Influence of the atmospheric turbulence over the optical cosmic investigations with ground based telescopes

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Abstract - **A research on the influence of the atmospheric turbulence over the optical images of cosmic objects, formed in the focal planes of ground-based telescopes is presented. Explicit final expressions about the average optical intensity, the contrast of the fluctuations and about the correlational radii of the fluctuations in front of the entrance aperture of the telescope and in its focal plane, are derived.**

Keywords - **Atmospheric turbulence, Atmospheric optics**

I. INTRODUCTION

The processing and interpretation of the data from the research of cosmic objects, using ground-based optical receiving systems and the relevant registration devices, are a very important scientific and scientifically applicable issue. These objects are sources of thermal radiation or disperse light falling diffusely on them. In both cases the stochastic structure of the radiations changes along with the distance from the objects that determine them. Their initial spatial coherece (SC) is practically zero. Their propagation in the cosmic space and in the atmosphere is accompanied by the "birth" and development of SC with diffraction origin. Exactly because of this SC the passing of the radiation through the turbulent earth atmosphere is related to the appearance of specific increasing phase and amplitude fluctuations, cross spatial ones in particular. As a general result, the images of the cosmic object which are formed in the focal plane of the receiving optical system, are strongly influenced by the atmospheric turbulence [1,2]. What is more, not only do significant declinations of the optical intensity of the images from its average distribution take place, but there are also important changes of the very average distribution, compared to what corresponds to non-turbulent atmosphere.

The influence of the turbulence over the images of the separate one-dimension sections of a cosmic object is different. It is stronger if the level of SC is higher along the axes of these sections, and the latter is inversely proportional to the size of the sections(of the plane angles in which they are seen). That is why, the influence of the turbulence can only be expressed in a different "dilution" of each section along the relevant axes in the image, i.e. in a decrease in resolution (in long expositions) - Fig. 1a, or in breaking the image into pieces (in short expositions) - Fig. 1b (by using dashed lines, an example image while turbulence is missing, is presented).

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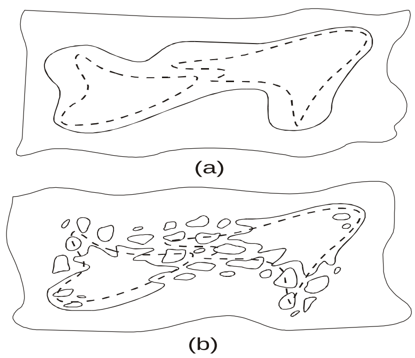
Fig.1.Examle of influenced of atmospheric turbulence image



(a - acase of long exposures; b - a case of short exposures)

Therefore, in order to restore the actual shape of the object on the basis of its image, it is necessary to know the character of the ifluence of the turbulence as well as its quantitative expressions.

The theoretical research of the process of the forming of the optical image when there is no turbulence, is done using methods which origin in the theory about diffraction and partial coherence [3], respectively in the theory about coordinate-invariant and frequence-ivariant linear systems [4]. The conclusions, however, do not lead to an explicit analytic solution even in the simplest geometrical configurations of the radiating object. The reason why this happens is the large number of consequent integrations with special functions.

When there is a turbulence layer in the transmission medium, the theoretical analysis becomes more complex. As a rule, the Rytov method of smooth perturbations (called MR - Method of Rytov) is used [5]. Of course, the derived results are not explicit analytical formulae, either [6,7]. What is more, these results concern only the distribution of the average optical intensity in the focal plane. In the last few years the interest to these problems has grown. This is related to the processing of the data derived from the experimental investigations of the solar corona during the total solar eclipse on 11th August 1999, which were done at the same time in many countries, including Bulgaria. This is also related to the improvement of adaptive optics systems [8]. The difficulties mentioned above, however, have not been overcome yet.

II. METHODS OF RESEARCH

The presented method has a higher analytical effectiveness

- it provides explicit analytical results, which is used in the next sections.

It is rational to use a current one-dimensional section of the two-dimensional projection of the object in the plane of observation, and then, to restore the two-dimensional image. The corresponding setup is shown in Fig. 2.

