The Influence of Uncertain Load Parameter Data on Distribution Network Power Flow Results

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***Abstract* – The influence of uncertain load parameter data on power flow results of radial middle-voltage distribution network is treated in this paper. The load flow calculation is performed on the basis of fuzzy logic, modeling load levels and real and reactive power sensitivities on voltage as fuzzy numbers. The parameters of fuzzy number functions are obtained using measurement data. The influences of supply voltage value and power sensitivities on voltage on the results are discussed.**

***Keyword* – power flow, distribution network, fuzzy logic, load parameters**

I. INTRODUCTION

Many papers have been dedicated to middle-voltage distribution network calculation. Necessary input data for this calculation are load parameters. However, these parameters are poorly known by rule, and they assuredly cause result mistakes. Therefore, great number of papers deals with load parameter determination [1, 2]. The number of paper destining a fuzzy logic approach to distribution network calculation increases, because uncertainty of load parameters can be taken into account in this way. The results of the calculation: load flows, branch currents, bus voltages, and power losses are fuzzy numbers that give the ranges of observed variables for every confidence level.

A fuzzy logic approach to load flow calculation of radial distribution networks is presented in [3]. This paper is based on the assumptions that the load at every node is a fuzzy variable with associated trapezoidal membership function and real and reactive power sensitivities on voltage are constants. Load representation by trapezoidal membership functions is capable when load data are obtained using utility worker experience. If a number of daily load curve measurements are available [4], the curve can be divided into five levels, and the load at each level modeled as one fuzzy variable. Since daily load curves are different for various load classes: residential, commercial and industrial, the measurements are performed at typical network buses, and results extrapolated to other buses that feed load of the same class.

There are different defuzzification methods, but it was found that the numbers obtained in this way are of low importance for engineers. Therefore, method for uncertainty level determination of fuzzy numbers on the basis of weighted intervals is suggested in [5]. It yields the bounds that are most reasonable for engineering decisions.

This paper models load values, as well as real and reactive power sensitivities on voltage as fuzzy variables. The assumptions are that daily load curves for different load classes are obtained by a number of measurements, and experiments of changing supply voltage for determining real and reactive power sensitivities on voltage are carried out. The results of power flow calculation in different operating conditions are analyzed on the example of IEEE test-network with 32 buses [6].

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II. BASES OF FUZZY SET THEORY

A fuzzy set can be interpreted as a set of continual confidence intervals $A^{(\alpha)}$ which determine greater and greater uncertainty ranges with descending confidence levels. Confidence interval $A^{(\alpha)}$ can be defined as

$$A^{(\alpha)} = \{x \in X : \ \mu_{\tilde{A}}(x) \geq \alpha\}. \tag{1}$$

It means that all values of fuzzy variable belonging to the interval $A^{(\alpha)}$ can be concerned possible with confidence level that is greater or equal to α. Confidence interval $A^{(\alpha)}$ has its lower and upper bound, $A^{(\alpha)} = \left[a_1^{(\alpha)}; a_2^{(\alpha)}\right]$, where $\alpha \in [0,1]$. Since $A^{(\alpha)}$ is decaying function of α, for every pair $\alpha_1, \alpha_2 \in [0,1]$

$$(\alpha_2 > \alpha_1) \ \Rightarrow \ A^{(\alpha_2)} \subset A^{(\alpha_1)}. \tag{2}$$

Therefore, basic arithmetic operations on fuzzy sets can be regard as generalization of interval arithmetic [7]. These arithmetic interval operations for every confidence level ($\alpha \in [0,1]$) and fuzzy sets $\tilde{A}$ and $\tilde{B}$ are:

$$\tilde{C} = \tilde{A}(+)\tilde{B} = \left[a_1^{(\alpha)} + b_1^{(\alpha)}; \ a_2^{(\alpha)} + b_2^{(\alpha)}\right], \tag{3}$$

$$\tilde{C} = \tilde{A}(-)\tilde{B} = \left[a_1^{(\alpha)} - b_2^{(\alpha)}; \ a_2^{(\alpha)} - b_1^{(\alpha)}\right], \tag{4}$$

$$\tilde{C} = \tilde{A}(\cdot)\tilde{B} = \left[a_1^{(\alpha)} \cdot b_1^{(\alpha)}; \ a_2^{(\alpha)} \cdot b_2^{(\alpha)}\right], \tag{5}$$

$$\tilde{C} = \tilde{A}(:)\tilde{B} = \left[a_1^{(\alpha)} / b_2^{(\alpha)}; \ a_2^{(\alpha)} / b_1^{(\alpha)}\right]. \tag{6}$$

