Diversity Systems in the Presence of Correlated Shadowing

Milan Živković, Nenad Milošević, Bojan Dimitrijević, Zorica Nikolić

Abstract - **In this paper we describe micro- and macrodiversity reception systems in an additive white Gaussian noise channel in the presence of Rayleigh fading and correlated log-normal shadowing employing BPSK signaling. The SC and MRC microdiversity system with various number of branches, *L*, along with dual macrodiversity system with correlated, power unbalanced branches is analyzed. System performances is analyzed using MGF (moment-generating function) approach where the dependence to BER of the average SNR/bit is used as the measure of the performances.**

Keywords – **BPSK signaling, Rayleigh fading, correlated log-normal shadowing, diversity combining.**

I. Introduction

Mobile communication channel can be modeled as additive white Gaussian noise channel subject to Rayleigh fading (received amplitudes has Rayleigh distribution) and log-normal shadowing (the mean of signal-to-noise ration has log-normal distribution). This scenario is typical for congested downtown areas with a large number of slow-moving pedestrians and vehicles. A powerful communication receiver technique that provides wireless channel improvement at relatively low cost is a well-known as diversity reception. Supplying to the receiver several replicas of the same information signal transmitted over independently fading channels, the probability that all the signal components will fade simultaneously is reduced considerably [1] and therefore, instant and mean SNR can be increased. The diversity reception is categorized as micro- and macrodiversity.

Microdiversity is a method for reducing the effect of instantaneous fading in which several uncorrelated faded signals are received at a radio port. There are several techniques for evaluating transmitted signal at the receiver. In this paper we consider MRC (*Maximal Ratio Combining*) and SC (*Selection Combining*) techniques for microdiversity combining. When SC is employed, among the *L* diversity branches, the branch providing the largest signal-to-noise ratio (or largest fading amplitude) is selected. Using MRC, the signals from all the branches are co-phased and individually weighed by fading factor to provide the optimal SNR at the output. Macroscopic diversity, again, is a method for reducing for reducing the effect of shadowing, in which several signals are received at different radio-ports, with differently experienced long-term shadowing. The most commonly used Macrodiversity technique is macrodiversity selection where selected signal originates from the port where the smallest long-term shadowing (the largest mean SNR) is present.

In this paper we will analyze systems with implemented micro- and macrodiversity techniques in an additive white Gaussian noise channel in the presence of Rayleigh fading with dual correlated, power unbalanced, log-normal shadowing and compare different microscopic selection methods where the dependence to BER of the average SNR/bit is used as the measure of the performances. Numerical evaluations of performances will be done using MGF (moment-generating function) approach [2], which represent a power tool that simplifies numerical calculations.

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II. System Model

Consider the system with two different radio-ports forming the microscopic diversity group. In order to mitigate the effect of shadowing (long-term attenuation) one can select signal originated from those port where the largest mean SNR is present. Consider, again, *L* independent microdiversity branches at every radio-port employ one of considered microdiversity techniques. If the transmitted signal is $x(t)$, the low-pass equivalent received signal in the lth branch of the k th port [3]

$$\omega_{kl} = \alpha_{kl} e^{j\phi_{kl}} \cdot x + \eta_{kl} \quad k = 1,2 \quad l = 1,\cdots,L \tag{1}$$

where

α_{kl} - fading amplitude (factor) in the lth branch of the k th port (nonnegative number).

x - $\sqrt{E_b}$ ili $-\sqrt{E_b}$ with *a priori* probability 1/2.

ϕ_{kl} - fading phase in the l th branch of the k th port

η_{kl} - additive complex Gaussian noise in the lth branch of the k th port

E_b - bit energy.

Corresponding signal in the lth branch of the k th port after cophasing is

$$r_{kl}(t) = \mathrm{Re}\left\{\omega_{kl} e^{-j\phi_{kl}}\right\} = \alpha_{kl} \cdot x + n_{kl} \quad k = 1,2 \quad l = 1,\cdots,L \tag{2}$$

where $n_{kl} = \mathrm{Re}\left\{\eta_{kl} e^{-j\phi_{kl}}\right\}$. We assume that $E\left\{n_{kl}^2\right\} = N_0/2$ for every k i l.

