The Probability Theory with Application in SCP Technology

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Abstract – **The probability theory is used to investigate some of the basic properties of the SCP technology. Central limit theorem is used too to prove the gain of the random phased antenna array, as well as the Gaussian probability properties of the antenna array output signals.**

Keywords – **SCP technology, Random phase process, Central limit theorem, Gaussian PDF**

I. Introduction

A description of a new radio-communication technology, based on random phased antenna arrays approach and Spatial Correlation Processing (SCP), as well as Matrix presentations of the signals and the basic signal processing procedures, are given in Ref. 1. A GSO,s system, based on SCP-CDMA approach, as well as Matlab computer simulations of the system spatial resolution properties are given in Ref.2 too. The goal of this report is to represent some of the basic SCP system phenomena by means of probability theory, obtaining better understanding for the SCP unique properties.

II. The random phase process

A basic property of the SCP approach is the random phase spread among the signals, received by the different antenna array elements. Consider (Ref.3) the theory of a random process, which in the best way describes the SCP signals. Consider the time function:

$$X(t) = A\cos(\omega_0 t + \theta) \qquad -\infty \prec t \prec \infty \tag{1}$$

Where A, ω_o are constants, θ is the random variable with uniform probability density function in the interval $[0, 2\pi]$. The collection of all possible such waveforms together with the underlying probability assignment is called a phase random process. Each realization of a particular waveform is referred as a sample function. In SCP case each such sample function corresponds to a particular signal, obtained from a given antenna array element. Uniform amplitude distribution (A=const.) is considered for simplicity.

The mean value of the random phase process, given with Eq. (1) is:

$$E[X(t)] = \int_0^{2\pi} A\cos(\omega_0 t + \theta)\frac{d\theta}{2\pi} = 0 \tag{2}$$

The mean square value is:

$$\begin{aligned} E\left[X^2(t)\right] &= \int_0^{2\pi} A^2 \cos^2(\omega_0 t + \theta)\frac{d\theta}{2\pi} = \\ &= \int_0^{2\pi} \frac{A^2}{2}\frac{d\theta}{2\pi} + \int_0^{2\pi} \frac{A^2}{2}\cos 2(\omega_0 t + \theta)\frac{d\theta}{2\pi} = \frac{A^2}{2} \end{aligned} \tag{3}$$

The variance is:

$$\sigma_x^2(t) = E\left[X^2(t)\right] - \{E[X(t)]\}^2 = \frac{A^2}{2} - 0 = \frac{A^2}{2} \tag{4}$$

The autocorrelation function is:

$$\begin{aligned} R_x(t, t+\tau) &= E[X(t)X(t+\tau)] = \\ &= \int_0^{2\pi} A^2 \cos(\omega_0 t + \theta)\cos[\omega_0(t+\tau) + \theta]\frac{d\theta}{2\pi} = \frac{A^2}{2}\cos\omega_0\tau \end{aligned} \tag{5}$$

III. The Central Limit Theorem (CLT)

If the random variables (Ref. 4, 7 and 8) $X_1, X_2, \ldots\ldots\ldots, X_n$ are independent and identically distributed with probability density functions $f_x(x)$, means μ_x and variance σ_x^2, then the probability distribution function of the sum $Y = X_1 + X_2 + \ldots.. + X_n$ is approximately Gaussian with mean $\mu_y = n\mu_x$ and variance $\sigma_y^2 = n\sigma_x^2$ as long as n is "large enough". This result is called the Central Limit Theorem.

Applying CLT to the SCP phase random process, we obtain for the output antenna signal the sum $Y = X_1 + X_2 + \ldots.. + X_n$ with mean $\mu_y = n\mu_x = 0$ and variance:

$$\sigma_y^2 = n\sigma_x^2 = n\frac{A^2}{2} \tag{6}$$

In this particular case (zero mean), the variance is equal to the mean square value (the energy of the signal, or to the autocorrelation value at zero time off-set) resp. of the individual antenna element and of the total SCP antenna.

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The equation (6) proves the basic SCP property that the total antenna gain is sum of the gains of the individual slot radiators.

The sum Y is Gaussian with Probability Distribution Function (PDF) (Ref.5 and 6), given by:

$$p_y(\alpha) = \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\alpha^2 / 2\sigma_y^2} \tag{7}$$

The Cumulative Distribution Function (CDF) of such process is given by:

$$F_y(\alpha) = \Pr[Y \le \alpha] = \int_{-\infty}^{\alpha} \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{\lambda^2}{2\sigma_y^2}} d\lambda = 1 - Q(\alpha/\sigma_y) \tag{8}$$

Where $Q(\alpha)$ is defined as Complementary Cumulative Probability Distribution Function (CCDF) of a standard zero-mean, unit variance Gaussian random variables, $G(0,1)$:

$$Q(\alpha) = \int_{\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{\lambda^2}{2}} d\lambda \tag{9}$$

The root mean square value of the envelope of such process (Rayleigh) is (Ref.5):

$$R_{rms} = \sqrt{2}\sigma_y \tag{10}$$

IV. Cross-correlation properties of orthogonal random phase-spread signals

Matrix presentation of the RLSA poly-phase signals, as well as matrix description of the SCP correlation processes are given in Ref.1 only for BPSK modulated signals. In the real broadband satellite systems QPSK modulation is used in order to double the system information capacity. An important issue of QPSK SCP systems is the isolation between ***I*** and ***Q*** channels. This isolation depends on the cross-correlation properties between ***I*** channel pilot signal P_I and ***Q*** channel information signal I_Q (and vice versa). For this reason the cross-correlation properties of orthogonal poly-phase spread signals are considered below, following Ref. 10.

Consider the above-mentioned pair of QPSK SCP signals $P_I(t)$ and $I_Q(t)$ that are related to a random wide-sense stationary process $X(t)$ as follows:

$$\begin{aligned} P_I(t) &= X(t)\cos(2\pi f_c t + \theta) \\ I_Q(t) &= X(t)\sin(2\pi f_c t + \theta) \end{aligned} \tag{11}$$

Where f_c is a carrier frequency, and the random variable θ is uniformly distributed over the interval $(0,2\pi)$. Moreover, θ is independent of $X(t)$. One cross-correlation function of $P_I(t)$ and $I_Q(t)$ is given by:

$$\begin{aligned} &R_{IQ}(\tau) = E[P_I(t) I_Q(t-\tau)] = \\ &E[X(t)X(t-\tau)\cos(2\pi f_c t + \theta)\sin(2\pi f_c t - 2\pi f_c \tau + \theta)] = \\ &E[X(t)X(t-\tau)]E[\cos(2\pi f_c t + \theta)\sin(2\pi f_c - 2\pi f_c \tau + \theta)] = \\ &0{,}5 R_X(\tau) E[\sin(4\pi f_c t - 2\pi f_c \tau + 2\theta) - \sin(2\pi f_c \tau)] = \\ &0{,}5 R_X(\tau)\sin(2\pi f_c \tau) \end{aligned} \tag{12}$$

Where in the last line we have made use of the phase uniform probability density distribution. At $\tau = 0$, the factor $\sin(2\pi f_c \tau)$ is zero and therefore:

$$R_{IQ}(0) = E[P_I(t) I_Q(t)] = 0 \tag{13}$$

This shows that the random variables, obtained by simultaneously observing the QPSK modulated signals P_I and I_Q at some fixed value of time t, are orthogonal to each other.

Particularly for the SCP system Eq.13 means that after the process of signal recovery and PSK demodulation the I and Q channels are isolated.

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