Influence of Fluctuations of Laser Beam Direction on the Bit-Error Rate in Digital Space Communication Systems

Erwin Ferdinandov[1], Tsvetan Mitsev[2], Slavyan Saparev[3], Boryana Pachedjieva[4]

Abstract – **BER dependence for ground-to-space digital communication systems on the dispersion of laser beam axis random deviations from the center of receiving antenna aperture due to the turbulence degree of the atmosphere are presented. An account is given on the influence of distance, atmospheric attenuation, and background spectral radiance. The effects of the turbulence noise and photo detection quantum noise on *BER* are compared.**

Keywords – **Free space optics, Bit-error rate, Atmospheric attenuation, Atmospheric turbulence, Quantum noise.**

I. INTRODUCTION

The deep space expansion of contemporary civilization is accompanied from growing interest to the ground-to-satellite, satellite-to-ground and intersatellite free-space communication systems. Special attention is paid on the laser space communications that offer narrow beam divergence and possibilities to create multichannel systems with extraordinary high data rate [1-4]. In our work [5] is presented a general model analytical description of such a communication system.

It is very important the fact that narrow field of view and respectively too small cross-section of laser beam in the field of the receiving antenna aperture increase significantly influence of random fluctuation of laser light direction on the quality of system parameters. This is quite essential for the ground-to-satellite systems in which turbulized atmosphere is an initial section of the propagation medium. Atmospheric turbulence determines laser beam extension analytically described in [6]. Most important are random angular deviations of the laser beam from its central axis. Due to the great length of the space section of propagation medium as well as the relatively small general (diffraction and turbulence) beam scattering result, even small angular fluctuations of direction of laser beam can cause major cross linear deviations of laser beam in relation to receiving antenna aperture. Since the optical intensity in the beam periphery is smaller than the one in its centre, turbulence cross deviations will cause random changes of the optical flow through receiving aperture. Therefore, a peculiar turbulence noise will be added to the pulse-code signal, i.e. there will be increase of the possibility of erroneous recovery of the binary code pulse in a particular one-bit time interval of the signal. This means that the value of bit-error rate (*BER*) will rise.

Here we provide theoretical research work of the relationship between *BER* and the dispersion of fluctuation of laser light direction in different atmospheric and background conditions for the ground-to-space digital laser communication system. Using the theory of turbulence and the theory of laser beam propagation in turbulent medium, basically presented in [7-9], an analytical quantity evaluation of the influence of turbulence noise on BER value is obtained. The developed analytical model describing connection between *BER* and fluctuations of laser beam direction, independently of their origin, one can use in design of digital space communication systems.

II. THEORY

The intensity of Gaussian laser beam which corresponds to binary 1 is given by expression

$$I(z,y)=\frac{2\Phi_L\tau_T\tau_A}{\pi\varepsilon r^2(z)}\psi(z,y),\quad \psi(z,y)=\exp\left[-2\frac{y^2}{r^2(z)}\right], \quad (1)$$

where z is the transmitter-receiver antennas distance, y is the random linear deviation of the cross section laser beam centre from the receiving antenna aperture centre, Φ_L is the laser pulse energy, τ_T is the transmitter antenna efficiency, τ_A is the atmosphere transmittance, $\varepsilon = 1 - e^{-2} = 0{,}865$, and the expression

$$r(z)=r_0\sqrt{1+\left(\frac{K\lambda}{\pi r_0^2}\right)^2 z^2} \quad (2)$$

describes the diffraction radius of laser beam on distance z from the emitting aperture, $r_0 = r(z = 0)$, $K > 1$ is the experimentally determined coefficient to give an account for additional broadening of laser beam in the Fraunhofer field due to the incomplete initial cross-spatial coherence of the laser radiation.

For the atmospheric transmittance determination the Elterman model for extinction is used. As a rule in this model light extinction in the optical wavelength range is mainly determine from the troposphere and we have

$$\tau_A = \tau_{aer}.\tau_{mol}, \quad (3)$$

and

$$\tau_{aer} = \exp\left\{-\frac{3{,}92}{S_m b(S_m)}\left[1-\exp\left(-b(S_m)z_A\right)\right]\right\}, \quad (4)$$

where $b\left[\mathrm{km}^{-1}\right]=1{,}44\left(S_m[\mathrm{km}]\right)^{-0{,}2}$, S_m is the meteorological visual range for $z = 0$, z_A is the troposphere upper limit, and $\tau_{mol}(t_{z=0} = 20^\circ\mathrm{C}, p_{z=0} = 900\ \mathrm{mbar}, z_A = 15\ \mathrm{km}) \approx 0{,}9$.

