Application of the Volterra kernels for calculation of the crossmodulation and modulation distortions

Oleg Borisov Panagiev[1]

Abstract - **In this article the crossmodulation (CMD) and modulation distortions (MD) will be viewed, because they have more significant influence on the quality of the image. The sequences of Volterra will be used as a mathematical apparatus, particularly the Volterra kernels, which are the coefficients of the polynomial of third order in the mathematical model.**

Key words - **crossmodulation (CMD), modulation distortions (MD), nonlinear products, Volterra kernels, Hybrid Fiber Coaxial(HFC) network.**

I. INTRODUCTION

During the service of the contemporary cable television (CATV) systems under certain conditions [1] in the active devices, used in the system, non-linear products (HP) are emerging. Thus the subscribers receive except the useful signal also parasitic (disturbing) signals, which in a number of cases of high non-linearity are prevailing over the useful signal (especially in case of transmitting of digital signals, as well as digital and analogue – Hybrid Fiber Coaxial network- HFC). Non-linearity is approximated most often with polynomial of third power, because the theoretical researches through mathematical models are comparatively feasible. The availability of HP leads to worsening of the ratio C/N and hence to decreasing of BER, what is with special importance in transmitting of digital signals [2].

Crossmodulation (CMD) and modulation distortions (MD) will be viewed, because they have more significant influence on the quality of the image. The Volterra kernels, which are the coefficients of the polynomial of third order in the mathematical model, will be used as a mathematical apparatus,. For the active devices (mostly the cable amplifiers and laser diodes), the Volterra kernels are their transfer functions of the corresponding order for the particular combination frequency.

II. CHARACTER OF THE CROSSMODULATION AND MODULATION DISTORTIONS

When the non-linear distortions in the system are described with polynomials of third and upper order, at specific amplitude and phase ratios useful information (image) is transferred from the carrying frequency of one channel to the carrying frequency of another channel. In case of signals, transferring an image, on the screen of the television receiver are seen both the desired and the disturbing images - and at specific quantitative ratios - just the disturbing. In addition to problems in the brightness signal, such are displaying in the color-different signals, as the color is worsened, changed or lost at all. In a number of cases the horizontal and/or vertical synchronization could be disrupted.

Object of the present article further down will be the qualitative ratios in cases of cross-modulation and modulation distortions (products).

1. Crossmodulation distortions

They appear when the non-linearity of CATV systems is from third and upper order, i.e. n=3,4....Their study is made for one tested channel [1], through which is transferred just carrying signal (harmonious), and all other signal are modulated with one and the same frequency (fig.1). Let the tested channel frequency is f_s, and the modulated frequencies are $(f_r \pm \Omega)$, $r = 1,2,3....(N-1) \neq s$.

At fig.1a is shown part of the spectrum of CATV systems until the moment of occurrence of cross-modulation distortions. The tested channel has higher frequency.

Fig.1b illustrates the spectrum of CATV systems after occurrence of CMD. Around f_s two new signals appear with frequencies $f_r \pm \Omega$ that are the disturbing (unwanted) signals. The most severe case is presented when all signals have equal phases, i.e. amplitudes are summed up.

If we assume that n=3 than the inter-relation could be written as follows:

$$f_s \pm (f_r \pm \Omega) \pm f_r \tag{1}$$

We open the parenthesis consecutively and perform the mathematical operations and as a result we get four new signals that fall into the test channel:

$$f_s + (f_r + \Omega) + f_r = f_s + f_r + \Omega + f_r = f_s + \Omega + 2f_r \tag{2}$$

$$f_s + (f_r - \Omega) + f_r = f_s + f_r - \Omega + f_r = f_s - \Omega + 2f_r \tag{3}$$

$$f_s - (f_r - \Omega) - f_r = f_s - f_r + \Omega - f_r = f_s + \Omega - 2f_r \tag{4}$$

$$f_s - (f_r + \Omega) - f_r = f_s - f_r - \Omega - f_r = f_s - \Omega - 2f_r \tag{5}$$

