Accuracy of a Processing Algorithm, Using the Space Correlation of Satellite Movement in GPS

Marin S. Marinov[1] and Georgi V. Stanchev[2]

Abstract– **This paper discuses using of Kalman filtering to improve the accuracy of user locating when GPS is used. The state and observation equation are derived. The improvement of the accuracy by proposed algorithm is more than 80 percent. An analysis of model parameter influence is made.**

Keywords– **GPS, Kalman filtering, GPS accuracy.**

I. INTRODUCTION

In the present days the use of Global Positioning System (GPS) increases in daily life and often the locating of slow moving objects is needed. In this case the alteration of distances between the user and satellites depends on satellites orbital movement, when short time period is considered. The satellites transmit ephemeris data and some of them are related to the orbital parameters. Thus the user can calculate coordinates of satellites not only at the current but also at future moments in Earth-Centered-Earth-Fixed (ECEF) coordinate system [4]. This information can be used in the processing algorithm in order to increase the accuracy of the locating.

The locating of slow moving users through use of Kalman filtering is considered in this paper. The measurement equation and state equation of Kalman filter are derived.

II. PROCESSING ALGORITHM

Pseudoranges are measured in real GPS receivers and then they are used to calculate user coordinates. The measurements are always accompanied with errors. When the statistical distribution of these errors is not known the method of Least Mean Squares appears to be useful to determine the user location [2], [3].

The distance errors in GPS are supposed to possess Gaussian distribution with zero mean value and variance σ_D^2 [4]. The coordinate errors hold the same distribution as the distance errors. Kalman filter is a very good tool for parameter estimation in presence of additive errors with normal distribution [1]. An observation and a state equation have to be determined before Kalman filtering can be used.

The distance between the ECEF coordinate system origin and the n^{th} satellite at the moment $k\Delta t$ is given by:

$$\mathrm{R_n(k)} = \sqrt{\mathbf{C}_\mathrm{n}(\mathrm{k})\mathbf{C}_\mathrm{n}^\mathrm{T}(\mathrm{k})}\ , \tag{1}$$

where:

$$\mathbf{C}_\mathrm{n}(\mathrm{k}) = [\mathrm{x_n(k)}, \quad \mathrm{y_n(k)}, \quad \mathrm{z_n(k)}] \tag{2}$$

is a vector of satellite coordinates; Δt is the time step of the measurement.

The distance from user to a given satellite is:

$$\mathrm{D_n(k)} = \sqrt{[\mathbf{C}_\mathrm{n}(\mathrm{k}) - \mathbf{C}_\mathrm{c}(\mathrm{k})][\mathbf{C}_\mathrm{n}(\mathrm{k}) - \mathbf{C}_\mathrm{c}(\mathrm{k})]^\mathrm{T}}\ , \tag{3}$$

where:

$$\mathbf{C}_\mathrm{c}(\mathrm{k}) = \mathbf{C}_0(\mathrm{k}) + \mathbf{C}_\delta(\mathrm{k}) \tag{4}$$

is a vector of calculated user coordinates;

$$\mathbf{C}_0(\mathrm{k}) = [\mathrm{x_0(k)}, \quad \mathrm{y_0(k)}, \quad \mathrm{z_0(k)}] \tag{5}$$

is a vector of true user coordinates;

$$\mathbf{C}_\delta(\mathrm{k}) = [\delta_\mathrm{x}(\mathrm{k}), \quad \delta_\mathrm{y}(\mathrm{k}), \quad \delta_\mathrm{z}(\mathrm{k})] \tag{6}$$

is a vector of the errors in the calculated user coordinates.

Both sides of Eq.(3) are raised to the second power then as a result the root in Eq.(3) disappears. However this leads to appearance of the second power of coordinate errors. The difference of the distance squares between the user and two satellites is obtained in order to avoid the squares of errors:

$$\begin{aligned}\Delta_\mathrm{nm}(\mathrm{k}) &= \mathrm{D_n^2(k) - D_m^2(k)} = \\ &= \mathrm{R_n^2(k) - R_m^2(k)} + 2[\mathbf{C}_\mathrm{m}(\mathrm{k}) - \mathbf{C}_\mathrm{n}(\mathrm{k})]\mathbf{C}_\mathrm{c}^\mathrm{T}(\mathrm{k})\end{aligned}\ . \tag{7}$$