[1] Erwin Ferdinandov is with the Faculty of Communication Techniques and Technologies, Kliment Ohridski blvd. 8, 1797 Sofia, Bulgaria.

[2] Tsvetan Mitsev is with the Faculty of Communication Techniques and Technologies, Kliment Ohridski blvd. 8, 1797 Sofia, Bulgaria, E-mail:Mitzev@tu-sofia.bg.

[3] Slavyan Saparev is with the Technical University-Branch Plovdiv, Canko Diustabanov St. 25, 4000 Plovdiv, Bulgaria.

[4] Boryana Pachedjieva is with the Technical University-Branch Plovdiv, Canko Diustabanov St. 25, 4000 Plovdiv, Bulgaria, E-mail: Pachedjieva@yahoo.co.uk.

The laser beam current diameter of $2r$ is significantly influenced by two groups of spatial turbulence inhomogeneities of the refraction index n of atmosphere air: relatively small inhomogeneities of $l<<2r$ and relatively big inhomogeneities of $l>>2r$. The influence of small inhomogeneities on beam structure expression in its not large additional extension with no change of the initial laser beam direction z. Quite essential to our analysis appear to be the big inhomogeneities, which determine the random angular deviations γ of laser beam from the axis z and practically do not affect $r(z)$. We shall ignore the influence of small inhomogeneities on the beam as well as the relatively weak connection between turbulence and diffraction changes of the spatial configuration of radiation. On one hand, the two currently ignored effects do not determine essential quantity changes, and on the other, their consequences are partially made up for.

The physical reason for the γ deviations lies in the turbulence determined random phase difference $\Delta\varphi_t$ between the optical oscillations in the central point $\rho = 0$ and the peripheral points $\rho = r(z)$ of the beam cross section. This difference is accumulated by radiation distribution through turbulent atmosphere layer. It can be localized equivalently in the conditional upper border plane $z = z_t$ of this layer and described by the structural function of cross phase distribution, calculated for $z = z_t$:

$$D_{\Delta\varphi_t} = \left\langle [\Delta\varphi_t(z_t, r(z_t))]^2 \right\rangle = \left\langle [\varphi_t(z_t, \rho = 0) - \varphi_t(z_t, \rho = r(z_t))]^2 \right\rangle. \quad (5)$$

The random phase difference $\Delta\varphi_t$ has a corresponding random cross plane oriented and random size deviation of the normal to the wave front of the beam at γ angle in relation to the axis z, i.e. one random oriented turn of the wave front of the random angle γ in relation to its centre. Of course, this turn is simply connected to the longitudinal shift $r(z_t)\gamma$ in the laser beam periphery. Therefore, introducing the wave vector module $k = 2\pi/\lambda$ we can write down

$$\Delta\varphi_t = kr(z_t)\gamma. \quad (6)$$

Replacing Eq.(6) in Eq.(5) results in

$$D_{\Delta\varphi_t} = k^2 r^2(z_t)\langle\gamma^2\rangle = k^2 r^2(z_t)\sigma_\gamma^2, \quad (7)$$

where σ_γ^2 is the γ dispersion (we have $\langle\gamma\rangle = 0$).

We use the well known formula for $D_{\Delta\varphi_t}$ of a spherical wave in homogeneously turbulized atmosphere. i.e. $C_n^2(z) = const$ in the interval $z \in [0, z_t]$ (with C_n^2 standing for the structural constant of n). For $z = 0$ this interval appears to be

$$D_{\Delta\varphi_t} = 1{,}095 C_n^2(0) k^2 z_t [r(z_t)]^{5/3}. \quad (8)$$

In real atmosphere, however, the following gradient is observed

$$C_n^2(z) = C_n^2(0) f(z), \quad (9)$$

where $f(z)$ is a fast decreasing function (example $f(z) \in [1; 0{,}1]$ for $z \in [0, 1 \text{ km}]$). Accepting the Gaussian model

$$f(z) = \exp\left(-\frac{z^2}{a^2}\right) \quad (10)$$

in Eq.(9) and replacing real atmosphere with an equivalent homogeneously turbulized layer at $C_{n,\text{eq}}^2 = C_n^2(0)$, we find out

$$z_{\text{t,eq}} = \int_0^\infty \exp\left(-\frac{z^2}{a^2}\right) dz = \frac{\sqrt{\pi} a}{2}. \quad (11)$$

With the condition $f(1 \text{ km}) = 0{,}1$ in Eq.(10), the calculation of Eq.(11) results in $z_{\text{t,eq}} \approx 0{,}6$ km.

