-2}$) are the values of the function Q[.].

Thresholds for comparison are calculated by $T_p = (C_p + C_{p+1})/2$, where C_p represents the gray values dividing the normalized histogram of the input halftone image $\boldsymbol{C}$ into n equal parts. The value of the filtered element $\mathbf{c}_f(k,l)$ in Eq. (2) is:

$$\mathbf{c}_f(k,l) = \mathbf{c}(k,l) + \mathbf{e}_0(k,l). \qquad (3)$$

The summarized error can be expressed as:

$$\mathbf{e}_0(k,l) = \sum_{(r,t)\in \boldsymbol{W}}\sum \mathbf{w}_{k,l}(r,t)\mathbf{e}(k-r, l-t) = \mathbf{W}_{k,l}^t \mathbf{E}_{k,l}\ , \qquad (4)$$

where $\mathbf{e}(k,l) = \mathbf{c}_f(k,l) - \mathbf{d}(k,l)$ is the error of the current filtered element when its value is substituted by q_p; $\mathbf{w}_{k,l}(r,t)$ are the filter weights defined in the certain causal two -dimensional window $\boldsymbol{W}$; $\mathbf{W}_{k,l}$ and $\mathbf{E}_{k,l}$ are the vectors of the weights and their summarized errors, respectively.

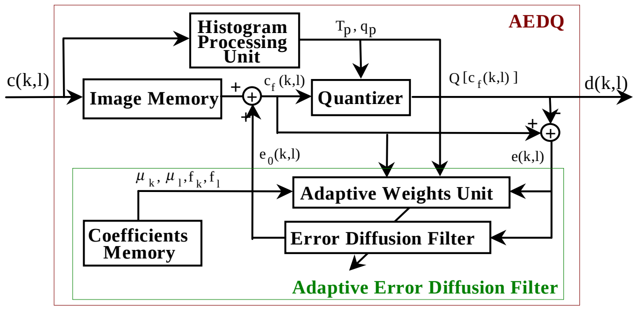

Fig.1 Adaptive error-diffusion quantiser

According to 2D-LMS algorithm [7] the adaptive error diffusion filter (AEDF) weights can be determined recursively:

$$\mathbf{W}_{k,l} = f_k \mathbf{W}_{k,l-1} - \mu_k \nabla_{k,l-1} + f_l \mathbf{W}_{k-1,l} - \mu_l \nabla_{k-1,l}\ , \qquad (5)$$

where: $\nabla_{k,l-1}$ and $\nabla_{k-1,l}$ are the gradients of the squared errors by the quantisation in horizontal and vertical directions; f_k, f_l - coefficients, considering the direction of the adaptation, where : $f_k + f_l = 1$; μ_k, μ_l - adaptation steps in the respective direction.

According to [5] the convergence and the stability of the AEDF adaptation process is given by the following condition:

$$\left|f_k - \mu_k \lambda_i\right| + \left|f_l - \mu_l \lambda_i\right| < 1\ , \qquad (6)$$

where λ_i are the eigenvalues of the gray-tone image covariance matrix .

Sequence (5) is 2D LMS algorithm of Widrow summary from which the following two particular cases should hold:

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First. If $f_k=1, \mu_k=\mu, f_l=\mu_l=0$ then the adaptive calculation of the weights is proceeded only in the horizontal direction:

$$\mathbf{W}_{k,l}=\mathbf{W}_{k,l-1}+\mu_k(-\nabla_{k,l-1})=\mathbf{W}_{k,l-1}-\mu\frac{\partial \mathbf{e}^2(\mathbf{k},\mathbf{l}-\mathbf{1})}{\partial \mathbf{W}_{k,l-1}}. \quad (7)$$

Second. If $f_l=1, \mu_l=\mu, f_k=\mu_k=0$ then the adaptive calculation is proceeded only in the vertical direction:

$$\mathbf{W}_{k,l}=\mathbf{W}_{k-1,l}+\mu_l(-\nabla_{k-1,l})=\mathbf{W}_{k-1,l}-\mu\frac{\partial \mathbf{e}^2(k-1,l)}{\partial \mathbf{W}_{k-1,l}}. \quad (8)$$