The distances $R_n(k)$ and $R_m(k)$ can be derived from ephemeris data, i.e. they are known and can be moved on the left side in Eq.(7):

$$\begin{aligned}\mathrm{E_{nm}(k)} &= \Delta_\mathrm{nm}(\mathrm{k}) - [\mathrm{R_n^2(k) - R_m^2(k)}] = \\ &= 2[\mathbf{C}_\mathrm{m}(\mathrm{k}) - \mathbf{C}_\mathrm{n}(\mathrm{k})][\mathbf{C}_0(\mathrm{k}) + \mathbf{C}_\delta(\mathrm{k})]^\mathrm{T}\end{aligned}\ . \tag{8}$$

The coordinates that should be estimated are three and consequently three observations like this described by Eq.(8) have to be used. Every GPS receiver uses constellation of four satellites at least and as a result there is no problem to obtain these observations. The matrix form of the observation equations in Kalman filtering can be obtained by Eq.(8):

$$\mathbf{E}(\mathrm{k}) = \mathbf{H}(\mathrm{k})\boldsymbol{\xi}(\mathrm{k}) + \mathbf{v}(\mathrm{k}) = [\mathrm{E_{12}(k)}, \quad \mathrm{E_{34}(k)}, \quad \mathrm{E_{13}(k)}]^\mathrm{T}, \tag{9}$$

where: $\mathbf{H}(\mathrm{k}) = 2[\mathbf{C}_\mathrm{D}(\mathrm{k}), \quad \mathbf{0}]$; **0** is zero matrix 3 by 3 dimensions; $\xi(\mathrm{k}) = [\mathbf{C}_0(\mathrm{k}), \quad \dot{\mathbf{C}}_0(\mathrm{k})]^\mathrm{T}$ is the state vector;

[1]Marin S. Marinov, PhD is with the Aviation Faculty, National Military University, 5856 Dolna Mitropolia, Bulgaria, e-mail: mmarinov2000@yahoo.com.

[2]Georgi V. Stanchev, PhD is with the Aviation Faculty, National Military University, 5856 Dolna Mitropolia, Bulgaria, e-mail: gstanchev@af-acad.bg.

$$\mathbf{C}_D(k) = [\mathbf{C}_2(k) - \mathbf{C}_1(k), \mathbf{C}_4(k) - \mathbf{C}_3(k), \mathbf{C}_3(k) - \mathbf{C}_1(k)]^T;$$

$$\mathbf{V}_v(k) = 4\sigma_D^2 \begin{bmatrix} D_1^2(k)+D_2^2(k) & 0 & D_1^2(k) \\ 0 & D_3^2(k)+D_4^2(k) & -D_3^2(k) \\ D_1^2(k) & -D_3^2(k) & D_1^2(k)+D_3^2(k) \end{bmatrix}$$

is the covariance matrix of the measurement error; $\sigma_D{}^2$ is the variance of distance errors.

The error elements on the right side of Eq.(9) are expressed by the distance errors:

$$\delta_{nm} = 2D_{0n}\delta_n - 2D_{0m}\delta_m, \tag{10}$$

where $D_{0n}(k)$ is the true distance to the n^{th} satellite and $\delta_{Dn}(k)$ is the measurement errors.

A model for user motion is needed to use Kalman filtering. The following model is useful for slow moving objects [5]:

$$\frac{d\xi(t)}{dt} = \mathbf{\Phi}\left[\mathbf{C}_0^T(t), \quad \dot{\mathbf{C}}_0^T(t)\right]^T + \mathbf{w}(t), \tag{11}$$

where: $\mathbf{w}(t) = [0,\ 0,\ 0,\ w_x(t),\ w_y(t),\ w_z(t)]$ is normally distributed noise component; $\mathbf{\Phi} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\alpha\mathbf{I} \end{bmatrix}$ is the transition matrix; α is coefficient reciprocal to the correlation time of the velocities in each coordinate; **I** is an identity matrix with 3 by 3 dimensions.

The state equation of Kalman filter is obtained directly from Eq.(11):

$$\xi(k) = \mathbf{F}\xi(k-1) + \mathbf{w}(k), \tag{12}$$

where: $\mathbf{F} = \begin{bmatrix} \mathbf{I} & (1-e^{-\alpha T})\alpha^{-1}\mathbf{I} \\ \mathbf{0} & e^{-\alpha T}\mathbf{I} \end{bmatrix}$ is the state transition matrix; T is the sampling time interval; $\mathbf{V}_w = \alpha T \sigma_v^2 \begin{bmatrix} \frac{2}{3}T^2\mathbf{I} & T\mathbf{I} \\ T\mathbf{I} & 2\mathbf{I} \end{bmatrix}$ is the covariance matrix of normally distributed noise component in Eq.(12); σ_v is standard deviation (STD) of the user velocity that is assumed to be equal for each of the coordinates.