Equalizing Eqs.(7) and (8) leads to

$$\sigma_\gamma^2 = 1{,}095 C_n^2(0) z_{\text{t,eq}} [r(z_{\text{t,eq}})]^{-1/3}. \quad (12)$$

For y we have the following relation

$$y = (z - z_{\text{t,eq}})\gamma, \quad \langle y \rangle = 0, \quad (13)$$

where z stands for the distance to space correspondent. Based on Eq.(13) we can write down

$$\sigma_y^2 = (z - z_{\text{t,eq}})^2 \sigma_\gamma^2 \quad (14)$$

and replacing Eq.(12) in Eq.(14) we receive

$$\sigma_y^2(z, C_n^2(0)) = 1{,}095 C_n^2(0) z_{\text{t,eq}} (z - z_{\text{t,eq}})^2 [r(z_{\text{t,eq}})]^{-1/3}. \quad (15)$$

The expression for signal power in the receiver aperture is $\Phi_L(z, y) = \pi R^2 \tau_R I(z, y)$. Here τ_R is the receiver antenna efficiency and R is the radius of the receiver aperture. Using Φ_L the cathode signal current of photomultiplier (PMP) is calculated by

$$i_S(z, y) = W(z)\psi(z, y), \quad (16)$$

where

$$W(z) = \frac{2}{\varepsilon r^2(z)} S_i \Phi_L \tau_T \tau_A \tau_R R^2, \quad (17)$$

and

$$S_i = \frac{e}{hc}\eta\lambda. \quad (18)$$

In Eqs.(17) and (18) S_i is the light output-current efficiency of PMP cathode, η is the quantum efficiency of PMP, $e = 1{,}6.10^{-19}$ C, $h = 6{,}626.10^{-34}$ Js, $c = 3.10^8$ m/s.

As the distance y fluctuate, the function $\psi(z, y)$ in Eqs.(1) and (16) is random and the current $i_S(z, y)$ fluctuate too. We accepted that the probability distribution of y is Gaussian with mean value $m_y = 0$ and dispersion σ_y^2. Using the expression for $\psi(z, y)$ given in Eq.(1) we find [6]

$$m_\psi(z, \sigma_y^2) = \frac{1}{\sqrt{1 + \frac{4\sigma_y^2}{r^2(z)}}},$$

$$\sigma_\psi^2(z, \sigma_y^2) = \frac{1}{\sqrt{1 + \frac{8\sigma_y^2}{r^2(z)}}} - m_\psi^2(z, \sigma_y^2), \quad (19)$$

and from Eq.(16)

$$m_{i_S}\left(z,\sigma_y^2\right)=W(z)m_\Psi\left(z,\sigma_y^2\right),\ \sigma_{i_S}^2\left(z,\sigma_y^2\right)W^2(z)\sigma_\Psi^2\left(z,\sigma_y^2\right),\quad(20)$$

where $m_{i_S}(z)$ is the mean signal current for the one bit time interval when we have binary 1.

Dispersion of current quantum fluctuations in the PMP cathode circuit is expressed as

$$\sigma_j^2\left(z,\sigma_y^2\right)=\sigma_{j_S}^2\left(z,\sigma_y^2\right)+\sigma_{j_b}^2+\sigma_{j_d}^2\ ,\qquad(21)$$

where

$$\sigma_{j_S}^2\left(z,\sigma_y^2\right)=2em_{i_S}\left(z,\sigma_y^2\right)\Delta f\ ,$$

$$\sigma_{j_b}^2(z)=2ei_b\Delta f\ ,\qquad \sigma_{j_d}^2(z)=2ei_d\Delta f\qquad(22)$$

are the dispersions of quantum fluctuations of signal current, background current, and dark current respectively, i_b is the mean value of background current, and i_d is the mean value of dark current, both last for the time interval with binary 1.