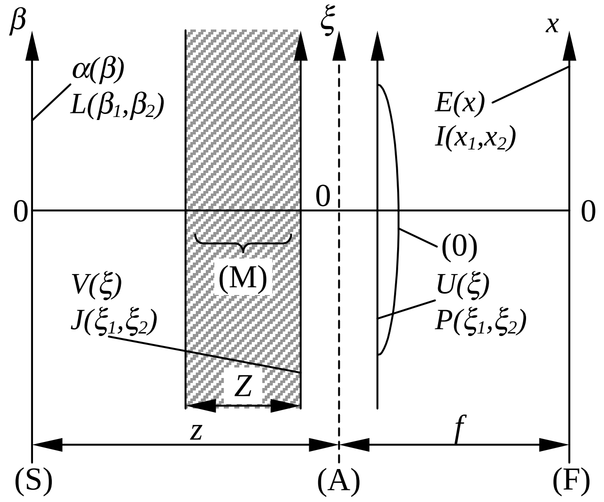


Fig. 2. Setup of the research

β denotes the axis of the one-dimensional cosmic object (S), (M) denotes the layer with width Z, which corresponds to the turbulence atmosphere, ξ denotes the axis of the one-dimensional input aperture (A) of the receiving telescope, (0) is the lens equivalent to the telescope, x is the axis of the one-dimensional focal plane (F), in which the image is formed. The complex amplitudes and the mutual intensities of the optical planes along the axes β, ξ before (A), ξ after (A), x are respectively $\alpha(\beta)$ and $L(\beta_1,\beta_2)$, $V(\xi)$ and $J(\xi_1,\xi_2)$, $U(\xi)$ and $P(\xi_1,\xi_2)$, $E(x)$ and $I(x_1,x_2)$. The methods known so far are used only for derivation of the distribution of the average optical intensity. What is more, they cannot lead to practically usable results, even in the simplest configurations of the radiation objects – mostly because of the large number of necessary consequent integrations of expressions which contain special functions. The aim of the method presented here is to deal with these disadvantages. Its most important part is the opportunity found for analytical representation of the influence of the turbulence as a "correction multplier" of the images which would be formed if turbulence was missing.

$$\langle J(\xi_1,\xi_2)\rangle = \langle J_0(\xi_1,\xi_2)\rangle Turb(\xi_1,\xi_2) \qquad (1)$$

To do this, we introduce mathematical models, the descriptions of which allow integrations with elementary quadratures.

In compliance with the latter we assume that the average intensity of $\alpha(\beta)$ can be described by Gaussian function of β , i.e.

$$\langle L(\beta)\rangle = \langle L(\beta_1,\beta_2)\rangle\big|_{\beta_1=\beta_2=\beta} = L_m \exp\left(-2\frac{\beta^2}{b^2}\right) \qquad (2)$$

where b is the half-size of the cosmic object.

Using [9] and [10] we derive

$$\langle J_0(\xi_1,\xi_2)\rangle = C_z^2 q L_m \int_{-\infty}^{\infty} \exp\left(-2\frac{\beta^2}{b^2}\right)\times$$

$$\times \exp\left[-j\frac{k}{z}(\xi_2-\xi_1)\beta\right] d\beta \ , \qquad (3)$$

where $q=\frac{\lambda}{\sqrt{\pi}}$, $C_z=\sqrt{\frac{k}{2\pi z}}$, $k=\frac{2\pi}{\lambda}$ [4,7].

We use this result which is subject to double integration. We deal with it as a marked integral over third variable and , by changing the sequence of the integral operators, we derive division of the first two integration variables.

When we use [9,10] the derivation of $\langle I_0(x)\rangle$ implies not complex, but continuous calculations and transformations. Here we will show only the final result. It is expressed by using new parameters which, however, have explicit physical sense. So we derive

$$\langle I_0(x)\rangle = \frac{aL_m}{\sqrt{2}f}\frac{1}{\sqrt{1+r^2}}\exp\left(-2\frac{x^2}{x_0^2}\right) \qquad (4)$$

where

$r=\frac{\lambda}{\pi a\theta}$, $\langle I_0(x_0)\rangle = \frac{\langle I_0(0)\rangle}{e^2}$, $x_0=\sqrt{1+r^2}\,x_b$, $x_b = f\theta$.

In the cases of strongly hindered analytical moving on, appropriate mathematical approximations are used, and the areas of their validity are carefully assessed.

III. Distribution of the Average Optical Intensity

The subject of the analysis in this section is the distribution of the mathematical expectation of the optical intensity, which is a major interest in registration with long exposures. The aim of the analysis is to derive practically usable results about the distribution of the average temporal intensity in an explicit way, which can also be used as initial basis for future research into the statistical structure of this intensity. The analysis is done with the help of the method suggested and developed in the previous section. The main part of the research here is the realization of our idea for analytic presentation of the influence of turbulence as a multiplicative correction of the images which would be formed if turbulence was missing.

The "multiplier of turbulence", which corresponds to this correction, is defined

$$Turb(\xi_1,\xi_2)=\left\langle \exp[\chi(\xi_1)+\chi(\xi_2)]\times \right.$$

$$\left. \times \exp\{-j[\Phi(\xi_1)-\Phi(\xi_2)]\}\right\rangle , \qquad (5)$$

where $\chi(\xi)=\ln\dfrac{V_m(\xi)}{V_{0m}(\xi)}$ and $\Phi(\xi)=\varphi_V(\xi)-\varphi_{0V}(\xi)$ are, respectively, the random perturbations of the logarithm of the amplitude and the phase variation at the point ζ [5,6].

