On Wavelet Selection for Nuclear Medicine Images Processing

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Abstract – **This paper presents some experimental results of NM image filtering by using several types of wavelets. For this purpose, one set of wavelets that belong to different wavelet families is chosen. Based on these experiments, the Coiflet 5 wavelet is identified as the best wavelet. The aim of the paper is identifying the best wavelet for filtering of NM images in order to develop sophisticated diagnostic software that could automatically offer the optimal positions and the shapes of the regions of interest needed for the heart studies.**

Keywords – **Nuclear medicine image, wavelets, filtering, compression.**

I. Introduction

Wavelet transforms have received significant attention recently from mathematicians, signal analysts and engineers as a new tool for feature extraction, signal and image compression, edge detection and de-noising. Unlike the traditional Fourier techniques, wavelets are localized both in time and frequency domain. This feature makes them suitable for the analysis of non-stationary signals. At present, there exist no theoretical results that can predict which wavelet is suitable for a particular type of signal. Usually, the best wavelet is chosen by comparing the performances of several types of wavelets.

One interesting area of wavelet applications is biomedical engineering. Wavelets have been applied to several problems in biomedical signals, but analysis of *Nuclear Medicine* (NM) images by using wavelet transforms has not be widely explored yet.

Nuclear medicine images are diagnostic digital images, which provide both anatomical and functional information. They present the projection of distribution of radioisotope(s) in a body of a patient after injection of adequate dose of radioisotope(s). The raw NM images are based directly on the total counts detected over a fixed observation period by computerized gamma cameras and have a low signal-to-noise ratio (SNR) due to the nature of the gamma ray emission process and the operational characteristics of the gamma cameras. Therefore, the NM image analysis must be preceded by a certain image preprocessing, which ought to provide an accurate recognition of anatomic data of the patient (the boundaries of the various objects – organs, on the images). This process can be much diversified, since it should be adjusted to the organs and tissues.

In this paper we demonstrate the potential use of different types of wavelets for chest-region NM image filtering. Based on experimental experiments, the Coif5 is identified as the best wavelet. The result can be used in noise removal in NM images and automatic extracting of the anatomic data from the chest region dynamical NM images. The paper is organized as follows. The wavelet theory is summarized in Section II. Section III outlines the scheme used in wavelet filtering of NM images. The experimental results are presented in Section IV. Filtering with different types of wavelets are applied to real images captured with our own gamma camera upgrading system, developed at the department of NM in Bitola. Section V concludes the paper.

II. An Overview of the Discrete Wavelet Transform

The Discrete Wavelet Transform (DWT) decomposes a signal into a set of orthogonal components describing the signal variation across the scale [3]. The orthogonal components are generated by dilations and translations of a prototype function ψ called *mother wavelet.*

$$\psi_{i,k}(t) = 2^{-i/2}\psi(t/2^{i} - k), \quad k,i \in Z \tag{1}$$

The above equation shows that the mother function is dilated by the integer i and translated by the integer k. In analogy with other function expansions, a function f may be written for each discrete coordinate t as sum of a wavelet expansion up to certain scale J plus a residual term, that is:

$$f(t) = \sum_{j=1}^{J} \sum_{k=1}^{2^{-j}M} d_{jk}\psi_{jk}(t) + \sum_{k=1}^{2^{-J}M} c_{Jk}\phi_{Jk}(t) \tag{2}$$

The estimation of d_{jk} and c_{Jk} is carried out through an iterative decomposition algorithm, which uses two complementary filters h_0 (low-pass) and h_1 (high-pass). Since the wavelet base is orthogonal, h_0 and h_1 satisfies the quadrature mirror filter conditions (QMF) [4]. Filter bank theory is closely related to wavelet decompositions and multiresolution concepts. For this reason, it is helpful at this point to view the scaling function ϕ as a low pass filter h_0 and wavelet function ψ as a high pass filter h_1. The mother and scaling functions are defined as follows [3]:

