# Model of a Differential BPSK Decoder with Matched Filter Included, Providing a Minimal ISI

Georgi V. Stanchev[1] and Marin S. Marinov[2]

*Abstract*– **This paper sets out a model of a differential BPSK decoder which includes a matched filter in order to provide a minimal inter-symbol interference and maximal signal to noise ratio at its output. This model is designed to reject the phase difference between the received carrier and the recovered one, too. The influence of the roll-off factor and impulse response limiting over system performance is indicated.**

*Keywords*– **differential BPSK decoder, minimal inter-symbol interference, matched filter, phase difference rejection.**

## I. Introduction

The digital methods for information exchange have established themselves in the field of communications because of their decisive superiority to analogue ones. Besides building of LAN and WAN wire networks, these methods are increasingly employed in mobile and satellite communications [1]. One of the main criteria of information transfer quality is the error probability, and values of this criterion depend on transmitted information – voice; images; video and data [4]. Because of increasing requirements to quality of the communications services it is necessary appropriate methods for digital signal processing to be researched. Attention should be paid to the special features of propagation medium as well as to the enforced limits on the frequency resources. Some useful methods are differential encoding/decoding and matched filtering. To obtain good results the models of transmitter and receiver should be designed and considered together [5].

## II. Transmitter model

The diagram shown in Fig.1 describes frequency band limiting of the transmitted digital signals in conditions of limited frequency resources.

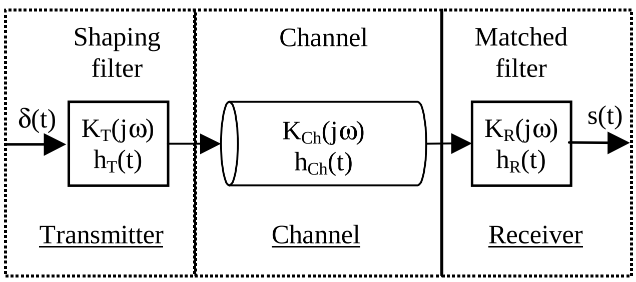


Fig.1. Band limiting of transmitted digital signals

The following notations are used in Fig.1: $K_T(j\omega)$ and $h_T(t)$ are respectively the frequency and the impulse responses of the shaping filter in the transmitter; $K_{Ch}(j\omega)$ and $h_{Ch}(t)$ are respectively the frequency and the impulse responses of the channel; $K_R(j\omega)$ and $h_R(t)$ are respectively the frequency and the impulse responses of the matched filter in the receiver.

The lack of inter-symbol interference (ISI) can be obtained if the impulse response of whole system is [3, 4]:

$$h(t)=\frac{\sin\left(\pi\frac{t}{T_C}\right)\cos\left(\pi\rho\frac{t}{T_C}\right)}{\pi\frac{t}{T_C}\left[1-\left(2\rho\frac{t}{T_C}\right)^2\right]}, \quad (1)$$

where $\rho$ is the roll-off factor with values from 0 to 1, and $T_C$ is the period of the impulse sequence. This impulse response corresponds to the raised-cosine frequency response of whole system:

$$K(f)=\begin{cases}1 & , \frac{f}{R_C}-\frac{1}{2}<-\frac{\rho}{2}\\ \cos^2\left[\frac{\pi}{2\rho}\left(\frac{f}{R_C}-\frac{1}{2}+\frac{\rho}{2}\right)\right] & , \left|\frac{f}{R_C}-\frac{1}{2}\right|\le\frac{\rho}{2}\\ 0 & , \frac{f}{R_C}-\frac{1}{2}>\frac{\rho}{2}\end{cases}, \quad (2)$$

where $R_C$ is the rate of the impulse sequence and

$$K(f)=K_T(f)K_{Ch}(f)K_R(f). \quad (3)$$

If $K_{Ch}(f)$ is much wider than $K_T(f)$ and $K_R(f)$ the following approximation takes place:

$$K(f)\approx K_T(f)K_R(f). \quad (4)$$

The matched filtering in the receiver requires that:

$$K_T(f)=K_{0T}K_0(f); \qquad K_R(f)=K_{0R}K_0(f), \quad (5)$$

and

$$h_T(t)=K_{0T}h_0(t); \qquad h_R(t)=K_{0R}h_0(t). \quad (6)$$

Without loss of any generality it can be assumed that $K_{0T}=K_{0R}=1$ and consequently $K_0(f)=\sqrt{K(f)}$. The impulse response, as it is shown in [2], can be derived by means of Fourier transform and Eq.7 and Fig.2 describe it:

[1]Georgi V. Stanchev, PhD is with the Aviation Faculty, National Military University, 5856 Dolna Mitropolia, Bulgaria, e-mail: gstanchev@af-acad.bg.

