An Overview of Microstrip Transmission Line Resonators

Dobri M. Dobrev and Marin V. Nedelchev

Abstract: This paper presents closed form formulas for the input admittance and the admittance slope parameter for different microstrip resonator types. Generalized formula for the admittance slope parameter of stepped impedance resonator is derived. Closed form formula for the input admittance for a hairpin resonator with stub elements is derived. The resonance conditions of different resonator types are discussed.

Keywords: Microstrip resonator, resonance condition, input admittance, admittance slope.

I. INTRODUCTION

Microstrip filters are widespread in the microwave devices for their compactness, reliability, simplicity and easy manufacturing. The resonator topology defines, in a great extent, the filter amplitude characteristic. The comparatively low unloaded Q factor of the resonators leads to high insertion loss in the passband and low steepness of the amplitude response of the filter. Using cross-coupled filters, dual-mode resonator filters or 0° feed structure of the filter solves these problems.

The cross-coupled filters have higher selectivity, because of the placing of transmission zeros in the stopband. The dual-mode resonators may be used as a doubly tuned resonant circuit and therefore the number of resonators required for a *n*-degree filter is reduced by half, resulting in a compact filter configuration. 0° feed structure means placing the input and the output feed points at the opposite locations about the center of the resonator. This creates two transmission zeros in the stopband.

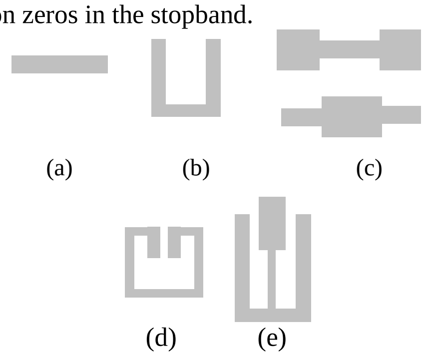

Fig.1. Different microstrip resonator types.
(a). $\lambda/2$ length (b) hairpin $\lambda/2$ length (c) Stepped impedance.
(d) Miniaturized hairpin (e) hairpin resonator with stub element

Dobri M. Dobrev – Assoc. Professor in Dept. of Radiotechnic in Faculty of Communications and Communication Technologies in TU –Sofia E-mail: dobrev@tu-sofia.bg
Marin Veselinov Nedelchev – PhD student in Dept. of Radiotechnic in Faculty of Communications and Communication Technologies in TU –Sofia E-mail mnedelchev@abv.bg

$\lambda/2$ length (Fig.1a) [1] and hairpin $\lambda/2$ length (Fig.1b) [2] are the most frequently used microstrip resonators. Because of their resonance response periodicity, the filters based on these resonators have also periodicity in the amplitude response. The spurious passbands are of the second or the third harmonic of the fundamental resonance. Breaking the regularity in the resonator structure results in breaking the periodicity in the filter amplitude response. This is desirable for filters placed after mixers, power amplifiers working in class C or filters in the resonance system of microwave oscillators, where suppressing the second and the third harmonic is necessary. Stepped impedance resonators (SIR) are proposed in [3] (Fig.1c). They have smaller dimensions than the $\lambda/2$ length resonator and the first spurious frequency is not at $2f_0$. In [4] is proposed miniaturized hairpin resonator (Fig.1d), where coupled lines are joined to the main resonator line to reduce the overall resonator dimensions. The first spurious frequency is not at $2f_0$ too. For increasing the filter amplitude response steepness, the authors of [5] propose a hairpin resonator with stub elements (Fig.1e). This resonator has two parallel and one serial resonant frequency.

The paper presents closed form formulas for the input admittance and the admittance slope parameter for different types of microstrip resonators. They are necessary for calculating the resonance frequencies and the coupling coefficients of the filter. A general formula for the SIR admittance slope parameter and a formula for the input admittance of the hairpin resonator for stub elements are derived.

II. BASIC STRUCTURES OF MICROSTRIP RESONATORS AND RESONANCE CONDITIONS.

A. *λ/2 length resonator*

The $\lambda/2$ length resonators are the most frequently used resonators in the microstrip filters, because of their simple structure and easy adjustment. A network model of $\lambda/2$ length resonator is shown on Fig.2. The resonator consists of a transmission line with characteristic impedance Z_c and physical length of $\lambda/2$.