Let's $a_{k1}, \alpha_{k2}, \cdots, \alpha_{kL}$ denote fading amplitudes correspondent to microdiversity branches of the k th port. We assume that they are statistically independent with Rayleigh probability density function (pdf) of the instant SNR, $\gamma_{kl} = \alpha_{kl}^2 \frac{E_b}{N_0}$, which has the form

$$p_{\gamma_{kl}}\left(\gamma_{kl} / \gamma_k\right) = \frac{1}{\gamma_k} e^{-\frac{\gamma_{kl}}{\gamma_k}} \tag{3}$$

which is conditioned on the local mean SNR at the k th port $\gamma_k = E\left\{\alpha_k^2\right\} \frac{E_b}{N_0} = z_k \frac{E_b}{N_0}$, where $z_k = E\left\{\alpha_k^2\right\}$ denotes mean-square amplitude values at the k th port. We assume dual branch, correlated, power unbalanced, macrodiversity system where the mean-square amplitude values follow a log-normal pdf [2]

$$p_{z_k}\left(z_k\right) = \frac{10/\ln 10}{\sqrt{2\pi}\sigma_{S_k} z_k} \exp\left(-\frac{\left(10\log_{10} z_k - \mu_k\right)^2}{2\sigma_{S_k}^2}\right) \tag{4}$$

where $\mu_k\left(dB\right)$ denotes mean, and $\sigma_S\left(dB\right)$ denotes standard deviation of the quantity $10\log_{10} z_k$. Let $z_{k\max}$ be the larger local mean-square value selected from the considered radio-ports, that is $z_{k\max} = \max\left\{z_1, z_2\right\}$. One can show that the pdf of $z_{k\max}$ has the form [4]

$$\begin{aligned} p_{z_{\max}}\left(z_{\max}\right) = {} & \frac{10/\ln 10}{\sqrt{2\pi}\sigma_{S_1} z_{\max}} \times \exp\left(\frac{\left(10\log_{10} z_{\max} - \mu_1\right)^2}{2\sigma_{S_1}^2}\right) \\ & \times \left(1 - Q\left(\left(\frac{1}{\sqrt{1-\rho^2}\sigma_{S_2}} - \frac{\rho}{\sqrt{1-\rho^2}\sigma_{S_1}}\right) 10\log_{10} z_{\max} \right.\right. \\ & \left.\left. - \frac{\mu_2}{\sqrt{1-\rho^2}\sigma_{S_2}} + \frac{\rho\mu_1}{\sqrt{1-\rho^2}\sigma_{S_1}}\right)\right) \\ & + \frac{10/\ln 10}{\sqrt{2\pi}\sigma_{S_2} z_{\max}} \times \exp\left(\frac{\left(10\log_{10} z_{\max} - \mu_2\right)^2}{2\sigma_{S_2}^2}\right) \\ & \times \left(1 - Q\left(\left(\frac{1}{\sqrt{1-\rho^2}\sigma_{S_1}} - \frac{\rho}{\sqrt{1-\rho^2}\sigma_{S_2}}\right) 10\log_{10} z_{\max} \right.\right. \\ & \left.\left. - \frac{\mu_1}{\sqrt{1-\rho^2}\sigma_{S_1}} + \frac{\rho\mu_2}{\sqrt{1-\rho^2}\sigma_{S_2}}\right)\right) \end{aligned} \tag{5}$$

Where ρ is correlation coefficient, and $Q\left(x\right)$ is Gaussian Q-function

$$Q\left(x\right) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt. \tag{6}$$