The state vector estimate is given by equation:

$$\hat{\xi}(k) = \tilde{\xi}(k) + \mathbf{K}(k)\left[\mathbf{E}(k) - \mathbf{H}(k)\tilde{\xi}(k)\right]. \tag{13}$$

The Kalman gain $\mathbf{K}(k)$ is described by:

$$\mathbf{K}(k) = \tilde{\mathbf{V}}_\xi(k)\mathbf{H}^T(k)\left[\mathbf{H}(k)\tilde{\mathbf{V}}_\xi(k)\mathbf{H}^T(k) + \mathbf{V}_v(k)\right]^{-1}, \tag{14}$$

where:

$$\tilde{\mathbf{V}}_\xi(k) = \mathbf{F}\mathbf{V}_\xi(k-1)\mathbf{F}^T + \mathbf{V}_w \tag{15}$$

is the predicted covariance matrix of the state vector.

The error covariance matrix is obtained by:

$$\mathbf{V}_\xi(k) = [\mathbf{I} - \mathbf{K}(k)\mathbf{H}(k)]\tilde{\mathbf{V}}_\xi(k). \tag{16}$$

The error covariance matrix is independent of the state vector estimate. It defines the time after which Kalman filter is in the stationary state. Diagonal elements of matrix $\mathbf{V}_\xi(k)$ are the variances of coordinate and velocity errors.

The most complex mathematical operation which has to be done during filtering is inverting of the matrix in Eq.(14). However the dimensions of the matrix are only 6 by 6 and there is no difficulty in inverting it.

III. Researches

The accuracy of proposed algorithm is studied at different standard deviations of distance errors σ_D. Different values for standard deviation of user velocity σ_v are used in order to estimate the influence of this parameter of user motion model. The sampling time Δt=0.01 sec. is used. Some of the results are shown in the following figures.

Standard deviations of estimates are shown in Fig.1. The results displayed in the figure are at STD of distance errors σ_D=18 m and the following values of model parameters: σ_v=6 m/s; α=0.2. These results are obtained by computer simulation of Eq.(16).

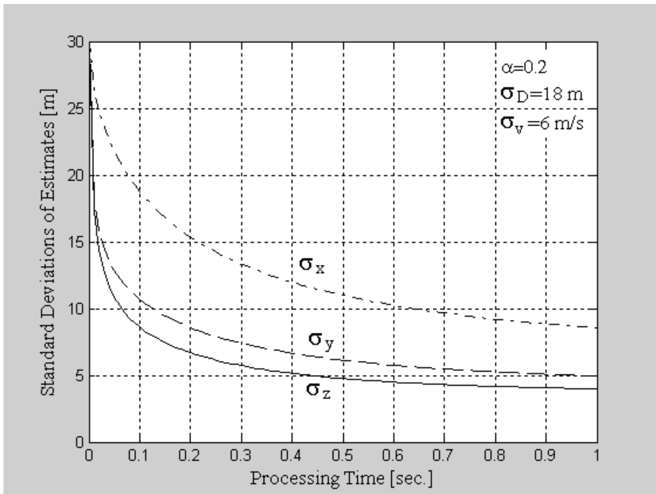

Fig.1. STD of estimates.

As one can see in Fig.1 the values of standard deviations first decrease fast and then the slopes of curves are smooth. The results indicate that the relative differences between STDs of estimates, when the processing time is T=0.5 sec. and when it is T=1 sec., are less than 3 dB. The studies show that the values of STDs of estimates are highest when the STD of velocities in the model is σ_v=6 m/s. The differences between minimal and maximal values of STDs of estimates for different velocity STDs are small.

The results for STD of z-coordinate estimate σ_z as a function of parameter α and STD of distance errors σ_D are shown in Fig.2. The worst case for velocities STD of σ_v=6 m/s is chosen. The STD of z-coordinate estimate is calculated for two values of processing time: T=0.05 sec. and T=1 sec. respectively.