The derivatives by the quantization error in the respective directions are determined by the Eqs. (1), (2), (3), (4) and (5). For the derivative in horizontal direction is obtained:

$$\frac{\partial \mathbf{e}^2(k,l-1)}{\partial \mathbf{W}_{k,l-1}}=2\mathbf{e}(k,l-1)\mathbf{E}_{k,l-1}\left[1-Q'_{\mathbf{c}_f}(k,l-1)\right] \quad (9)$$

where:

$$Q'_{\mathbf{c}_f}(k,l-1)=\begin{cases}0, & \text{if: } \mathbf{c}_f(k,l-1)\neq T_p\\ q_{p+1}-q_p, & \text{if: } \mathbf{c}_f(k,l-1)=T_p.\end{cases}$$

In the same way for the derivative in the vertical direction is obtained:

$$\frac{\partial \mathbf{e}^2(k-1,l)}{\partial \mathbf{W}_{k-1,l}}=2\mathbf{e}(k-1,l)\,\mathbf{E}_{k-1,l}\left[1-Q'_{\mathbf{c}_f}(k-1,l)\right] \quad (10)$$

For the AIHF weights the condition must be hold:

$$\sum\sum_{(r,t)\in \boldsymbol{W}} \mathbf{w}_{k,l}(r,t)=1, \quad (11)$$

which guarantees that $\mathbf{e}(k,l)$ is not increased or decreased by its passing through the error filter.

On the basis of analysis, made in Eqs. (7) to (11) the sequence for the components of $W_{k,l}$ is

$$\begin{aligned}w_{k,l}(r,t)=\ & f_k w_{k,l-1}(r,t)\\ & -2\mu_k e(k,l-1)e(k-r,l-t-1)\left[1-Q'_{\mathbf{c}_f}(k,l-1)\right]\\ & +f_l w_{k-1,l}(r,t)\\ & -2\mu_l e(k-1,l)e(k-r-1,l-t)\left[1-Q'_{\mathbf{c}_f}(k-1,l)\right].\end{aligned} \quad (12)$$

III. Experimental Results

An error diffusion filter with 4 coefficients has been used for the evaluation of the efficiency of the developed filter. The spatial disposition, shown on Fig.2, and the initial values of weights correspond to these in the Floyd-Steinberg filter [1].

	l - 2	l - 1	l	l + 1	l + 2
k-2	$w^e(.)$	$w^e(.)$	$w^e(.)$	$w^e(.)$	$w^e(.)$
k-1	$w^e(.)$	w(1,1)	w(1,0)	w(1,-1)	
k	$w^e(.)$	w(0,1)	Current		

$\boldsymbol{W}^e$ $\boldsymbol{W}$

Fig.2. Spatial disposition of the weights $w_{k,l}(r,t)$ in $\boldsymbol{W}$.

For the calculation of each one $w_{k,l}(r,t)$ are used the weights $w^e_{k,l}(.)$ from the extended window $\boldsymbol{W}^e$.

The analysis of 2D variation of peak signal to noise ratio (PSNR) depending on parameters f and μ ($f_k=f, f_l=1-f, \mu_k=\mu_l=\mu$.) is made in [7] and [8].

The coefficients f and μ are changed in the following way: $f=0.0$ to 1.0 with step 0.1 and $\mu=1.0\text{x}10^{-6}$ to $2.0\text{x}10^{-6}$ with step $1.0\text{x}10^{-8}$.

The peak signal to noise ratio is determined by the equation

$$PSNR=10\,lq\frac{m^2}{\sum_{k=0}^{N}\sum_{l=0}^{M}[e(k,l)]^2}\ ,\ dB.$$

The examination of the function PSNR (f, μ) showed that the most proper mean values of f and μ are $f=0.7, \mu=1.67\text{x}10^{-6}$. In this case AIHF leads to increasing of PSNR with about 0.6 dB in comparison with the 4 coefficient $(f=1, \mu=0)$ non-adaptive filter of Floyd and Steinberg.

On Fig.3 the experimental results from the simulation of AEDQ for the test image "LENNA" with: M=N=512, m=256 levels, n=2 bits, $f_k=0.7$, $f_l=0.3$, $\mu_k=\mu_l=1.67\text{x}10^{-6}$ are presented.

