Determinating the amplitudes of intermodulation products of higher order by means Volterra kernels

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***Abstract* - Nonlinear distortions and the resulting nonlinear products in the systems for cable television present a fundamental problem for the quality receiving of the transmitted signals by such systems. In this paper we describe an algorithm for computation of the amplitudes of the nonlinear products of higher order.**

***Key words* - intermodulation distortions, nonlinear products, cable television products, binomial coefficients, Volterra kernels.**

I. INTRODUCTION

The quality receiving of transmitted products in a current CATV system is mainly determined by the possibility for nonlinear distortions in such a system. In many papers /1,2,3/ the resulting nonlinear products are considered up to distortions of third order and limited number of frequencies /channels/.

In the general case the combination frequencies are determined by the formula:

$$f = r_1 f_1 + r_2 f_2 + r_3 f_3 + ... = \sum_{i=1}^{N} r_i f_i \qquad (1)$$

where r_i are arbitrary integers, possibly equal to zero. If the transmission characteristic of the system is of n-th order, then the coefficients $r_1, r_2, r_3 ...$, need to satisfy the inequality $|r_1| + |r_2| + |r_3| + ... \leq n$

When it is needed to compute the amplitudes of the intermodulation products of a very high order n, in the case of many frequencies (channels) N, using trigonometric formulas and solving multiple integrals is too complicated and cumbersome.

Below we describe an algorithm for computation of the amplitude of each nonlinear intermodulation product in the presence of arbitrarily large number of sinusoidal signals at the input. Our treatment of the relation between each intermodulation product and the parameters of the input signals for the system (as well as its transfer characteristics, given by the Volterra kernels of the related order) is based upon the use of binomial coefficients.

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II. DESCRIPTION OF THE ALGORITHM.

Assume that the nonmodulated signal at the input of the system is given by the formula

$$x(t) = \sum_{i=1}^{N} A_i \cos(\omega_i t + \theta_i), \qquad (2)$$

and its nonlinearity is described by the polynomial

$$y(t) = \sum_{n=1}^{\infty} |H_n(f)| . x^n(t) \qquad (3)$$

The amplitudes of each nonlinear product of the intermodulation satisfy the following equation:

$$A = \frac{S}{2^{n-1}} |H_n| . \prod_{i=1}^{N} A_i^{d_i} \qquad (4)$$

where A_i is the amplitude of the i-th input frequency;

d_i - is the coefficient of the i-th input frequency in the nonlinear product;

i - taking values from 1 to N, labels the input frequencies;

$|H_n|$ - is the Volterra kernel of n-th order for the corresponding nonlinear product;

$|\mathrm{H}_n| \equiv |H_n(f)|$

n -is the degree of the polynomial, describing the nonlinearity of the system /coincides with the order of the nonlinear product/;

S - is the coefficient that characterizes the number of the nonlinear products with equal frequency at the output of the system.

$$S = \prod_{i=1}^{N} C_{p_i}^{d_i} \qquad (5)$$

which represents a product of binomial coefficients [5], obtained by the following formula:

$$C_{p_i}^{d_i} = \frac{p_i (p_i - 1)(p_i - 2).....[p_i - (d_i - 1)]}{d_i!} \qquad (6)$$

$$p_i = n - \sum_{i=1}^{i} d_{i-1} \qquad (7)$$

$$|H_n| = \frac{y_n^{(n)}(t)}{n!} \qquad (8)$$

where $y_n^{(n)}(t)$ is the n-th derivative of the n-th output signal with respect to x(t).

III. AN EXAMPLE:

Let n=3 and N=3, and let the nonlinear product be $f_1 \pm f_2 \pm f_3$.

Then:

1. First we determine the numbers d_i for each frequency

$$d_1 = 1; d_2 = 1; d_3 = 1$$

2. Second we compute the values of p_i. Since the nonlinear product is composed from three frequencies, we look for p_1, p_2, and p_3.