$$\psi(t) = \sum_{n} 2^{1/2} h_1 \psi(2t - n) \tag{3}$$

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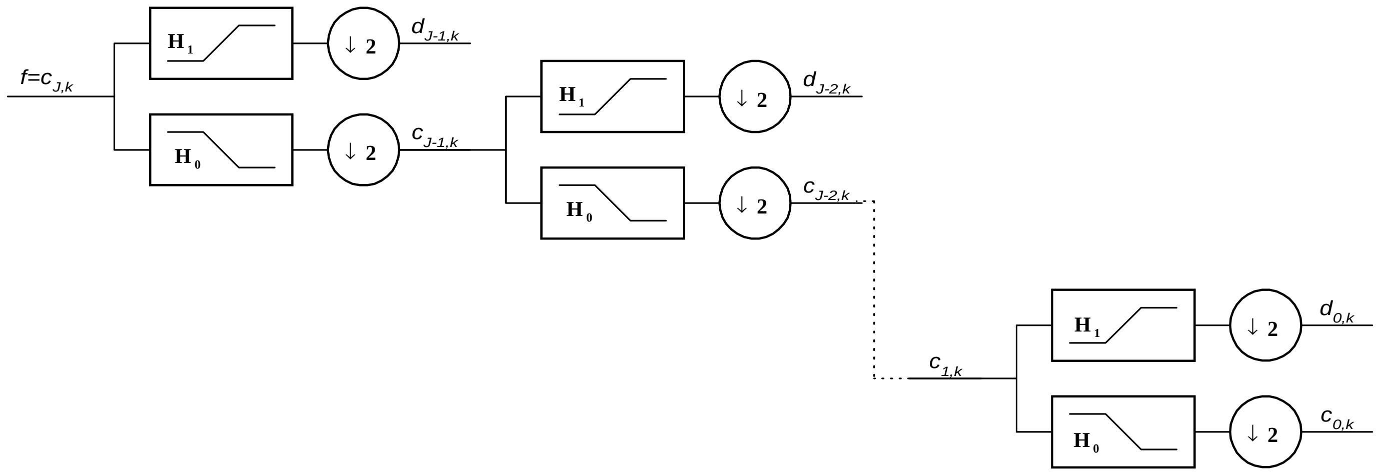

Fig. 1. Discrete Wavelet Transform Tree

$$\phi(t) = \sum_n 2^{1/2} h_0 \phi(2t-n) \tag{4}$$

For computation of wavelet transform, the following pyramidal algorithm is used:

The QMF bank decomposes the signal into low and high frequency components respectively. Convolving the signal with h_1 gives a set of wavelet coefficients $c_{J,k}$, while the convolution with h_0 gives the approximation coefficients $d_{j,k}$. Because of the redundancy of information, these filters are down-sampled, throwing away every other sample at each operation, thus halving the data each time. The approximation coefficients $d_{j,k}$ are then convolved again with the filters h_0 and h_1 to form the next level of decomposition. The backward algorithm simply inverts the process. It combines two linear filters with up-sampling operation. Fig. 1 shows the operation involved in the wavelet decomposition and synthesis of the signal.

At present, there exist no theoretical results that can predict which wavelet is suitable for a particular type of signal. Usually, the best wavelet is chosen by comparing the performances of several types of wavelets.

III. Filtration of NM Images

To demonstrate the potential of wavelets for NM image filtering, we use the most popular form of wavelet-based filtering, commonly known as *Wavelet Shrinkage*. For this purpose, we choose one set of wavelets that belong to different wavelet families including the following wavelet bases Daubechies (Db6 and Db10), Biorthogonal (Bior3.5 and Bior6.8), Coiflets (Coif3 and Coif5) and Symlets (Sym4 and Sym7). In order to identify the best wavelet, *Non-zero pixels compression ratio* (NZPCR) and *Percent root-mean-square difference* (PRD) are defined as follows. The non-zero pixels compression ratio is closely intertwined with the threshold in the wavelet shrinkage, because the bigger the threshold is, the bigger the non-zero pixels compression ratio is. So, we want to find the best wavelet for a set non-zero pixels compression ratio (threshold). As the best wavelet for a set NZPCR, we identify the one for which the PRD is minimal.

First, we calculate the wavelet coefficients of the observation, w_i, and then we filter contribution of a particular wavelet basis function in the signal expansion by weighting the corresponding coefficient w_i by a number h_i; $0 \le h_i \le 1$. That is, we modify the wavelet coefficients according to:

$$\hat{w}_i = w_i \cdot h_i. \tag{5}$$

In the wavelet shrinkage program that we use, the shrinkage filter corresponds to the “hard threshold” nonlinearity

$$h_i^{(\text{hard})} = \begin{cases} 1, & \text{if } |w_i| \ge \tau \\ 0, & \text{if } |w_i| < \tau \end{cases}. \tag{6}$$

After obtaining the filtrated coefficients, we calculate the non-zero pixels compression ratio and Percent root-mean-square difference that are defined below.

The Non-zero pixels compression ratio is defined as:

$$\text{NZPCR} = \frac{\sum \omega_f}{\sum \omega_{\hat{f}}} \tag{7}$$

where $\sum \omega_f$ is number of coefficients different than zero before, and $\sum \omega_{\hat{f}}$ is number of coefficients different than zero after applying the wavelet shrinkage. The threshold value can be adjusted to alter the number of preserved coefficients, thereby changing the non-zero pixels compression ratio.