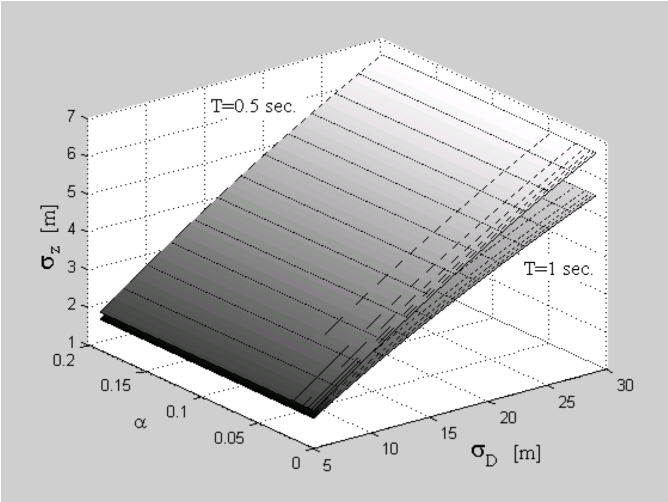

Fig.2. STD of z-coordinate estimate at σ_v=6 m/s.

The results indicate that the influence of parameter α is insignificant. The STD of *z*-coordinate estimate increases when the STD of distance errors increases. The relation between both STDs is not proportional. It is obvious that the accuracy is better when the processing time is longer.

The fact that the STDs of estimates for different coordinates are not equal is due to the user relative geometry. The influence of the geometry on accuracy can be estimated, using concept of dilution of precision (DOP) [4]. If the user's location is calculated directly from measured distances then the STDs of coordinate estimates are calculated by the following equations:

$$\sigma_x=\text{XDOP}\ \sigma_D;\ \ \sigma_y=\text{YDOP}\ \sigma_D;\ \ \ \sigma_z=\text{ZDOP}\ \sigma_D, \qquad (17)$$

where XDOP, YDOP and ZDOP are delusion of precisions of each coordinate respectively. The results shown in all figures are at the following values of DOPs: XDOP=4.2407; YDOP=2.0134; ZDOP=1.8466.

The improvement in accuracy (in percent), when the proposed Kalman filter is used, is shown in Fig.3 and Fig.4.

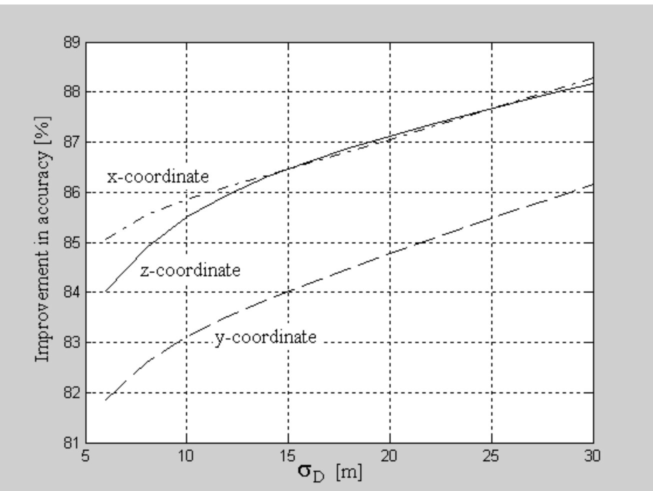

Fig.3. Improvement in accuracy at σ_v=6 m/s and T=0.5 sec.

The improvement is more than 80% in case of processing with proposed Kalman filter. The improvement grows when STD of distance errors increases.

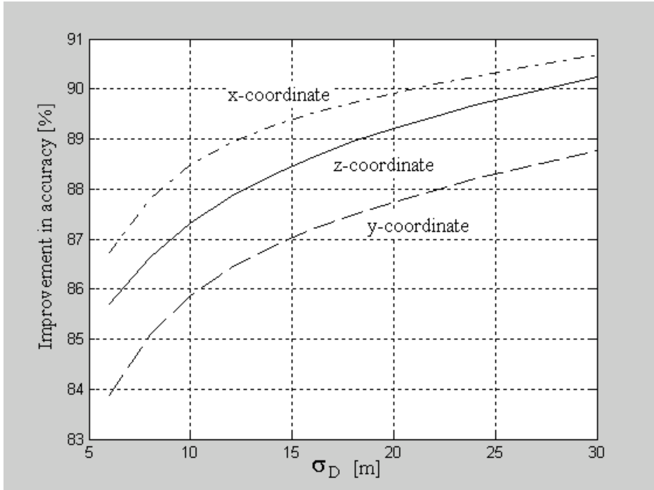

Fig.4. Improvement in accuracy at σ_v=6 m/s and T=1 sec.