We use the MR of the optical field [5] as well as the "heuristic" theory about the distribution of partially coherent optical radiation in turbulence medium and we derive an expression about "the multiplier of turbulence"

$$Turb(\Delta\xi)=\exp(-\gamma^2)+[1-\exp(-\gamma^2)]\exp\left[-\eta\frac{(\Delta\xi)^2}{\rho_{cV_0}^2}\right], \quad (6)$$

where γ^2 is determined by MR [5,9,10], and η is a function of γ, i.e.

$$\eta=\ln\left\{[\exp(\gamma^2)-1]\left[\exp\left(\frac{\gamma^2}{e^2}\right)-1\right]^{-1}\right\} . \qquad (7)$$

By using [10] we quantitatively define the margins of physical validity of the analysis concerning the angle of observation of the object. These margins are compared to the interval of values of the angle in question, in which the influence of turbulence is essential (Fig.3)

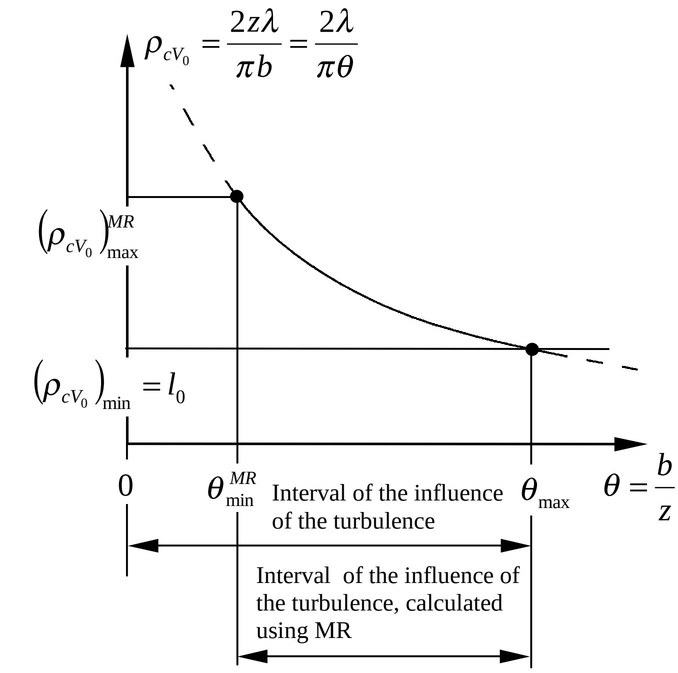


Fig. 3. Margins of the physical validity of the analysis

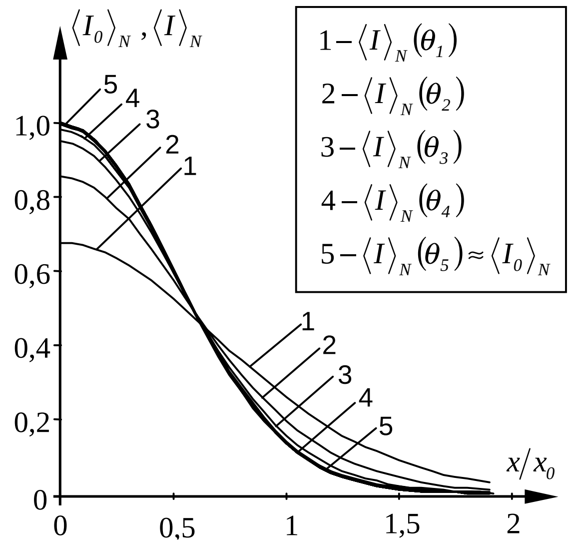
Explicit final formulae are derived about the distribution of the average temporal optical intensity in the focal plane in the telescope. In these formulae parameters take part as independent variables. They have explicit physical sense and they are easily calculated.

$$\langle I(x)\rangle=\langle I_0(x)\rangle\exp(-\gamma^2)+\frac{aL_m}{\sqrt{2}f}\times$$

$$\times\sqrt{\frac{\frac{2}{\eta}}{1+\frac{2}{\eta}(1+r^2)}}\;[1-\exp(-\gamma^2)]\times$$

$$\times\exp\left[-2\frac{\frac{2}{\eta}(1+r^2)}{1+\frac{2}{\eta}(1+r^2)}\frac{x^2}{x_0^2}\right] . \qquad (8)$$

By making example calculations () using (8) it is shown that turbulence has a stronger influence over the images of the sections of the objects whose angle sizes are smaller – exactly these images undergo larger turbulence expansions (Fig.4).