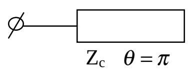

Fig.2 Network model of $\lambda/2$ resonator

The resonator physical length is unacceptable high for low frequencies. Compact filters based on $\lambda/2$ resonators are obtained in the X-band. The formula for the input admittance is:

$$Y_{in} = jy_c tg\theta \tag{1},$$

where θ is the electrical length of the resonator, and $y_c = 1/Z_c$

When $\theta = \pi$ rad , then $Y_{in} = 0$ and the resonance is parallel. The admittance slope parameter is defined in [6]:

$$b = \frac{\omega_0}{2} \frac{\partial Y_{in}}{\partial \omega}\bigg|_{\omega=\omega_0}$$, where ω_0 is the resonant frequency.

The admittance slope parameter of *λ/2* resonator is [6]:

$$b = \frac{\pi}{2} y_c \tag{2}$$

An electromagnetic (EM) simulation is made and the resonance response is shown on Fig.3.

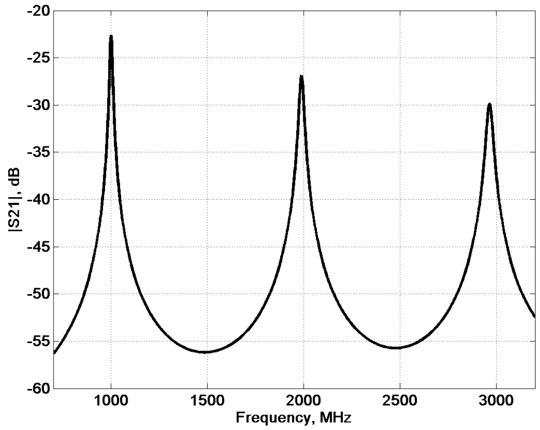

Fig.3 EM simulated frequency response on substrate *h=1.5mm* and $\varepsilon_r=4.5$

It is seen the periodicity of the resonance characteristic and the first spurious frequency is twice greater than the fundamental frequency. This results in the periodicity of the filter based on the *λ/2* resonator.

B. Stepped Impedance Resonator

Network model of SIR is shown on Fig.4. SIR consists of transmission line with characteristic impedance Z_1 and $2\theta_1$ long, loaded in its ends by open stubs with characteristic impedance Z_2 and θ_2 long. The discontinuity, introduced by the impedance difference, leads to decreasing the overall resonator length in comparison to *λ/2* resonators and shifts the first spurious frequency from $2f_0$.

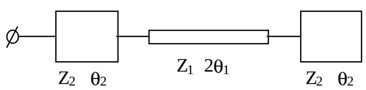

Fig.4 Network model of SIR

The formula for the input admittance of SIR is derived in [3]:

$$Y_{in} = jY_2 \frac{2\left(Rtg\theta_1 + tg\theta_2\right)\left(R - tg\theta_1 tg\theta_2\right)}{R\left(1-tg^2\theta_1\right)\left(1-tg^2\theta_2\right) - 2\left(1+R^2\right)tg^2\theta_1 tg^2\theta_2} \tag{3}$$

where $R = Y_1/Y_2 = Z_2/Z_1$ is the impedance ratio.

The condition for the fundamental parallel resonance is:

$$R = tg\theta_1 tg\theta_2 \tag{4}$$

The general formula for admittance slope parameter of SIR is:

$$b = -Y_2 \frac{\left(tg\theta_1 + \cot g\theta_1\right)\left(\frac{2R\theta_1}{\sin 2\theta_1} + \frac{\theta_2 \sin\theta_1}{\cos^3\theta_1}\right)}{\left(1-tg^2\theta_1\right)\left(1-R^2\cot g^2\theta_1\right) - 2\left(1+R^2\right)} \tag{5}$$

The normalized first spurious frequency over the fundamental resonance frequency as a function of the impedance ratio is shown on Fig.5.

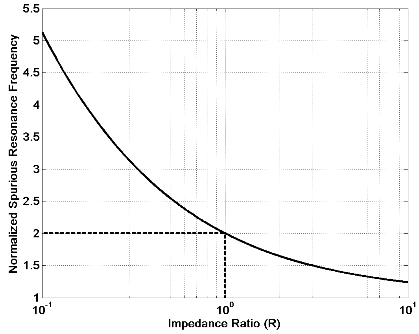

Fig.5 First spurious resonance frequency of SIR

It is obvious that for $R = Y_1/Y_2 = 1$, $f_s = 2f_0$, e.g. SIR degenerates to *λ/2* resonator.