[1]O. B. Panagiev is a system engineer at the Technical University of Sofia, e-mail: ctv@alpha.tu-sofia.bg

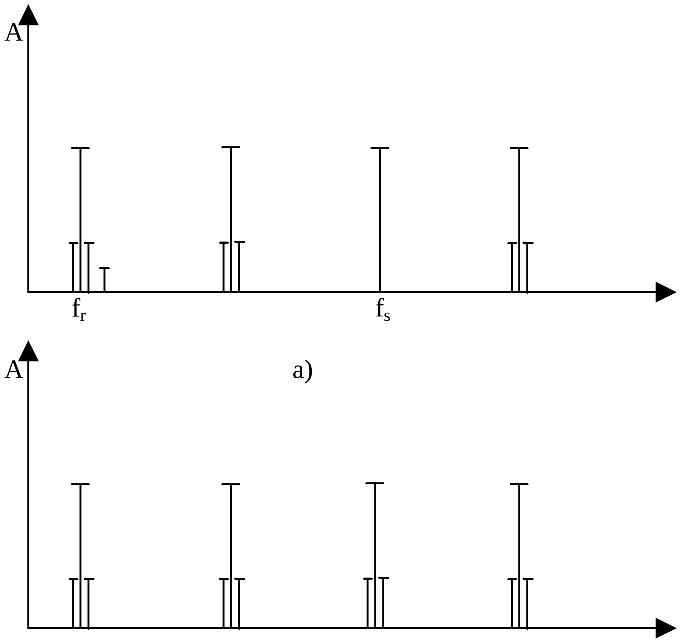
$$f_s-(f_r+\Omega)+f_r=f_s-f_r-\Omega+f_r=f_s-\Omega \quad (6)$$

$$f_s-(f_r-\Omega)+f_r=f_s-f_r+\Omega+f_r=f_s+\Omega \quad (7)$$

$$f_s+(f_r+\Omega)-f_r=f_s+f_r+\Omega-f_r=f_s+\Omega \quad (8)$$

$$f_s+(f_r+\Omega)-f_r=f_s+f_r+\Omega-f_r=f_s+\Omega \quad (9)$$

Resulting frequencies obtained from formulae (2) ÷ (5) fall out of the tested channel and will not be object of further considerations.



a)

b)

Fig.1.

Frequencies obtained from formulae (6) ÷ (9) fall into the tested channel and present nonlinear products of cross-modulation. In case of equal phases they superposition and their joint presentation is given on fig.1b.

2. Modulation distortions

These distortions arise when the carrier frequency (f_s) of a given channel aggregates with a signal from other channel with double modulation frequency (f_r), that is $f_s \pm 2\Omega$ (fig.2a.). These are wide-band distortions that influence both the tested channel and the adjacent channels. This fact is especially embarrassing for the present-day CATV systems where the channels are transmitted one next to other (channel beside channel) and especially for HFC network (fig.2b.), where analogue channels are transmitted up to about 470 MHz, and digital channels with M-QAM - above 470 MHz [2].

fig.2

When modulation distortions arise in CATV systems at n=3, the interrelation between the carrier frequency of the tested channel and the modulated carrier frequency of the disturbing channel is shown by the expression:

$$f_s \pm (f_r \pm \Omega) \pm (f_r - \Omega) \quad (10)$$

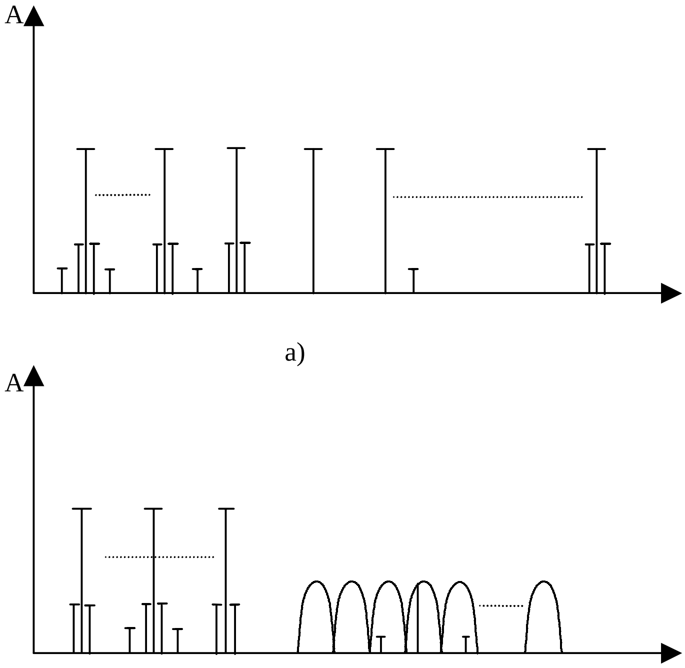
After consecutive performance of the mathematical operations in (10), four new signals are derived with frequencies as follows:

$$f_s+(f_r+\Omega)+(f_r-\Omega)=f_s+2f_r \quad (11)$$

$$f_s-(f_r+\Omega)+(f_r-\Omega)=f_s-2f_r \quad (12)$$

$$f_s+(f_r+\Omega)+(f_r-\Omega)=f_s+2\Omega \quad (13)$$

$$f_s-(f_r+\Omega)+(f_r-\Omega)=f_s-2\Omega \quad (14)$$



a)

b)

Fig.2.

Expressions (11) and (12) are of no interest because the signals are out of the tested channel while expressions (13) and (14) are signals with frequencies falling into the tested channel and its adjacent channels (fig.2).

III. CROSSMODULATION AND MODULATION DISTORTIONS AT TWO MODULATED FREQUENCIES FOR N=3

In order to determine CMD and MD during signal transmitting in CATV systems, trigonometrical transformations and Volterra kernels of the appropriate level will be used. Let non-linearity is described by polynomial of degree 3 (Eq.15) and the nonlinear products that fall into the tested channel are as a result of two modulated channels.

$$y(t)=\sum_{n=1}^{3} y(t)=y_1(t)+y_2(t)+y_3(t)=\sum_{n=1}^{3}\left|H_n(f)\right|x^n \qquad (15)$$

where $\left|H_n(f)\right|$ is the Volterra kernel

$x(t)=\sum_{N=1}^{3} X_N(t)$ is the input signal with one non modulated and two modulated frequencies.

$$x_N(t)=A_s\cos\omega_s t+\sum_{r=1}^{2} A_r(1+m\cos\Omega t)\cos\omega_r t \qquad (16)$$

Because CMD and MD are of third or higher order, only $y_3(t)$ is of interest

$$y_3(t)=\left|H_3(f)\right|x(t)^3 \qquad (17)$$

$$y_3(t)=\left|H_3(f)\right|\begin{bmatrix} A_s\cos\omega_s t+ \\ A_1(1+m\cos\Omega t)\cos\omega_1 t+ \\ A_2(1+m\cos\Omega t)\cos\omega_2 t \end{bmatrix}^3 \qquad (18)$$

Direct raise to the third power of the expression enclosed in parentheses leads to complex and numerous addends. For this reason the author proceed in the following way:

- addends in (18) are replaced as follows:

$$A_s\cos\omega_s t=a \qquad (19)$$

$$A_1(1+m\cos\Omega t)\cos\omega_1 t=b \qquad (20)$$

$$A_2(1+m\cos\Omega t)\cos\omega_2 t=c \qquad (21)$$

- the expression enclosed in parentheses is raised to the third power and (18) take the following form:

$$y_3(t)=\left|H_3(f)\right|\begin{pmatrix} a^3+b^3+c^3+3a^2b+3a^2c+ \\ 3b^2a+3b^2c+3c^2b+6abc \end{pmatrix} \qquad (22)$$

- Each addend from the parentheses in (22) is calculated separately by using the relations from trigonometric formulae. New replacements are made in order to simplify and visualize mathematical operations. All addends that are not object of this article are omitted from the final results.