The improvement at processing time *T*=1 sec. is bigger in comparison with processing time *T*=0.05 sec. The values of accuracy improvement reach more than 90% for *x* and *z* coordinates. The studies display that the user's relative geometry does not affect on the improvement degree in accuracy.

The results from a computer simulation are shown in the following figures. A user motion is generated for the equal values of velocities in each coordinate, i.e. $V_x=V_y=V_z$. These results are obtained by simulation of Eq.(13) and Eq.(15).

Mean values of estimate errors as a function of user velocity at σ_v=0.1 m/s and α=0.2 are displayed in Fig.5.

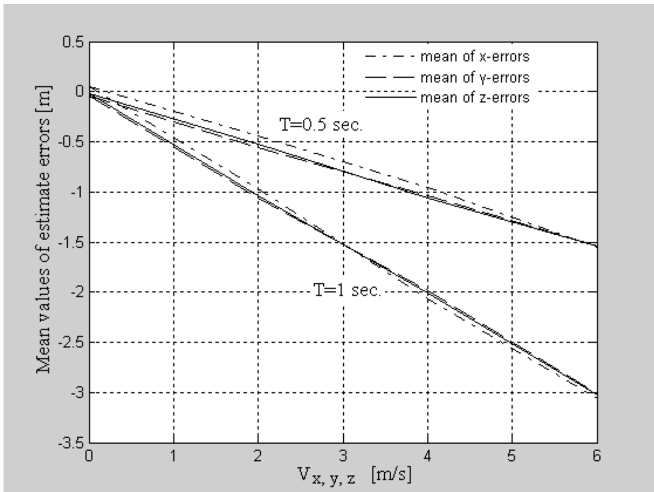

Fig.5. Mean values of estimate errors at σ_v=0.1 m/s and α=0.2.

It is clearly seen that the bias of estimates grows up when the user velocity increases. This is because of differences between model and real user motion. The bias at processing time *T*=1 sec. is bigger than at processing time *T*=0.5 sec. Maximal bias is about -3 m and -1.5 m for processing time *T*=1 sec. and *T*=0.5 sec. respectively. The mean values of estimate errors are almost equal for all three coordinates. Even with these model parameters the bias is small enough.

The STD of *x*-coordinate estimate at σ_v=0.1 m/s and α=0.2 is shown in Fig.6.

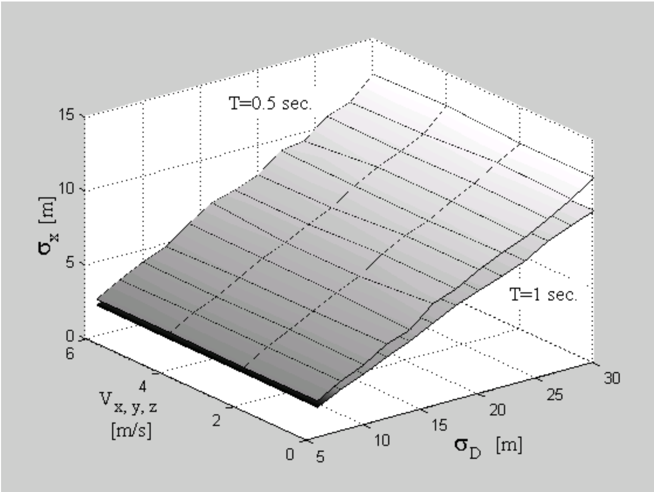

Fig.6. STD of x-coordinate estimate at σ_v=0.1 m/s and α=0.2.

It is obvious that the influence of user velocities $V_{x,\ y,\ z}$ on the STD of *x*-coordinate estimate is insignificant. The standard deviation is smaller when the processing time is longer. The results from simulation indicate that the improvement of accuracy, when processing time is *T*=1 sec., is below 3 dB in comparison with the case of processing time *T*=0.5 sec. The STD of *x*-coordinate estimate is less than 15 m.

Mean values of estimate errors as a function of user velocity at σ_v=6 m/s and α=0.2 are displayed in Fig.7 for two values of processing time.