For the most practical cases it is preferable to choose $\theta_1 = \theta_2 = \theta$. The resonance condition becomes:

$$\theta_0 = arctg\sqrt{R} \tag{6}$$

where θ_0 is the lines' length for the resonance frequency.

The expression for the admittance slope parameter is derived from (5):

$$b = 2\theta_0 Y_2 \tag{7}$$

When $\theta_0 = \pi/4$ rad, e.g. the resonator physical length is *λ/2*, the admittance slope is (2), as it is derived in [6].

The SIR filters have a transmission zero for the serial resonance frequency. They are suitable for applications with mixers and power amplifiers.

C. Miniaturized Hairpin Resonator

The miniaturized hairpin resonators proposed in [4] are much smaller than the conventional hairpin resonator. Network model is shown on Fig.6

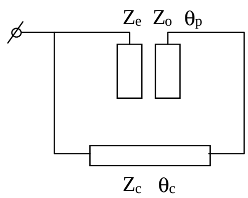

Fig.6 Network model of a miniaturized hairpin resonator

The coupled lines acts as a capacity connected to both ends of the main resonator line. This reduces the resonator dimensions and makes the resonator usable up to 3GHz. The resonator dimensions become too small above this frequency and it makes the resonator sensitive to manufacturing tolerances. A network model of miniaturized hairpin resonator is shown on

Fig.6. This resonator is a modification of SIR. The input admittance and the admittance slope parameter are derived in [7]:

$$Y_{in} = -j\frac{\begin{bmatrix} -(Z_e - Z_o)\cot g\theta_p + (Z_e + Z_o)\cot g\theta_p \cos\theta_s + \\ +\left(\frac{Z_e Z_o}{Z_c}\cot g^2\theta_p - Z_c\right)\sin\theta_s \end{bmatrix}}{2Z_e Z_o \cot g^2\theta_p \cos\theta_s - Z_c(Z_e + Z_o)\cot g\theta_p \sin\theta_s} \quad (8)$$

$$b = -\frac{1}{2}\frac{A+B}{C} \quad (9)$$

where

$$A = \frac{(Z_e - Z_o)\theta_p}{\sin^2\theta_p} + \frac{(Z_e + Z_o)\theta_p \cos\theta_s}{\sin^2\theta_p} + (Z_e + Z_o)\theta_s \sin\theta_s \cot g\theta_p$$

$$B = \theta_s \cos\theta_s\left(\frac{Z_e Z_o}{Z_c}\cot g^2\theta_p - Z_c\right) + \frac{2Z_e Z_o \theta_p \cot g\theta_p \sin\theta_s}{Z_c \sin^2\theta_p}$$

$$C = 2Z_e Z_o \cot g^2\theta_p \cos\theta_s - Z_c(Z_e + Z_o)\sin\theta_s \cot g\theta_p$$

and Z_e and Z_o are the even and odd mode impedances of the coupled lines;

θ_p and θ_c are the electrical length of the coupled lines and the main resonator line respectively.

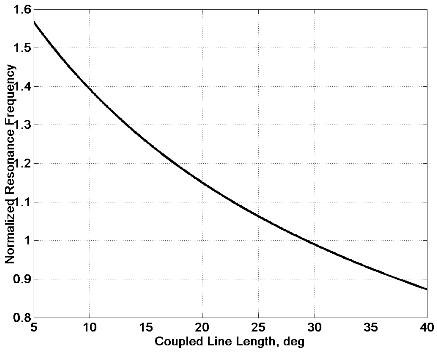

Fig.7 Normalized resonance frequency of a miniaturized hairpin resonator

Fig.7 shows the normalized resonance frequency as a function of the coupled line length for $Z_e=75\Omega$, $Z_o=33.3\Omega$, $\theta_c=100$deg.

In [8] closed form formulas are derived for the fundamental and the first spurious resonance frequency:

$tg\dfrac{\theta_c}{2} = \dfrac{Z_o}{Z_c}\cot g\theta_o$, for the fundamental frequency, where θ_0 is the odd mode electrical length of the coupled lines.

$cotg\dfrac{\theta_c}{2} = -\dfrac{Z_e}{Z_c}\cot g\theta_e$, for the first spurious frequency, where θ_e is the even mode electrical length of the coupled lines.

D. Hairpin resonator with stub elements

A network model of hairpin resonator with stub elements is shown on Fig.8.

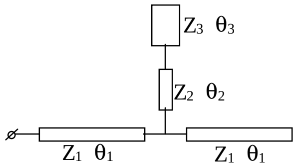

Fig.8 Network model of a hairpin resonator with stub elements.