Percent root-mean-square difference measures the numerical distortion between the original and the reconstructed NM image. We analyze PRD in relation to a set threshold:

$$\text{PRD} = \sqrt{\frac{\sum \left(f_i - \hat{f}_i\right)^2}{\sum f_i^{\,2}}} \times 100 \tag{8}$$

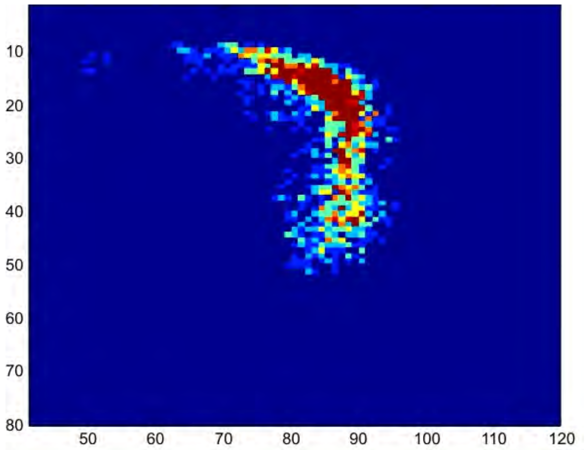

Fig. 2 The analyzed NM image

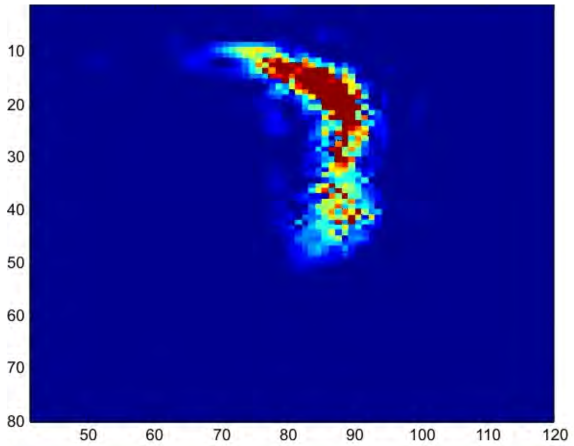

Fig.3   The filtrated NM image

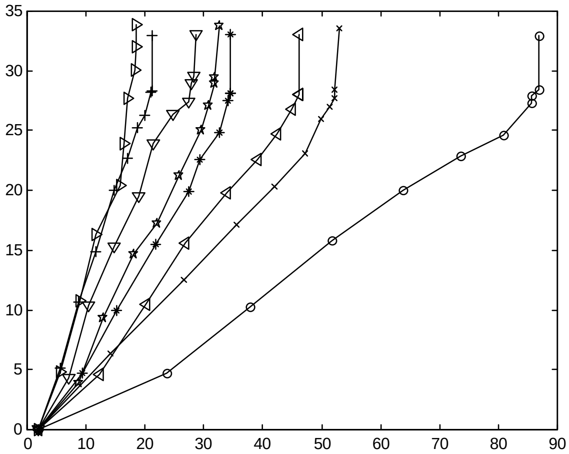

Fig.4   PRD versus NZPCR

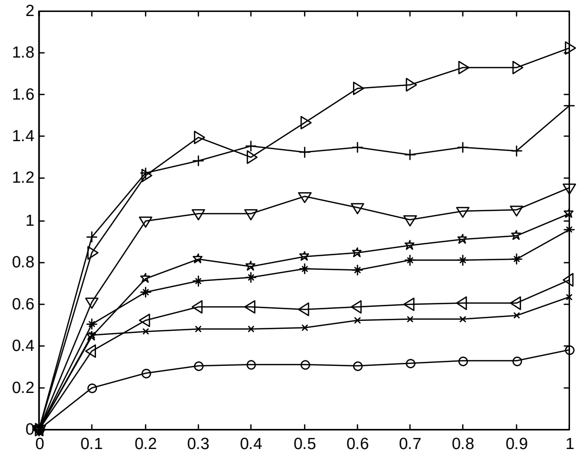

Fig. 5   PRD/CR versus threshold

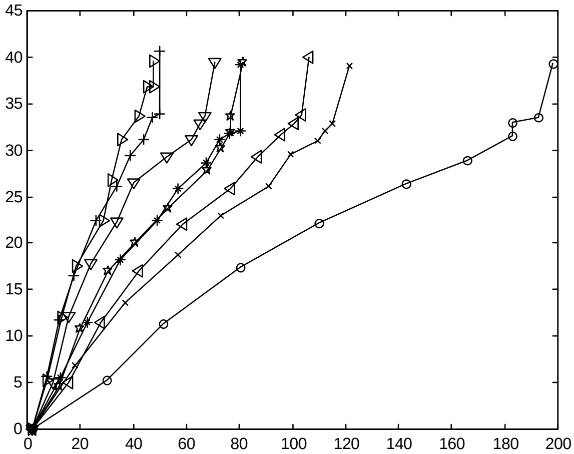

Fig.6   PRD versus NZPCR

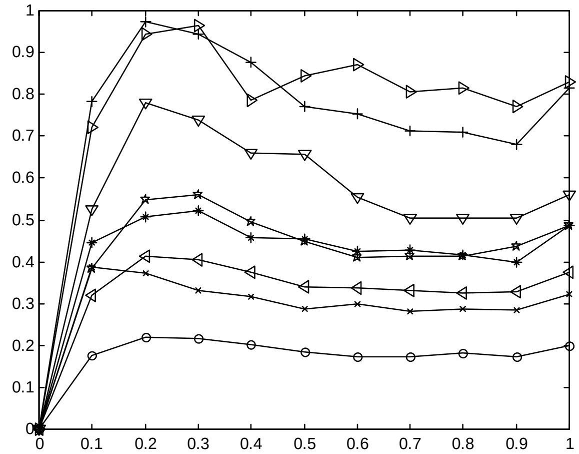

Fig. 7   PRD/NZPCR versus threshold

Legend for Figs. 5-8: db6 v, db10 x, bior3.5 >, bior6.8 *, coif3 <, coif5 o, sym4 +, sym7 p where f_i is a pixel of the original image f, and $\hat{f}_i$ is the corresponding pixel of the filtered image $\hat{f}$.

IV. EXPERIMENTAL RESULTS

As a sample image in our experiments we use one NM image matrix of resolution 128x128. Fig. 2 shows the NM image used for the experimentation. The image contains part of the vein and part of the heart of a patient. Autocorrelation low-pass filtering technique was applied to the image in order to remove salt and pepper noise. But, it still contains rather high level of noise due to: a) mixing the radionuclide with the blood and the spreading of this mixture, b) hydrodynamic processes in the blood vessels caused by the pumping work of the heart and c) by the randomness of the gamma rays emission and their detection by the gamma camera.

Taking into consideration this, we want to adequately preprocess the raw images in order to extract the anatomy information about the position of the vena cava superior and the heart. According to this information, the optimal position (and the shape) of the regions of interest (ROI's) for the heart study can be proposed [2].

A number of experiments were performed to identify the best wavelet for this type of images. In our experiments, first, we apply the wavelet shrinkage program to the image. We use the following set of wavelet bases Daubechies (Db6 and Db10), Biorthogonal (Bior3.5 and Bior6.8), Coiflets (Coif3 and Coif5) and Symlets (Sym4 and Sym7). As a result, all the wavelet detail coefficients with magnitudes larger than the threshold value are retained with full accuracy and the remaining coefficients are truncated.