$p_1 = 3 - (d_{1-1}) = 3 - d_0$ but $d_0 = 0$

Therefore $p_1 = 3$

$$p_2 = 3 - (d_{1-1} + d_{2-1}) = 3 - (d_0 + d_1)$$
$$= 3 - (0 + 1) = 2$$
$$p_3 = 3 - (d_{1-1} + d_{2-1} - d_{3-1})$$
$$= 3 - (d_0 + d_1 + d_2) = 3 - (0 + 1 + 1) = 1$$

3. Next we compute the binomial coefficients $C_{p_i}^{d_i}$

$$C_{p_1}^{d_1} = C_3^1 = \frac{3}{1!} = 3;\ C_{p_2}^{d_2} = C_2^1 = \frac{2}{1!} = 2;$$
$$C_{p_3}^{d_3} = C_1^1 = \frac{1}{1!} = 1$$

4. We compute the coefficient S:

$$S = C_{p_1}^{d_1} C_{p_2}^{d_2} C_{p_3}^{d_3} = C_3^1 C_2^1 C_1^1 = 6$$

5. Next we compute the Volterra kernel for the corresponding nonlinear product. From formula (3) with $n = 3$ we deduce

$y_3(t) = |H_3(f)| x^3(t)$, therefore

$y_3^{(3)}(t) = \left[|H_3(f)| x^3(t) \right]^{(3)}$, that is

$y_3^{(3)}(t) = 6 |H_3(f)|$. This is substituted in (8) and gives

$$|H_3| = \frac{6|H_3(f)|}{3!} = \frac{6|H_3(f)|}{1.2.3} = |H_3(f)|$$

6. Finally we compute the amplitude of the nonlinear product with frequency $f = f_1 \pm f_2 \pm f_3$.

$$A = \frac{6}{2^{3-1}} |H_3(f_1 \pm f_2 \pm f_3)| A_1 A_2 A_3$$
$$= \frac{6}{2^2} |H_3(f_1 \pm f_2 \pm f_3)| A_1 A_2 A_3$$
$$= \frac{3}{2} |H_3(f_1 \pm f_2 \pm f_3)| A_1 A_2 A_3 \qquad (9)$$

The expression /8/ is identical with the corresponding expression in Table 1,[1], determined by using trigonometric identities and by evaluating multiple integrals.

TABLE I

nonlinear products		frequency	amplitudes
order	type		
4	composite	$2f_1 \pm 2f_2$	$3/4\|H_4\|.A_1^2.A_2^2$
		$2f_1 \pm 2f_3$	$3/4\|H_4\|.A_1^2.A_3^2$
		$2f_2 \pm 2f_3$	$3/4\|H_4\|.A_2^2.A_3^2$
		$3f_1 \pm f_2$	$1/2\|H_4\| A_1^3.A_2.$
		$3f_1 \pm f_3$	$1/2\|H_4\| A_1^3.A_3$
		$3f_2 \pm f_1$	$1/2\|H_4\| A_2^3.A_1$
		$3f_2 \pm f_3$	$1/2\|H_4\| A_2^3.A_3$
		$3f_3 \pm f_1$	$1/2\|H_4\| A_3^3.A_1$
		$3f_3 \pm f_2$	$1/2\|H_4\| A_3^3.A_2$
		$2f_1 \pm f_2 \pm f_3$	$3/2\|H_4\|.A_1^2.A_2.A_3$
		$f_1 \pm 2f_2 \pm f_3$	$3/2\|H_4\|.A_1.A_2^2.A_3$
		$f_1 \pm f_2 \pm 2f_3$	$3/2\|H_4\|.A_1.A_2.A_3^2$

In a similar manner, following the above algorithm, one can determine the amplitudes of each combination of frequencies for arbitrary values of n and N without the need of complicated trigonometric computations. Same intermodulation products for n=4 and N=3 are presented in Table I.

IV. CONCLUSION

The suggested algorithm can be applied to determine the amplitudes of intermodulation products for large values of n and N for arbitrary combinations of frequencies (see Eq.1). A program presenting in a tabular and a graphical form the results from formula (4) will be the subject of a forthcoming publication.

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