[2]Marin S. Marinov, PhD is with the Aviation Faculty, National Military University, 5856 Dolna Mitropolia, Bulgaria, e-mail: mmarinov2000@yahoo.com.

$$h_0(t) = R_C \frac{\sin\left[\pi(1-\rho)\frac{t}{T_C}\right] + 4\rho\frac{t}{T_C}\cos\left[\pi(1+\rho)\frac{t}{T_C}\right]}{\pi\frac{t}{T_C}\left[1-(4\rho\frac{t}{T_C})^2\right]}. \quad (7)$$

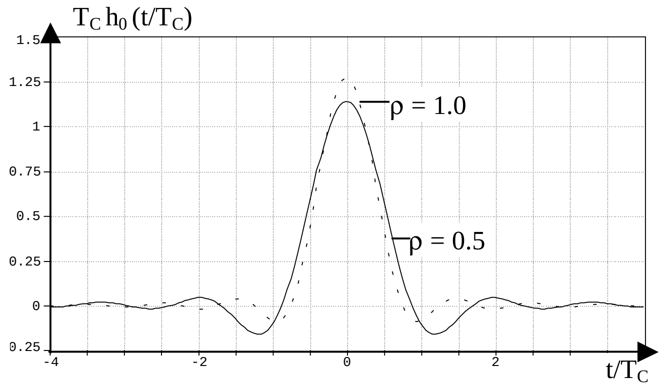


Fig.2. Impulse responses of the shaping and matched filters.

It is important to keep the joint phase characteristic of the shaping and matched filters linear.

These filters are not realizable because their impulse responses are infinite and they are not causal. An window-function *w(t)* is used in order to limit the impulse response:

$$w(t) = \begin{cases} 1 & , \ -kT_C \le t \le kT_C \\ 0 & , \ -kT_C > t > kT_C \end{cases} \quad (8)$$

and impulse response of a realizable filter is given by:

$$H(t) = h_0(t)w(t). \quad (9)$$

The particular impulse response that every single impulse $b_n$ causes on the shaping filter output is:

$$H_n(t) = h_0(t-nT_C)w(t-nT_C)b_n, \quad n = 0, \pm 1, 2, 3, ... \quad (10)$$

The block-diagram, where the general processes in the transmitter are described, is shown in Fig. 3. The transmitter includes a differential encoder, a shaping filter with impulse response *H(t)* and a modulator.

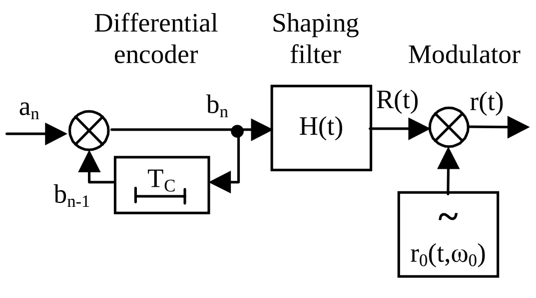


Fig.3. Model of the transmitter.

The total impulse response at the shaping filter output is:

$$R(t) = \sum_{n=-\infty}^{\infty} H_n(t);$$
$$R(t) = \sum_{i=-[k]}^{[k+1]} H_{n+i}(t), \quad nT_C \le t \le (n+1)T_C. \quad (11)$$

At the modulator output, where the signal is up-converted to the carrier frequency, the output signal is:

$$r(t) = R(t)\cos(\omega_0 t). \quad (12)$$

For the time being a single path Gaussian channel is presupposed.