The mean dark current is defined by

$$i_d=\frac{i_{da}}{G_i}\ ,\qquad(23)$$

where i_{da} is the PMP anode dark current, and G_i is the PMP current gain. They are catalogue parameters.

It is easy to find the expression

$$i_b=\pi^2L_{b\lambda}\tau_R(\Delta\lambda)_{IF}S_iR^2R_p^2\frac{1}{f_{eq}^2}\ ,\qquad(24)$$

where $L_{b\lambda}$ is the background spectral radiance, $(\Delta\lambda)_{IF}$ is the optical bandwidth of the interference filter before the PMP, R_p is the radius of the PMP aperture, f_{eq} is the equivalent focal length of the receiver antenna.

As we mentioned above $i_S(z, y)$ is the random function of y. That is why $m_{i_S}\left(z,\sigma_y^2\right)$ is figured in (11) instead of $i_S(z, y)$.

To calculate *BER* we use the expression

$$BER\left(z,\sigma_y^2\right)=\frac{1}{2}\mathrm{erfc}\left[\frac{m_{i_S}\left(z,\sigma_y^2\right)}{2\sqrt{2}\sqrt{N\sigma_j^2\left(z,\sigma_y^2\right)+\sigma_{j_S}^2\left(z,\sigma_y^2\right)}}\right],\quad(25)$$

where

$$\mathrm{erfc}(x)=\int_x^\infty \exp\left(-\xi^2\right)d\xi$$

is the tabulated function, and N is the excess-noise factor of the PMP due to the amplification.

In order to estimate the effect of turbulence noise for the increase of *BER* we use the Eqs.(15) and (25). The following relation is made up

$$\beta\left(z,C_n^2(0)\right)=\frac{BER\left(z,C_n^2(0)\right)}{(BER)_0(z)}\ ,\qquad(26)$$

where $(BER)_0$ corresponds to *BER* without considering turbulence. The Turbulent-to-Quantum Noise Ratio (*TQNR*) is estimated by the formula

$$TQNR\left(z,C_n^2(0)\right)=\frac{\sigma_{i_S}\left(z,C_n^2(0)\right)}{\sqrt{N}\sigma_j\left(z,C_n^2(0)\right)}\ .\qquad(27)$$

III. CALCULATIONS

On the basis of the relations (1-25) an example calculation was done. The input parameters are: τ_T = 0,8; τ_R = 0,6; r_0 = 10 cm; Φ_L = 0,5 W; λ = 0,53 μm (yttrium aluminium garnet with 3-valent neodymium); Δf = 100 MHz; K = 6; $(\Delta\lambda)_{IF}$ = 20 Å; R = 4 cm; η = 0,1; R_p = 3 mm; i_{da} = 10 nA; G_i = 10^7; f_{eq} = 0,5 m; N = 1,5.

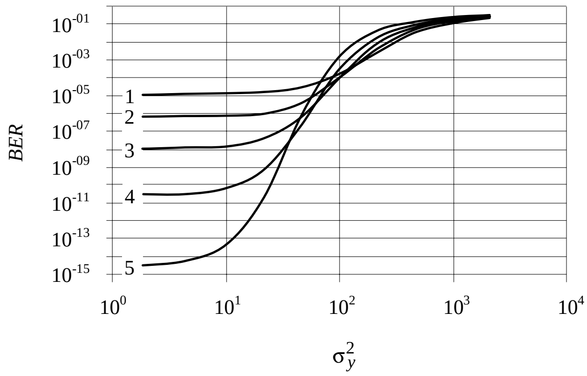

Fig. 1. *BER* as a function of σ_y^2 of random deviations *y* for S_m = 10 km, $L_{b\lambda}$ = 10^{-2} W/(m².sr.Å) and distances *z*: 1 - 6200 km; 2 - 5600 km; 3 - 5000 km; 4 - 4400 km; 5 - 3800 km