In this paper, following interval operations are also used for each α:

$$\tilde{C} = \tilde{A}^{\tilde{B}} = \left[\min a_i^{\,b_j};\;\; \max a_i^{\,b_j}\right], \quad i,j=1,2, \qquad (7)$$

$$\left|\tilde{C}\right| = \left[\min\left(c_{Ri}^2 + c_{Ij}^2\right)^{1/2};\;\; \max\left(c_{Ri}^2 + c_{Ij}^2\right)^{1/2}\right], \quad i,j = 1,2. \qquad (8)$$

Here a_i and b_j are the bounds of confidence interval, $A^{(\alpha)}$ and $B^{(\alpha)}$, respectively, and c_{Ri} and c_{Ij} are the bounds of $C_R^{(\alpha)}$ and $C_I^{(\alpha)}$ that are confidence intervals of complex variable real and imaginary parts.

III. Load Parameter FN Functions

The assesment is that the load at 10kV voltage level can be devided into classes. Mean daily load curves of each class are obtained from great number of measurements during certain season and can be extrapolated to other nodes with load of the same class. However, declinations of these curves are objectivelly possible. Therefore, it is advicable to use lingvistic description of load level for every class expressed as percentage of transformer rated power: very small (VS) - 0-20%S_n, small (S) - 20-40%S_n, middle (M) - 40-60%S_n, large (L) 60-80%S_n, and very large (VL) - 80-100%S_n. Then, fuzzy variable x assigned to each load level and each load class may be modeled as normalized fuzzy number (FN) as depicted in Fig. 1. Kernel X_K of this triangular FN denotes mean value obtained from daily load curve for corresponding load level. Lower and upper bounds, X_{0l} and X_{0u}, are specified in previous paragraph for each load level. Interval of possible values of x with lower and upper bound, $X_{\alpha l}$ and $X_{\alpha u}$, is assigned to every confidence level.

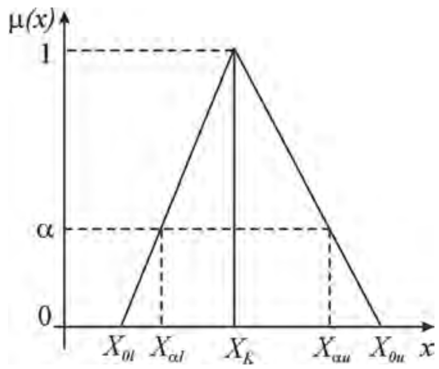

Fig. 1. Characteristic function of a FN

Also, fuzzy numbers can be assigned to real and reactive power sensitivities on voltage. Their bounds are minimal and maximal values obtained by measurements at different day intervals at nodes with certain load class. Kernels are mean values of real and reactive load sensitivities on voltage.

The uncertainty grade of every variable x, which is presented as fuzzy number in Fig. 1, may be determined as

$$UG = \frac{100}{X_K}\int_0^1 \left(X_{\alpha u} - X_{\alpha l}\right)(1-\alpha)\,d\alpha\,. \qquad (9)$$

It gives the interval of variable x values obtained by weighting all possible intervals of variable values by the uncertainty levels characterizing these intervals. Lower and upper bound relative declinations of X_K are

$$LU = \frac{100}{X_K}\int_0^1 \left(X_K - X_{\alpha l}\right)(1-\alpha)\,d\alpha\,, \qquad (10)$$

$$UU = \frac{100}{X_K}\int_0^1 \left(X_{\alpha u} - X_K\right)(1-\alpha)\,d\alpha\,. \qquad (11)$$

On the basis of former definitions, $LU+UU=UG$. Finally following bounds for variable x values may be taken as most reasonable for engineering decisions made under uncertainty

$$X_K\left(1-\frac{LU}{100}\right) \le x \le X_K\left(1+\frac{UU}{100}\right). \qquad (12)$$

IV. Algorithm

Input parameters for radial distribution network load flow calculation are: 1. Load class parameters – histogramic daily load curves, real and reactive power sensitivities on voltage obtained by measurements and power factor of every load class; 2. Network parameters – source voltage, sending and receiving node of each branch, its resistance and reactance, transformer rated power and class of the load connected to the branch receiving node.