## III. Receiver model

The examined model of the receiver includes the following processes: band converting; matched filtering; sampling and differential decoding. Because of non-coherent band converting, two channels of signal processing, an in-phase I-channel and a quadrature Q-channel, are used – Fig. 4. The carrier is recovered at accuracy to phase.

$$r_I(t) = 2\cos(\omega_0 t + \psi); \qquad r_Q(t) = 2\sin(\omega_0 t + \psi). \quad (13)$$

After down converting of the received signal *r(t)* into baseband signal it becomes $R_I(t)$ and $R_Q(t)$ respectively in the inphase and quadrature channels:

$$R_I(t) = R(t)\cos\psi + R(t)\cos(2\omega_0 t + \psi);$$
$$R_Q(t) = R(t)\sin\psi + R(t)\sin(2\omega_0 t + \psi). \quad (14)$$

The matched filters have the same limited impulse response as the shaping filter in the transmitter. The signals at the filter outputs are:

$$S_I(t) = R_I(t) * H(t) = \int_{t-kT_C}^{t+kT_C} R_I(\tau)H(t-\tau)d\tau;$$
$$S_Q(t) = R_Q(t) * H(t) = \int_{t-kT_C}^{t+kT_C} R_Q(\tau)H(t-\tau)d\tau. \quad (15)$$

In essence the matched filter is low-pass filter and components with doubled carrier frequency are not presented at its output. In fact the roll-off factor of 0 is corresponding to an ideal low-pass filter. It is taken for granted that the synchronization system is working in an ideal manner. Moreover the symmetric impulse response of the applied filters, i.e. *H(t)=H(-t)*, is taken into consideration. Then the signal of the model output, at the moments of interest, is:

$$S(nT_C) = S_I(nT_C)S_I[(n-1)T_C] + S_Q(nT_C)S_Q[(n-1)T_C] =$$
$$= (\cos^2\psi + \sin^2\psi)\int_{(n-k)T_C}^{(n+k)T_C} \sum_{i=-[k]}^{[k+1]} H_{n+i}(\tau)H(\tau - nT_C)d\tau \quad (16)$$
$$\times \int_{(n-1-k)T_C}^{(n-1+k)T_C} \sum_{i=-[k]}^{[k+1]} H_{n-1+i}(\tau)H[\tau-(n-1)T_C]d\tau.$$

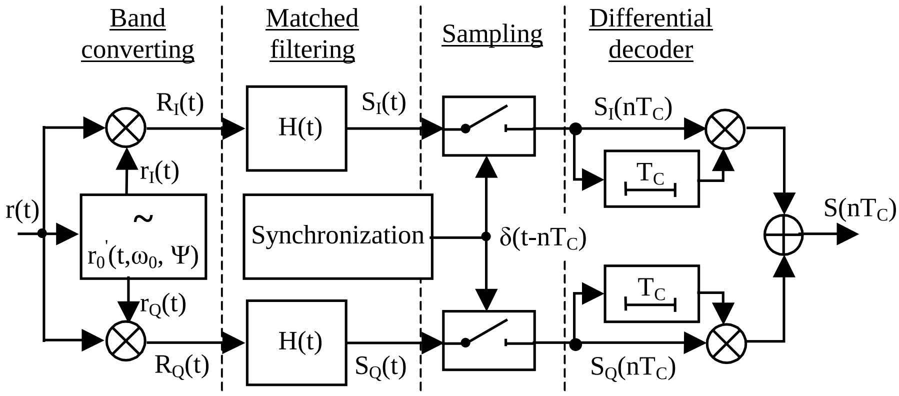


Fig.4. Model of the receiver.

In this way the rising to the second power of the signals in the channels, in order to reject the random phase difference between the received and recovered carrier, and the differential decoding are combined in one and the same process.

## IV RESEARCHES

In order to study the models, described above, two computer programs were created. Walsh sequences were used as a kind of impulse sequence and the influence of the roll-off factor and "windowing" to system performances was examined.

Some of the results are presented in the following figures.

In the Fig. 5 the magnitude of output signal is shown, when the volume of the Walsh functions is 8, the roll-off factor is 0.5 and the relative window width is 6. Every element of the sequence is evaluated separately. Minimal, maximal and average magnitudes of the output signal are calculated, too.

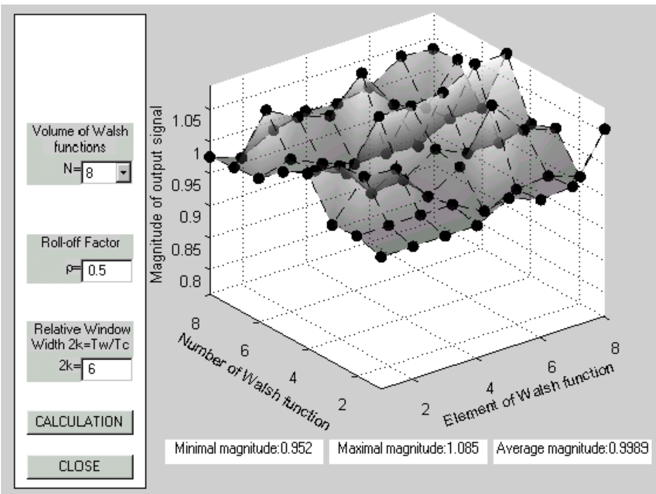


Fig.5. Magnitude of the output signal.