The average variation of weights $w_{k,l}(r,t)$ is presented on Fig.4. The given results show that the weights point to a value for which the mean square error (MSE) is minimized for the current image. A second image transform with scanning the pixels in reverse direction is performed for further decrease of mean square error. The initial values of the weights are equal to the last ones obtained from the direct transform. In this case a better convergence of the adaptation is obtained, the phase distortions are decreased and PSNR increased in addition with about 0.2 dB.

IV. Conclusion

The developed generalized AEDQ results in the following particular cases: the wide-spread non-adaptive error diffusion filter of Floyd and Steinberg (for n=2, $f_k=1, \mu_k=\mu_l=f_l=0$); adaptive error diffusion using the weights only in the horizontal (from the same image row - $f_k=1, f_l=0$) or only in the vertical direction (from the previous image row - $f_l=1, f_k=0$). The adaptive filter provides minimum reconstruction error, uniform distribution of the arranged structures in the homogeneous areas and precise reproduction of edges in the output multilevel images. The coefficients f_k, f_l, μ_k, μ_l must be selected on the basis of PSNR analysis and keeping of Eq. (7) as is done in [7]. The developed AEDQ is appropriate for realization on special VLSI circuit to accelerate calculation of image transform.

The presented error diffusion filter can be used for transformation of color palettes or brightness of pixels in