The following algorithm can describe the calculation procedure:

1. FN function parameters, X_K, X_{0l} and X_{0u}, of five load levels are determined for every load class (mostly five). FN function parameters of real and reactive load sensitivities on voltage are provided for every load class;
2. For specified hour of the day, on the basis of daily load curves and load class distribution among network nodes, one FN function of load level is assigned to each consumer. FN functions of real and reactive power sensitivities on voltage are assigned to each consumer according to the load class connected to the node;
3. Flat voltage profile is adopted, and confidence level α is set to zero;
4. Lower and upper bounds of load level and bounds of real and reactive power sensitivities on voltage are determined for α at each network node regarding FN function parameters obtained in step 1. Following calculation is performed on the basis of Eqs. (3) – (8);
5. In ν^{th} iteration, load current components are calculated according to formulae

$$I_{pR}(i)^{(\alpha)\nu} = \frac{P_{pp}(i)^{(\alpha)\nu-1}\cdot U_R(i)^{(\alpha)\nu-1} + Q_{pp}(i)^{(\alpha)\nu-1}\cdot U_I(i)^{(\alpha)\nu-1}}{\left(U_R(i)^{(\alpha)\nu-1}\right)^2 + \left(U_I(i)^{(\alpha)\nu-1}\right)^2}\,, \qquad (13)$$

$$I_{pI}(i)^{(\alpha)\nu} = \frac{P_{pp}(i)^{(\alpha)\nu-1}\cdot U_I(i)^{(\alpha)\nu-1} - Q_{pp}(i)^{(\alpha)\nu-1}\cdot U_R(i)^{(\alpha)\nu-1}}{\left(U_R(i)^{(\alpha)\nu-1}\right)^2 + \left(U_I(i)^{(\alpha)\nu-1}\right)^2}\,, \qquad (14)$$

where $I_{pR}(i)^{(\alpha)\nu}$ and $I_{pI}(i)^{(\alpha)\nu}$ are real and imaginary components of i^{th} node load current, $U_R(i)^{(\alpha)\nu-1}$ and $U_I(i)^{(\alpha)\nu-1}$ are real and imaginary voltage components from previous iteration, and $P_{pp}(i)^{(\alpha)\nu-1}$ and $Q_{pp}(i)^{(\alpha)\nu-1}$ real and reactive power of the load from previous iteration. These powers are calculated with respect of real and reactive power at rated voltage ($P_p(i)^{(\alpha)}$ and $Q_p(i)^{(\alpha)}$), absolute voltage value at the node from previous iteration ($U(i)^{(\alpha)\nu-1}$) and sensitivities of real and reactive power on voltage corresponding the load connected to i^{th} node ($k_{pu}(i)$ and $k_{qu}(i)$)

$$P_{pp}(i)^{(\alpha)\nu-1} = P_p(i)^{(\alpha)} \cdot \left(\frac{U(i)^{(\alpha)\nu-1}}{U_n} \right)^{k_{pu}(i)} , \qquad (15)$$

$$Q_{pp}(i)^{(\alpha)\nu-1} = Q_p(i)^{(\alpha)} \cdot \left(\frac{U(i)^{(\alpha)\nu-1}}{U_n} \right)^{k_{qu}(i)} ; \qquad (16)$$

6. Branch current components are obtained starting from the end toward the source node

$$I_R(i)^{(\alpha)\nu} = I_{pR}(i)^{(\alpha)\nu} + \sum_{j\in J} I_R(j)^{(\alpha)\nu} , \qquad (17)$$

$$I_I(i)^{(\alpha)\nu} = I_{pI}(i)^{(\alpha)\nu} + \sum_{j\in J} I_I(j)^{(\alpha)\nu} , \qquad (18)$$

where J is the set of branches that are directly feed from i^{th} node;