$\theta_1=\theta_{\min}=0{,}93.10^{-5}$; $\theta_2=1{,}89.10^{-5}$; $\theta_3=3{,}85.10^{-5}$;
$\theta_4=7{,}82.10^{-5}$; $\theta_5=\theta_{\max}=1{,}59.10^{-4}$.

Fig. 4. Distribution of the average optical intensity

IV. Fluctuations of the Optical Intensity

The subject of the analysis in this section are the fluctuations of the optical intensity determined by turbulence, which are a major interest in registration with short exsposures and they have not been dealt with in the literature so far. The aim of the analysis is to derive practically applicable results which describe the statistical characteristics of the intensity in an explicit way.

The dispersion of the fluctuations of the optical intensity along the radial axis *x* in the focal plane is determined by the expression

$$\sigma_I^2(x) = \langle I^2(x)\rangle - \langle I(x)\rangle^2 , \quad (9)$$

where

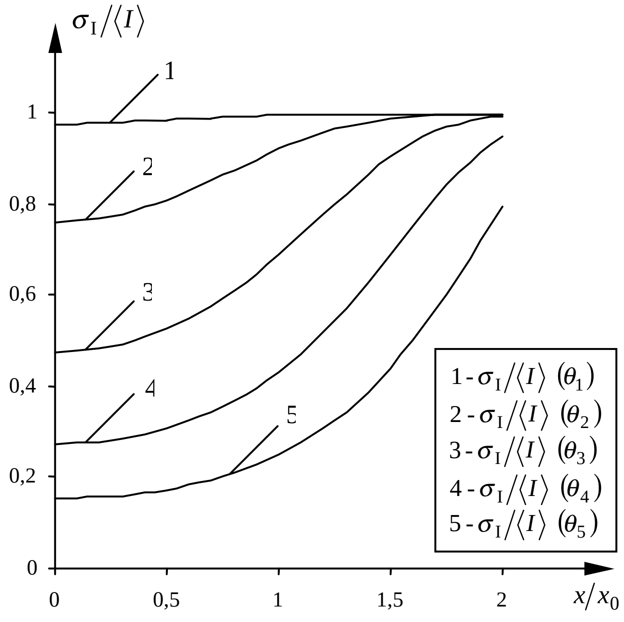
$$I(x) = E(x)E^*(x) = |E(x)|^2 \quad (10)$$

is the intensity itself, and $E(x)$ is its corresponding optical field. The analysis is based on a method presented in section two. By using the results about the average intensity from the previous paragraph and, assuming some approximations and simplifying concepts with quantitatively assessed consequences, which at the same time contribute to the engineering effectiveness of the analytical results, we derive explicit final expressions about the contrast of fluctuations (about the distribution of the mean quadratic deviation, compared with the distribution of the mathematical expectation), about the correlational radii of the fluctuations in front of the entrance aperture of the telescope and in its focal plane [11]:

$$\sigma_I^2(x) = \langle I(x)\rangle^2 - \left[\exp\left(-2\gamma^2\right)\right]\langle I_0(x)\rangle^2 , \quad (11)$$

$$\rho_{cE} = \frac{\lambda f}{\pi}\frac{2}{a}\sqrt{\frac{1}{2} + \frac{1}{\left(1-0{,}2\gamma^2\right)^2}\left(\frac{a}{\rho_{cV_0}}\right)^2} . \quad (12)$$

By exemplary calculations it is shown that even in medium turbulence, the marginal part of an image (not only the one that corresponds to the turbulent expansion, but also the one that is determined by diffraction) can be registered with relatively reduced brightness, or not be registered at all, in a large number of short exposures (Fig.5).



$\theta_1 = \theta_{min} = 0{,}93.10^{-5}$; $\theta_2 = 1{,}89.10^{-5}$; $\theta_3 = 3{,}85.10^{-5}$; $\theta_4 = 7{,}82.10^{-5}$; $\theta_5 = \theta_{max} = 1{,}59.10^{-4}$.

Fig. 5. Distribution of the contrast of of fluctuations

What is more, it is most probable that, at the same time, in the diametrically opposed marginal part of the same image, the opposite result be observed - increase of the brightness of the image compared to the average brightness.

V. Conclusions

A conclusion can be made, that the presented research has a much wider compass than its application in processing the images of cosmic objects. The derived results can successfully be used in the research of the distribution of laser radiation in the atmosphere, laser communication systems, satellite communications, etc. For this expansion of the research, some additional assumptions are necessary, which correspond to the specifics of the spatial structure of the researched radiation.

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