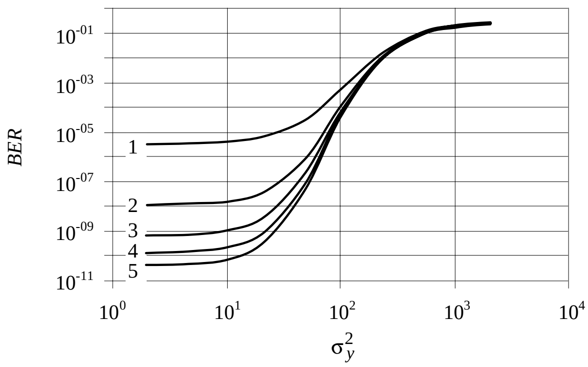

Fig. 2. *BER* as a function of dispersion σ_y^2 of random deviations *y* for *z* = 5000 km, $L_{b\lambda}$ = 10^{-2} W/(m².sr.Å) and visibilities S_m: 1 - 5 km; 2 - 10 km; 3 - 15 km; 4 - 20 km; 5 - 25 km

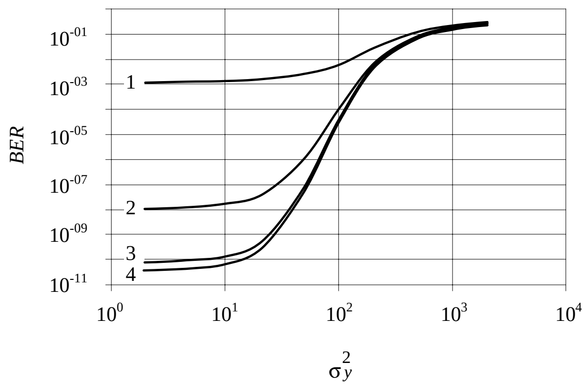

Fig. 3. *BER* as a function of dispersion σ_y^2 of random deviations *y* for *z* = 5000 km, S_m = 10 km and background spectral radiances $L_{b\lambda}$: 1 - 10^{-1} W/(m².sr.Å); 2 - 10^{-2} W/(m².sr.Å); 3 - 10^{-3} W/(m².sr.Å); 4 - 10^{-4} W/(m².sr.Å)

The *BER* results are shown plotted as a function of dispersion of random deviations y for different distances z (Fig.1), different meteorological visual ranges S_m (Fig.2), and different background spectral radiances $L_{b\lambda}$ (Fig.3). The dependence of *BER* on $C_n^2(0)$ for different distances z is shown in Fig.4. Plots of the function Eqs.(26) and (27) are shown in Fig.5 and Fig.6 respectively.

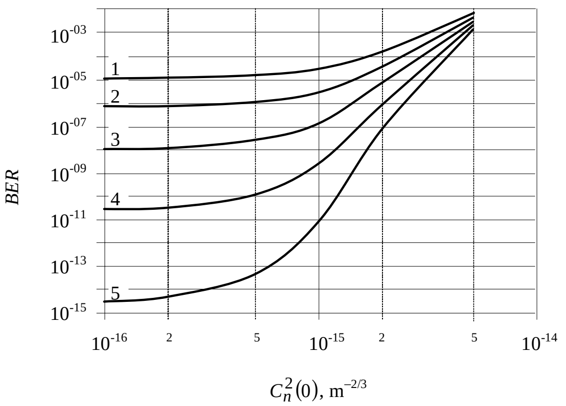

Fig. 4. *BER* as a function of structural constant $C_n^2(0)$ of the atmospheric turbulence for distances z:
1 - 6200 km; 2 - 5600 km; 3 - 5000 km; 4 - 4400 km; 5 - 3800 km

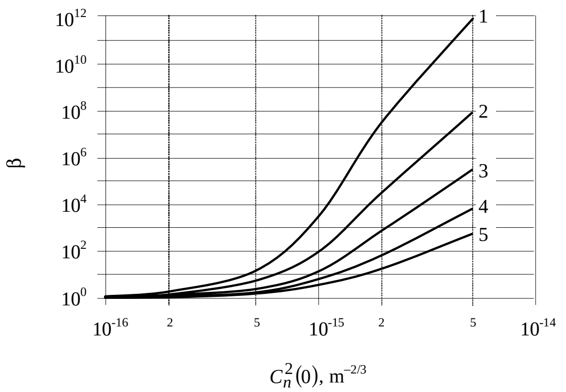

Fig. 5. β as a function of structural constant $C_n^2(0)$ of the atmospheric turbulence for distances z:
1 - 3800 km; 2 - 4400 km; 3 - 5000 km; 4 - 5600 km; 5 - 6200 km