Next, we calculate the rate distortion curves (PRD versus non-zero pixels compression ratio) for eight wavelets by using decomposition at level one. The curves are shown in Fig. 4. The ratio PRD/CR versus threshold is illustrated on Fig. 5. From the both figures, it can be noticed that for any set threshold or non-zero pixels compression ratio, the percent root-mean-square difference is smallest for the Coiflet 5 wavelet. So, in terms of the best wavelet within the chosen set of wavelets, the Coiflet 5 wavelet performs better results than the other wavelets. General characteristic of Coiflets wavelets is that they are compactly supported wavelets with highest number of vanishing moments for both ϕ and ψ for a given support width, which is useful for compression purpose.

The rate distortion curves, obtained for the same eight wavelets when decomposition at level two is used, are shown in Fig. 6. Now, the wavelet shrinkage program is applied to the detail coefficients from the both levels: one and two. The ratio PRD/CR versus threshold is illustrated on Fig. 7. Again Coiflet 5 is identified as the optimal wavelet.

The filtrated image obtained by using wavelet filtration at level 2 by the optimal wavelet and a threshold equal to 30% of intensity of the pixel with maximal intensity is shown in Fig. 3.

V. CONCLUSION

In this paper we demonstrate the potential use of different types of wavelets for chest-region NM image filtering. Based on these experiments, the Coiflet 5 wavelet is identified as the best wavelet from one set of wavelets that belong to different wavelet families. The result can be used in noise removal in NM images and automatic extracting of the anatomic data from the chest region dynamical NM images. Anatomical data can be determined in order to upgrade the software with an expert system that could identify the optimal positions and shapes of the regions of interest needed for the heart study.

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