1. CROSSMODULATION DISTORTIONS

$$[b]^3_{CMD}=\tfrac{9}{8}A_1 m\left(A_1^2+m^2\right)\cos(\omega_1\pm\Omega)t \qquad (23)$$

$$[c]^3_{CMD}=\tfrac{9}{8}A_2 m\left(A_2^2+m^2\right)\cos(\omega_2\pm\Omega)t \qquad (24)$$

$$\left[3b^2c\right]_{CMD}=\tfrac{9}{3}A_1^2A_2\left(4m+m^3\right)\cos(2\omega_1\pm\omega_2\pm\Omega)t+\tfrac{3}{16}A_1^2A_2\left(12m+m^3\right)\cos(2\omega_2\Omega)t \qquad (25)$$

$$\left[3c^2b\right]_{CMD}=\tfrac{9}{32}A_2^2A_1\left(4m+m^3\right)\cos(2\omega_2\pm\omega_1\pm\Omega)t+\tfrac{3}{16}A_2^2A_1\left(12m+m^3\right)\cos(2\omega_1\pm\Omega)t \qquad (26)$$

$$\left[3b^2a\right]_{CMD}=\tfrac{3}{4}A_1^2A_2 m\cos(2\omega_1\pm\omega_s\pm\Omega)t+\tfrac{3}{2}A_1^2A_s m\cos(2\omega_s\pm\Omega)t \qquad (27)$$

$$\left[3c^2a\right]_{CMD}=\tfrac{3}{4}A_2^2A_3 m\cos(2\omega_2\pm\omega_s\pm\Omega)t+\tfrac{3}{2}A_2^2A_s m\cos(2\omega_s\pm\Omega)t \qquad (28)$$

$$\left[3a^2b\right]_{CMD}=\tfrac{3}{8}A_s^2A_1 m\cos(2\omega_s\pm\omega_1\pm\Omega)t+\tfrac{3}{4}A_s^2A_1 m\cos(2\omega_1\pm\Omega)t \qquad (29)$$

$$\left[3a^2c\right]_{CMD}=\tfrac{3}{8}A_s^2A_2 m\cos(2\omega_s\pm\omega_2\pm\Omega)t+\tfrac{3}{4}A_s^2A_2 m\cos(2\omega_2\pm\Omega)t \qquad (30)$$

$$[6abc]_{CMD}=\tfrac{3}{2}A_sA_1A_2 m\cos(\omega_s\pm\omega_1\pm\omega_2\pm\Omega)t \qquad (31)$$

Amplitudinal and frequency ratios obtained in expressions from (23) to (31) are CMD in the system for assigned initial conditions. Most of them are out of the tested channel, others are there only for particular conditions and the rest are in the tested channel.

Analyzing (23), (24) and the second addends in (25), (26), (29) and (30) we see that CMD concern the channels whose signals are modulated that is they are of no interest for the current work.

Analyzing the first addends in (25) and (26), a conclusion could be made that under some conditions these CMD could be in the tested channel. These conditions are as follows:

$$2\omega_1\pm\omega_2=\omega_s \qquad (32)$$

$$2\omega_2\pm\omega_1=\omega_s \qquad (33)$$

The second addends in (27) and (28) fall completely in the tested channel and represent the main CMD. Considering this fact, expression (22) get the following form:

$$y_{CMD}(t)=\tfrac{3}{2}A_s\left(A_1^2+A_2^2\right)m\left|H_3(\omega_s\pm\Omega)\right|\cos(\omega_s\pm\Omega)t \qquad (34)$$