The additional stub elements are connected in the middle of the *λ/2* line and have two resonances-a parallel and a serial. The frequency of the parallel resonance of the stub is chosen close to the passband of the filter. Two parallel and one serial resonance are the advantages of this resonator. The stubs and the resonator are connected in parallel in order to decrease the overall resonator losses. This resonator may be used for narrow bandwidth filters and high selectivity. Usually these filters are built by two or three resonators, because higher number of resonators makes difficult the resonance controlling and filter adjustment. The formula for the input admittance is:

$$Y_{in} = -jY_1\frac{2Atg\theta_1 + B}{Atg^2\theta_1 + Btg\theta_1 - A} \quad (10),$$

where $A = Y_1(Y_2 - Y_3 tg\theta_2 tg\theta_3)$

$B = Y_2(Y_3 tg\theta_3 + Y_2 tg\theta_2)$.

It is assumed that the main resonator line and the stub elements do not have influence. When $\theta_1 = \pi/2$ rad, then the input admittance is zero for the resonance frequency. The serial and the second parallel resonance are as a result of the common action of the stubs and the right *λ/4* open-end line (Fig.8). They are transformed through the approximately *λ/4* length line for the frequencies of parallel and serial resonances. The resonance frequencies are shifted, because of the inaccurate transformation (θ_1 is not $\pi/2$ for the serial and the parallel resonance). Fig.9 shows EM simulation of hairpin resonator with stub elements. The serial and both parallel resonances are clearly seen.

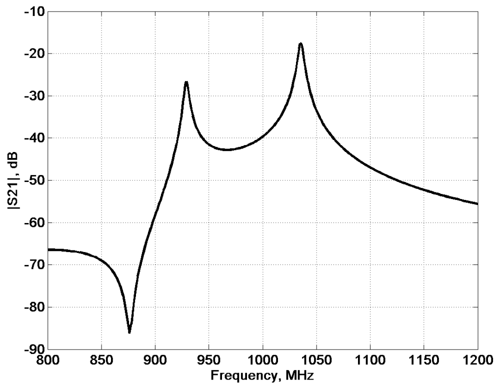

Fig.9 EM simulated frequency response on substrate $h=1.5mm$ and $\varepsilon_r=4.5$

V. CONCLUSION

The paper describes the most frequently used microstrip transmission line resonators. Closed form formulas for the input admittance and the admittance slope parameter are presented for all of the resonators. General formula for the admittance slope parameter and a formula for the input admittance of the hairpin resonator with stub elements are derived. The frequency range and the desired filter characteristics conduct the proper resonator choice for the filter design. In many applications is suitable the use of two different resonator types. This allows the combining the advantages of different resonators for obtaining the filter characteristic.

REFERENCES

[1] S.B. Cohn. Parallel-Coupled Transmission-Line-Resonator Filters. IEEE Transactions on MTT, 1958 pp.223-231.

[2] Cristal, E.G., Frankel, S., Hairpin-Line and Hybrid Hairpin-Line/Halfwave Parallel-Coupled Filters, IEEE Trans. On MTT Nov. 1972.

[3 Makimoto, M., Yamashita, S., Bandpass Filters Using Parallel Coupled Stripline Stepped Impedance Resonators, IEEE Trans. on MTT-28, 1980, pp.1413-1417.

[4] Sagawa, M. Takahashi, K., Makimoto, M., Miniaturized Hairpin Resonator Filters and Their Application to Receiver Front-End MIC's, IEEE Trans. On MTT-37, Dec. 1989, pp.1991-1997

[5] . A. A. Aleksandrovsky, B. A. Belyaev, and A. A. Leksikov, Synthesis and Selective Features of Microstrip Filters Based on Hairpin Resonators with Stub Elements, Journal of Communications Technology and Electronics, Vol. 48, No. 4, 2003

[6] Matthaei, G., Young, L., Jones, E.M.T., Microwave Filters, Impedance-Matching Networks, and Coupling Structures, Artech House, Norwood, MA, 1980

[7 Iliev I. G., CAD of Microwave Bandpass Filters Based on Miniature U Resonators, TELEKOM-95, Varna, pp.90-96, 1995

[8] Lee, S. -Y., Tsai, C. -M. A New Network Model for Miniaturized Hairpin Resonators and its Applications, IEEE MTT-S, Digest, 2000, pp.1161-1164.