7. Voltages are calculated starting from the source node towards the end one

$$U_R(i)^{(\alpha)\nu} = U_R(j)^{(\alpha)\nu} - \left(R(i) I_R(i)^{(\alpha)\nu} - X(i) I_I(i)^{(\alpha)\nu} \right), \qquad (19)$$

$$U_I(i)^{(\alpha)\nu} = U_I(j)^{(\alpha)\nu} - \left(R(i) I_I(i)^{(\alpha)\nu} + X(i) I_R(i)^{(\alpha)\nu} \right), \qquad (20)$$

where j is the sending node of a branch i, and $R(i)$ and $X(i)$ are i^{th} branch resistance and reactance;

8. Iterative procedure is finished when the differences between lower (1) and upper (2) bounds of the confidence interval for real and imaginary voltage components in two neighbourhood iterations are less or equal to specified number ε

$$\left| U_{R1(2)}(i)^{(\alpha)\nu} - U_{R1(2)}(i)^{(\alpha)\nu-1} \right| \le \varepsilon ,\ i = 1,\ldots,N , \qquad (21)$$

$$\left| U_{I1(2)}(i)^{(\alpha)\nu} - U_{I1(2)}(i)^{(\alpha)\nu-1} \right| \le \varepsilon ,\ i = 1,\ldots,N . \qquad (22)$$

Then, α is increased by $\Delta\alpha$, and calculation continues with step 4. If conditions are not fulfilled, go to step 5;

9. If $\alpha > 1$, FN functions of certain output variables are plotted. Uncertainty grade and bounds of these variables are calculated according to Eqs. (9) – (12). If $\alpha \le 1$, go to step 4.

Based on this algorithm, estimation of daily real and reactive energy losses may be realized, too.

V. RESULTS

The calculations are performed in different operating conditions of distribution IEEE test-network with 32 buses. For easier result comparison here is presented results of the cases when all network consumers belong to the same class – daily load curve and real and reactive power sensitivities on voltage are the same.

It is assumed that the load belongs to large (L) level. Lower and upper bounds for proper FN function are 0.6 and 0.8, respectively, and data from daily load curve yield the kernel value amounts 0.675. Literature data provide different real and reactive power sensitivities on voltage. However, here are presented results for those that characterize constant power, constant current or constant impedance load type (k_{pu}=k_{qu}=0, 1 or 2, respectively). The results for sensitivities as deterministic values are compared with the cases when sensitivities are modeled as fuzzy numbers with:

1. X_K =0, LU=-0.5 and UU=0.5,
2. X_K =1, LU=0.5 and UU=1.5,
3. X_K =2, LU=1.5 and UU=2.5.

The supply voltage is modeled as deterministic variable for easier inference and it is varied in the wide range from 0.95U_n to 1.1U_n.

In Figs. 2, and 3, current of the first branch for real and reactive power sensitivities on voltage modeled as deterministic variables and as fuzzy numbers are presented. Supply voltage of the results in Figs. 2 and 3 is assumed to be 1.1U_n and 0.95U_n, respectively. Both figures present the spread of FN functions - increase of uncertainty grade when sensitivity coefficients are fuzzy numbers. Also, the greatest first branch current for supply voltage 1.1U_n is when the load is of constant impedance type, while for 0.95U_n it is for constant power load type. It was found that the network is practically indifferent on load type when supply voltage is 1.05U_n, because in this case the average bus voltage in the network is U_n. In this case kernel values of I_{max} differ less than 1.93% from kernel mean value (134.54A) obtained for mentioned load types. These declinations exceed 6.6% for 1.1U_n and 8.2% for 0.95U_n.

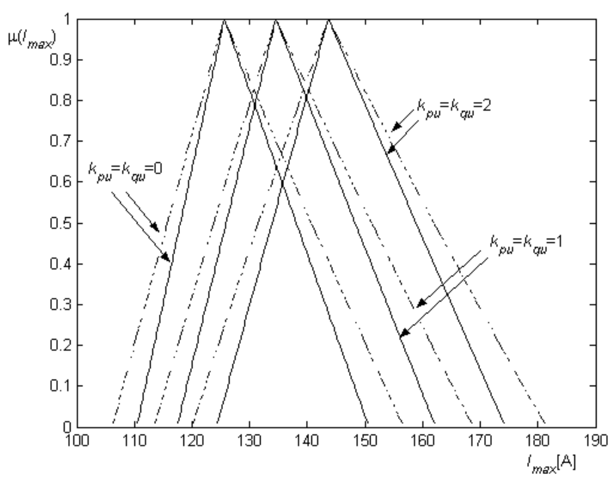

Fig. 2. I_{max} for U_0=1.1U_n and power sensitivities on voltage as deterministic (———) and fuzzy variables (– · – · – ·)