The program interface allows other values of the Walsh functions volume, the roll-off factor and the relative window width to be examined. The studies showed that the magnitude of the output signal depends on the values of the roll-off factor and the window width. For example if only the roll-off factor is changed to 0, the minimal magnitude becomes 0.4124, maximal magnitude becomes 1.5 and the average magnitude becomes 1.012.

The magnitude of the output signal for all possible numbers of Walsh functions at the determinate volume of 32 was examined and the results are shown on following figures.

In Fig. 6 the results for minimal magnitude of the output signal as a function of the roll-off factor and the relative window width are presented. Program interface shows the minimal and maximal values of all minimal magnitudes and the calculated average magnitude, too. The minimal magnitude is the most important one because it is closely related to the risk the output signal to be wrong.

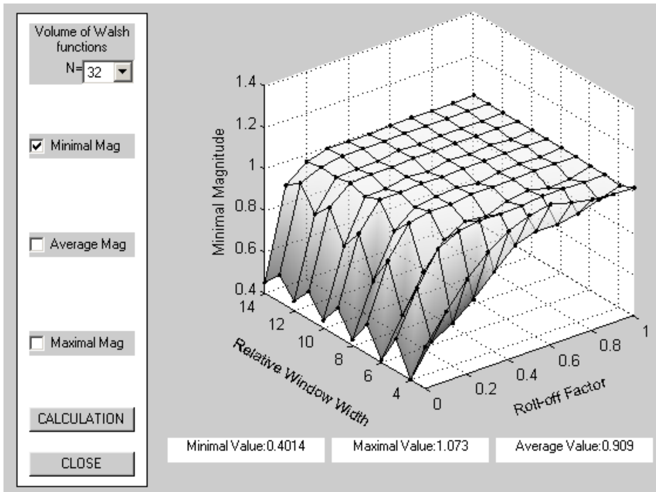


Fig.6. Minimal magnitude of the output signal.

The results show that the minimal magnitude is the lowest when the roll-off factor is zero. The value of the minimal magnitude increases when the roll-off factor grows in value.

The changes of minimal magnitude with the window width changing are due to the rejection of the impulse response tails. Because of finite time duration of the impulse response the perfect conditions are not satisfied and there is a residual inter-symbol interference. The studies show that at values of the roll-off factor greater than 0.2 the changes of the minimal magnitude are too small. The minimal, maximal and average values of minimal magnitude at other volumes of the Walsh functions slightly differ from these above.

In Fig.7 the results for maximal magnitude of the output signal as a function of the roll-off factor and the relative window width are presented. The studies show that alterations of maximal magnitude are more significant at the lowest values of the roll-off factor.

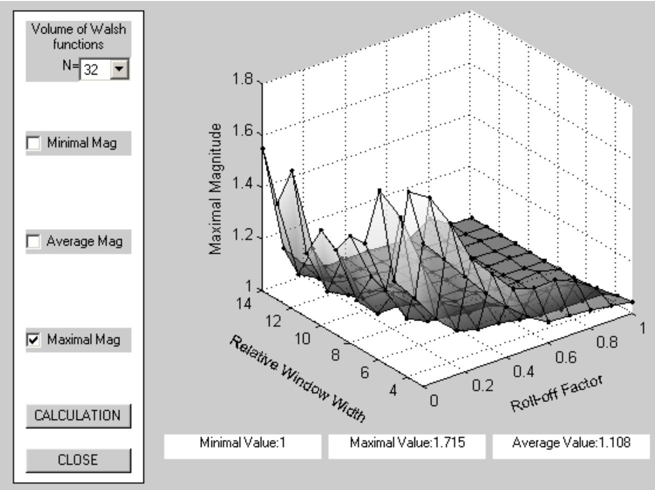


Fig.7. Maximal magnitude of the output signal.

It is obvious that the values of maximal magnitude are smaller for relative window widths from 7 to 9, when the roll-off factor is below 0.3. The minimal, maximal and average values of maximal magnitude at other volumes of Walsh functions slightly differ from these shown in Fig.7.