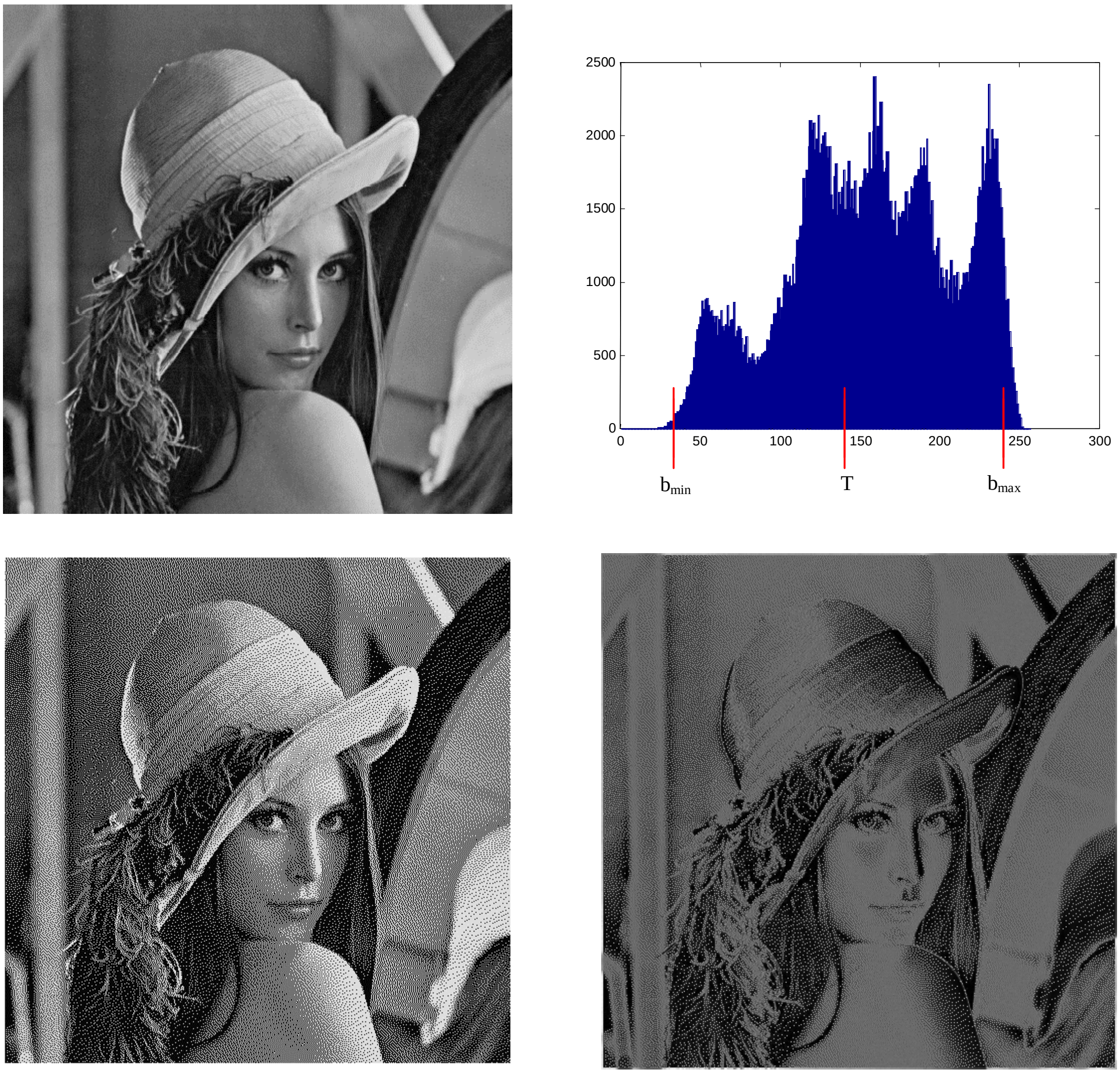

Fig.3. Experimental results - input image "LENNA", histogram of image, output bi-level image end error image.

multimedia systems, for printing color and halftone images and transmission by facsimile devices.

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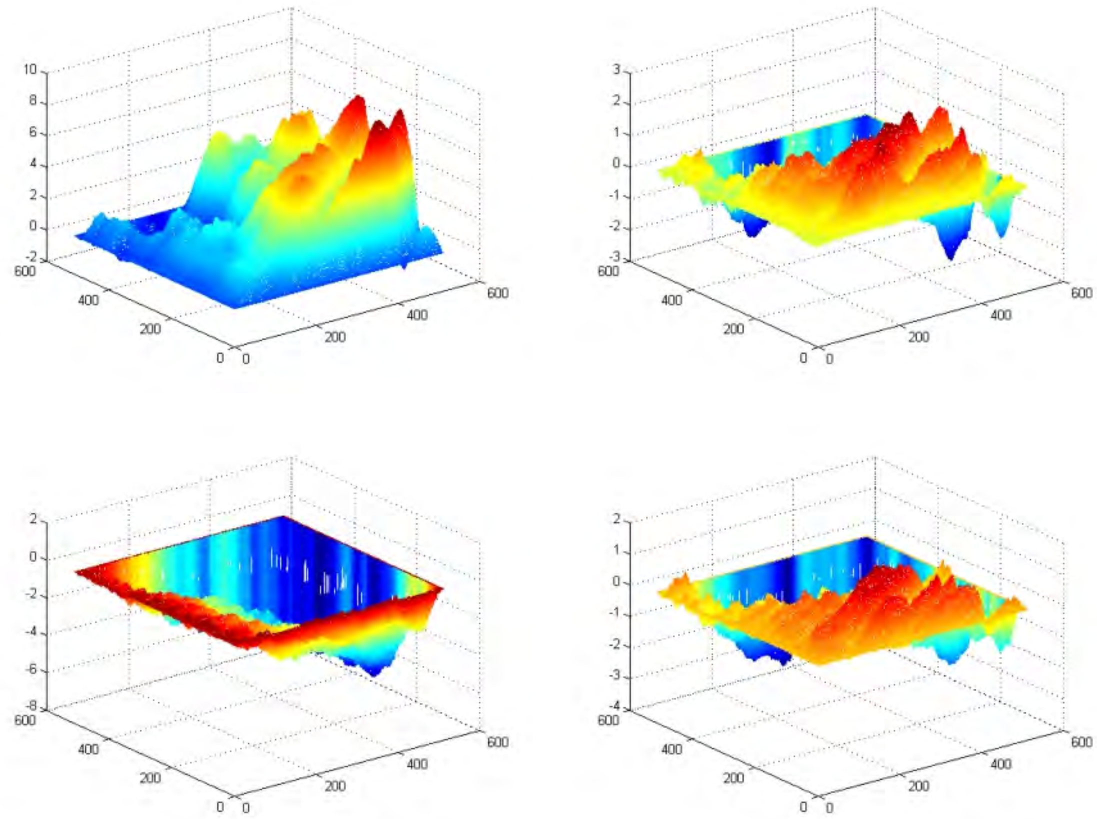

Fig.4. Variation of weights $w_{k,l}(r,t)$ for the test image "LENNA".