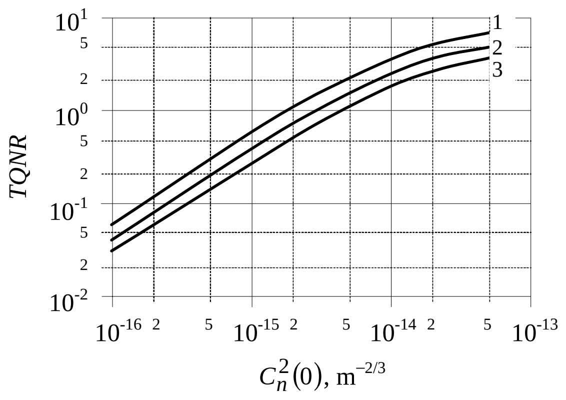

Fig. 6. Turbulent-to-Quantum Noise Ratio as a function of structural constant $C_n^2(0)$ of the atmospheric turbulence for distances z:
1 - 3800 km; 2 - 5000 km; 3 - 6200 km

IV. Conclusion

The *BER* values increase significantly with increase of the dispersion of random linear deviation y of the cross section laser beam centre from the receiving antenna aperture centre as is seen from the plots. It is easy to explain *BER* increase with change for the worse of the meteorological conditions and with reinforcement of the background spectral radiance (Fig.2 and Fig.3). Peculiar behavior of the curved lines $BER(\sigma_y^2)$ with z = vary on Fig. 1 is due to the circumstance that for small values of z the diffraction broadening of the laser beam is more weak and its peripheral parts more probably fall into receiver antenna aperture. Fig.4 and Fig.5 clearly show that *BER* and β increase with $C_n^2(0)$ relatively slowly in the example interval $C_n^2(0) < 5.10^{-16}$ m$^{-2/3}$. However, further increase of turbulization extent results in sharp increase of both gradients. Quite interesting appears to be the fact that the relative effect of turbulence noise is bigger at small distances. Such strange at first sight behavior of β can be easily explained if bearing in mind that, for example, for $C_n^2(0) \approx 10^{-15}$ m$^{-2/3}$ we have $(BER)_0 \approx 10^{-12}$ for z = 3800 km and $(BER)_0 \approx 10^{-5}$ for z = 6200 km. The curves in Fig.6 show that at relatively weak turbulence the quantum noise dominates over the turbulence one while at relatively strong turbulence it is the other way round. The two types of noise are commensurable for $C_n^2(0) \approx 3.10^{-15}$ m$^{-2/3}$.

References

[1] H. Hemmati, at al., "Comparative Study of Optical and Radio Frequency Communication Systems for a Deep Space Mission", JPL TDA Prog. Rep., pp. 42-128, February 1997.

[2] R. Strickland, M. Lavan, E. Woodbridge and V. Chan, "Effects of Fog on the Bit-Error Rate of a Free-Space Laser Communication System", Applied Optics, vol. 38, no. 3, pp. 424-431, 1999.

[3] D. Begley, "Laser Cross-Link Systems and Technology", IEEE Comm. Mag., Free Space Laser Comm., pp. 126-132, August 2000.

[4] K. Wilson, M. Enoch, "Optical Communications for Deep Space Missions", IEEE Comm. Mag., Free Space Laser Comm., pp. 134-139, August 2000.

[5] E. Ferdinandov, Ts. Mitsev, "Link Range of Free Space Laser Communication System", IEEE Trans., Microwave Review, vol 9. no. 2, pp. 41-42, 2003.

[6] S. Arnon, "Effects of atmospheric turbulence and building sway on optical wireless-communication systems", Optics Letters, vol. 28, no. 2, pp. 129-131, 2003.

[7] S. Panchev, *Random Functions and Turbulence*, New York, Toronto, Sydney, Braunschweig, Pergamon Press Oxford, 1971.

[8] A. Gurwich at al., *Laser radiation in turbulized atmosphere*, Moscow, Science Press, 1974. (russian)

[9] E. Ferdinandov, *Laser radiation in radiotechnics*, Sofia, Science Press, 1981. (bulgarian)