In Fig.8 the results for average magnitude of the output signal as a function of the roll-off factor and the relative window width are presented.

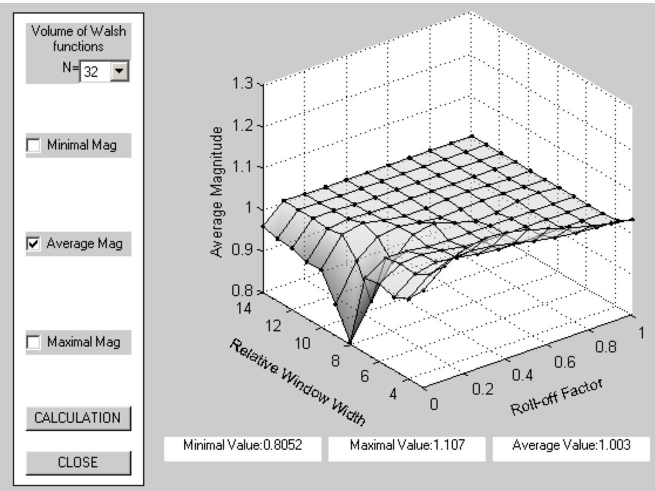


Fig.8. Average magnitude of the output signal.

The studies show that when the roll-off factor is below 0.3 there is a significant decreasing of the average magnitude at relative window widths from 7 to 9. Some decreasing of average magnitude occurs at all window widths if the roll-off factor is below 0.1.

In Fig.9 the lines of the worst and the best cases of the average magnitude are shown. It is clearly seen that the worst case should be taken in account when the roll-off factor is less than 0.3, because when it is greater than 0.3 the values of the average magnitude in the worst case differ very slightly from the other ones. The best case is always when the window width is minimal, but this means maximal frequency bandwidth of the signal which is not often acceptable.

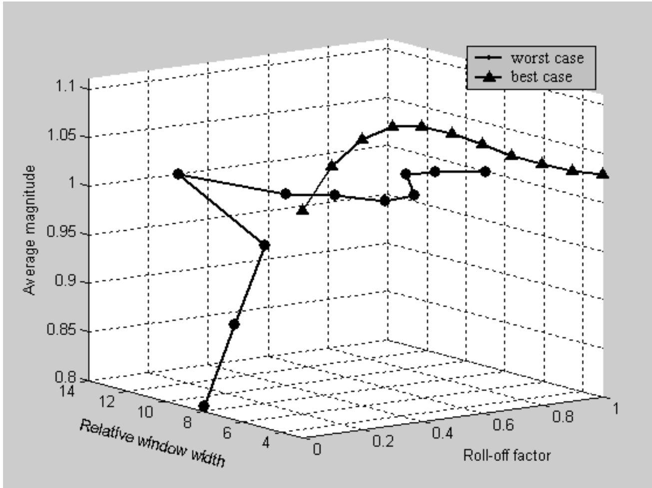


Fig.9. Worst and best case lines.

The results from Fig.9 prove once again that it is better to use roll-off factor greater than 0.3 if it is possible.

## V. CONCLUSION

The attained results prove that the proposed model of decoder can be used successfully for decoding of differential binary phase shift keying signals. It combines the advantages of matched forming and filtering of signals, providing a maximal signal to noise ratio.

The shaping filter and matched filter possess finite impulse responses and consequently these filters can always be made realizable. The limiting of the impulse responses in time domain makes the decoder performance worse, but by proper choice of the roll-off factor and relative window width a performance slightly different from the perfect one can be achieved.

The results show that the roll-off factor should always be chosen to be more than 0.1. When this factor is between 0.1 and 0.3 the relative window width should not be between 7 and 9.

Because both the roll-off factor and the relative window width influence on the frequency bandwidth of the emitted signals they should match the channel bandwidth requirements.

## REFERENCES

[1] D. Roddy, *Satellite Communication,* New York, McGraw-Hill, 1995.

[2] G. Stanchev, *Doctoral thesis: Study of Satellite Digital Communication System Stability, Using Spread Spectrum Signals,* Dolna Mitroplia, 2000.

[3] N. Durchev, G. Bichev. *Telecommunications Systems with Pulse Code Modulation,* Sofia, Tehnika, 1986.

[4] S. Haykin, *Communication Systems,* New York, John Wiley & Sons Inc., 1994.

[5] Y. Okunev, *Phase and Phase-Difference Modulation in Digital Communications,* Artech House, London, 1997.