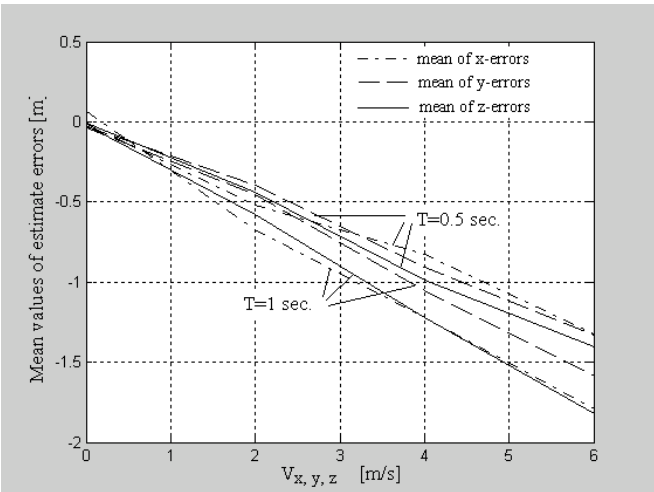

Fig.7. Mean values of estimate errors at σ_v=6 m/s and α=0.2.

It is readily seen that the bias of estimates grows up when the user velocity increases. The difference between biases at processing times *T*=1 sec. and *T*=0.5 sec. is not so much. The bias of coordinate estimates is less significant than in the case shown in Fig.5. The bias is below 1 m when the user velocities are less than 3 m/s. The mean values of estimate errors are almost equal for all three coordinates.

The STD of *x*-coordinate estimate at σ_v=6 m/s and α=0.2 is shown in Fig.8.

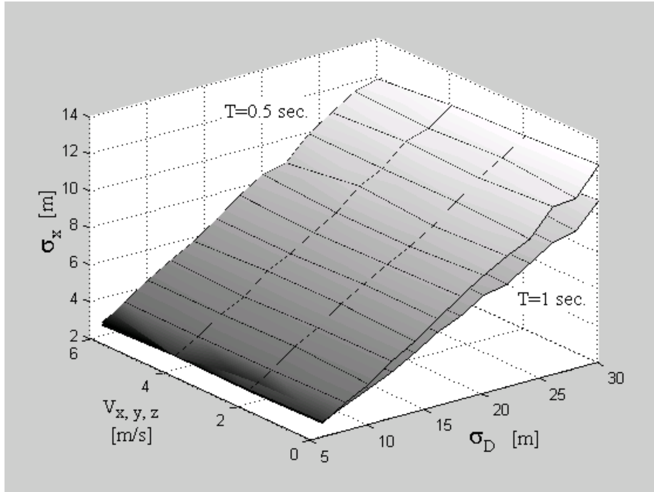

Fig.8. STD of x-coordinate estimate at σ_v=6 m/s and α=0.2.

The influence of user velocities $V_{x,\ y,\ z}$ on the STD of *x*-coordinate estimate is small but clearly seen in Fig.8. The accuracy increases for bigger values of user velocities. The results from simulation indicate that the improvement of accuracy, when processing time is *T*=1 sec., is below 3 dB in comparison with the case of processing time *T*=0.5 sec. The STD of *x*-coordinate estimate is below 14 m. The studies show that the grater value of σ_v in the model better matches the real user motion.

IV. Conclusion

The proposed model of user motion and the formed observation equation permit using of the Kalman filtering. The general advantage of formed observation is that any approximation is not implemented and thus any additional errors are not introduced in the processing algorithm. The results prove that the improvement in accuracy is great in comparison with direct user's location calculating. The studies show that by careful choosing of the model parameters an additional improvement in accuracy can be achieved. The DOPs usually are less than 2.5 and in the most cases the use of proposed algorithm will lead to errors of a few meters only.

References

[1] A. P. Sage, J. L. Melse, *Estimation Theory with Application to Communication and Control*, Moscow, 1976.

[2] M. Marinov, G. Stanchev, *Use of Space Correlation of Satellite Move in GPS*, Proceeding of XXXVII International Scientific Conference ICEST, 16-18 October 2003, Sofia, Bulgaria, 2003, pp.132-135.

[3] M. Marinov, G. Stanchev, *Use of Joint Space Correlation of Satellite Constellation Move in GPS*, Proceeding of XXXVII International Scientific Conference ICEST, 16-18 October 2003, Sofia, Bulgaria, 2003, pp.128-131.

[4] *Understanding GPS Principles and Applications*, Editor E. D. Kaplan, Artech House, London, 1996.

[5] V. Ivanov, M. Marinov, *Stochastic Theory of Aircraft Radio Engineering Devices*, Sofia, Bulgaria, 1999.