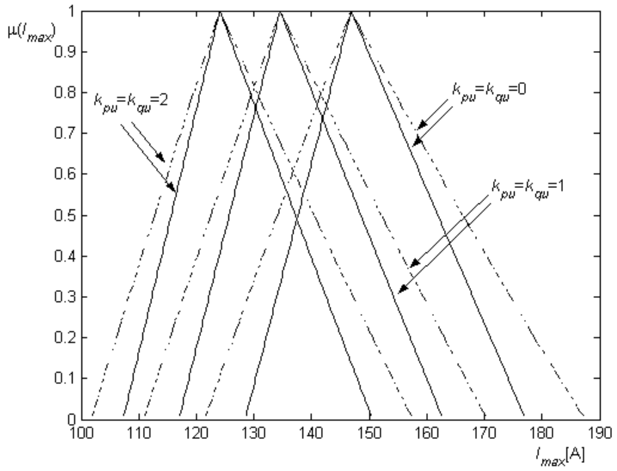

Fig. 3. I_{max} for U_0=0.95U_n and power sensitivities on voltage as deterministic (———) and fuzzy variables (– · – · – ·)

TABLE I

CALCULATION RESULTS

Quantity	U_0	Power sensitivities	Determin. or fuzzy	Kernel	*LU*	*UG*
I_{max} [A]	1.1	k_{pu}=k_{qu}=0	Determin.	125.6	3.98	10.54
			Fuzzy	125.6	5.12	13.15
		k_{pu}=k_{qu}=2	Determin.	143.67	4.46	11.45
			Fuzzy	143.67	5.49	14.00
	0.95	k_{pu}=k_{qu}=0	Determin.	146.86	4.13	10.89
			Fuzzy	146.86	5.71	14.59
		k_{pu}=k_{qu}=2	Determin.	124.07	4.46	11.45
			Fuzzy	124.07	5.96	14.73
U_{min} [p.u.]	1.1	k_{pu}=k_{qu}=0	Determin.	0.9591	0.29	0.46
			Fuzzy	0.9591	0.34	0.56
		k_{pu}=k_{qu}=2	Determin.	0.9542	0.35	0.58
			Fuzzy	0.9542	0.42	0.69
	0.95	k_{pu}=k_{qu}=0	Determin.	0.9444	0.41	0.67
			Fuzzy	0.9444	0.56	0.93
		k_{pu}=k_{qu}=2	Determin.	0.9542	0.35	0.58
			Fuzzy	0.9542	0.45	0.77

Numerical results of first branch current (I_{max}) and electrically most distant node voltage (U_{min}) for 1.1U_n and 0.95U_n supply voltage and power sensitivities on voltage as deterministic and fuzzy variables are presented in Table I. These results show noticeable increases of uncertainty grade and lower and upper bound declinations when power sensitivities on voltage are treated as fuzzy numbers. For example, *UG* of I_{max} changes more than 2.5%, while *LU* and *UU* change more than 1 and 1.5%. As real power losses are functions of square currents, increases of their *UG*, *LU* and *UU* are significantly greater and exceed approximately 2%, 3% and 5%, respectively. All mentioned increases would be more important under assumption that measurement based data of real and reactive power sensitivities on voltage in various seasons are not available. Also, results of the paper show that uncritical usage of literature data of these sensitivities can cause unacceptable power flow result mistakes.

VI. CONCLUSION

Presented distribution network load flow calculation treat uncertain load parameters as fuzzy numbers. Calculation results are the most reasonable ranges of output data. They show essential importance of real and reactive power sensitivities on voltage and supply voltage value on load flow results. Modeling of real and reactive power sensitivities on voltage as fuzzy numbers causes significant increase of uncertainty grade and lower and upper bound declinations of load flow output data. Suggested methodology may be used for energy loss estimation.

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