When conditions (32) or (33) are fulfilled, the generalized expression for CMD is:

$$y_{CMD}(t)=\tfrac{3}{2}\left[\left(A_1^2+A_2^2\right)m+\tfrac{3}{16}A_{1(2)}^2A_{2(1)}\left(4m+m^3\right)\right]\left|H_3(\omega_s\pm\Omega)\right|\cos(\omega_s\pm\Omega)t \qquad (35)$$

2. MODULATION DISTORTIONS

In the solution of expression (22) along with the other non-linear distortions there are modulation distortions distributed the following way in the addends:

$$\left[b^3\right]_{MD}=\tfrac{9}{16}m^2A_1^3\cos(\omega_1\pm2\Omega)t \qquad (36)$$

$$\left[c^3\right]_{MD}=\tfrac{9}{16}m^2A_2^3\cos(\omega_2\pm2\Omega)t \qquad (37)$$

$$\left[3b^2c\right]_{MD}=\tfrac{9}{16}A_1^2A_2m^2\cos(2\omega_1\pm\omega_2\pm2\Omega)t+\tfrac{9}{8}A_1^2A_2m^2\cos(2\omega_2\pm2\Omega)t \qquad (38)$$

$$\left[3c^2b\right]_{MD}=\tfrac{9}{16}A_2^2A_1m^2\cos(2\omega_2\pm\omega_1\pm2\Omega)t+\tfrac{9}{8}A_2^2A_1m^2\cos(2\omega_1\pm2\Omega)t \qquad (39)$$

$$\left[3b^2a\right]_{MD} = \tfrac{3}{16} A_1^2 A_s m^2 \cos\left(2\omega_1 \pm \omega_s \pm 2\Omega\right)t + \tfrac{3}{8} A_1^2 A_s m^2 \cos\left(2\omega_s \pm 2\Omega\right)t \quad (40)$$

$$\left[3c^2a\right]_{MD} = \tfrac{3}{16} A_2^2 A_s m^2 \cos\left(2\omega_2 \pm \omega_s \pm 2\Omega\right)t + \tfrac{3}{8} A_2^2 A_s m^2 \cos\left(2\omega_s \pm 2\Omega\right)t \quad (41$$

$$\left[6abc\right]_{MD} = \tfrac{3}{8} A_1 A_2 A_3 m^2 \cos\left(\omega_s \pm \omega_1 \pm \omega_2 \pm 2\Omega\right)t \quad (42)$$

Modulation products (MP) in expressions (36), (37) and (42) are out of the tested channel but the ones in (36) and (37) fall in the channels adjacent to the channels with modulated carrier, and the non-linear products in (42) in some cases could fall out of CATV systems frequency range. Assertions concerning MP in expression (42) apply fully to the first addends in expressions (38), (39), (40) and (41). Only in (38) and (39) under some conditions MP could fall in the spectrum of the tested channel.

The main modulation products that fall into the tested channel are given by the second addends in (40) and (41).

Taking into account the above considerations it could be written:

$$y_{MD}(t) = \tfrac{3}{8} A_s \left(A_1^2 + A_2^2\right) m^2 \left|H_s\left(\omega_s \pm 2\Omega\right)\right| \cdot \cos\left(\omega_s \pm 2\Omega\right)t \quad (43)$$

When condition /32/ or /33/ are fulfilled, the generalized expression for MD is the one below:

$$y_{MD}^*(t) = \tfrac{3}{8} \left[A_s\left(A_1^2 + A_2^2\right) + 3A_1^2 A_2\right] m^2 \left|H_3\left(\omega_s \pm 2\Omega\right)\right| \cos\left(\omega_s \pm 2\Omega\right)t \quad (44)$$

IV. Conclusion

Expressions /35/ and /44/ show the products of CMD and MD of third order. Obtaining NP (of higher order) that are of special importance for HFC network, by trigonometric transformations is rather complex and sluggish calculation process.

A method will be proposed in a subsequent publication that uses the relation between inter-modulation, cross-modulation and modulation distortions, which is interpreted using Volterra kernels applied during calculation of corresponding coefficients.

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