II. THE SYSTEM PERFORMANCE

The average bit error probability (BER) of overall system is derived by averaging bit error probability conditioned on instantaneous γ_b, over the corresponding pdf

$$P_b\left(e\right) = \int_0^\infty P_b\left(e / \gamma_b\right) p_{\gamma_b}\left(\gamma_b\right) d\gamma_b \tag{7}$$

where the conditioned BPSK BER is given as

$$P_b\left(e / \gamma_b\right) = Q\left(\sqrt{2\gamma_b}\right) \tag{8}$$

We use MGF approach to analyze the proposed system performance, which is based on using the moment-generating function of instantenous γ_b [2]

$$M_{\gamma_b}\left(s\right) = \int_0^\infty p_{\gamma_b}\left(\gamma_b\right) e^{s\gamma_b} d\gamma_b \tag{9}$$

Moreover, a finite integral form of Gaussian Q-function is used for system analysis [2]

$$Q\left(x\right) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\phi}\right) d\phi \tag{10}$$

where argument of the function becomes integrand of the finite integral, instead of lower limit of infinite integral as it is in the classical form (6). Substituting (8) in (7), using modified form of Gaussian Q-function (10) we get

$$P_b\left(e\right) = \frac{1}{\pi} \int_0^{\pi/2} \int_0^\infty e^{-\frac{\gamma_b}{\sin^2\phi}} p_{\gamma_b}\left(\gamma_b\right) d\gamma_b d\phi \tag{11}$$

where the inner integral presents familiar form of moment-generating function (9), so we have

$$P_b\left(e\right) = \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_b}\left(-\frac{1}{\sin^2\phi}\right) d\phi. \tag{12}$$

As we consider system with both macro- and microdiversity reception, pdf of instantaneous γ_b has the form

$$p_{\gamma_b}\left(\gamma_b\right) = \int_0^\infty p\left(\gamma_b / z_{\max}\right) p\left(z_{\max}\right) dz_{\max} \tag{13}$$

The corresponding moment-generating function one can get as

$$M_{\gamma_b}\left(s\right) = \int_0^\infty \left(\int_0^\infty p\left(\gamma_b / z_{\max}\right) e^{s\gamma_b} d\gamma_b\right) p\left(z_{\max}\right) dz_{\max} \tag{14}$$

which is according to (9)

$$M_{\gamma_b}\left(s\right) = \int_0^\infty M_{\gamma_b}\left(s / z_{\max}\right) p\left(z_{\max}\right) dz_{\max} \tag{15}$$

where $p\left(z_{\max}\right)$ is given in (5).

The form of $M_{\gamma_b}\left(s / z_{\max}\right)$ depends on applied microdiversity technique. When MRC is applied, it has form

$$M_{\gamma_b}\left(s / z_{\max}\right) = \left(1 - s\gamma_{\max}\right)^{-L} \tag{16}$$

In order to simplify the further evaluations we assume, without the loss of generality, that $\mu_1 = \mu_2 = 0$. Then, MGF in (15) has form

$$\begin{aligned} M_{\gamma_b}\left(s\right) = \int_0^\infty \left(1 - s\gamma_{\max}\right)^{-L} \Bigg\{ & \frac{10/\ln 10}{\sqrt{2\pi}\sigma_{S_1} z_{\max}} \times \exp\left(\frac{\left(10\log_{10} z_{\max}\right)^2}{2\sigma_{S_1}^2}\right) \\ & \times \left(1 - Q\left(\left(\frac{1}{\sqrt{1-\rho^2}\sigma_{S_2}} - \frac{\rho}{\sqrt{1-\rho^2}\sigma_{S_1}}\right) 10\log_{10} z_{\max}\right)\right) \\ & + \frac{10/\ln 10}{\sqrt{2\pi}\sigma_{S_2} z_{\max}} \times \exp\left(\frac{\left(10\log_{10} z_{\max}\right)^2}{2\sigma_{S_2}^2}\right) \\ & \left(1 - Q\left(\left(\frac{1}{\sqrt{1-\rho^2}\sigma_{S_1}} - \frac{\rho}{\sqrt{1-\rho^2}\sigma_{S_2}}\right) 10\log_{10} z_{\max}\right)\right) \Bigg\} \end{aligned} \tag{17}$$

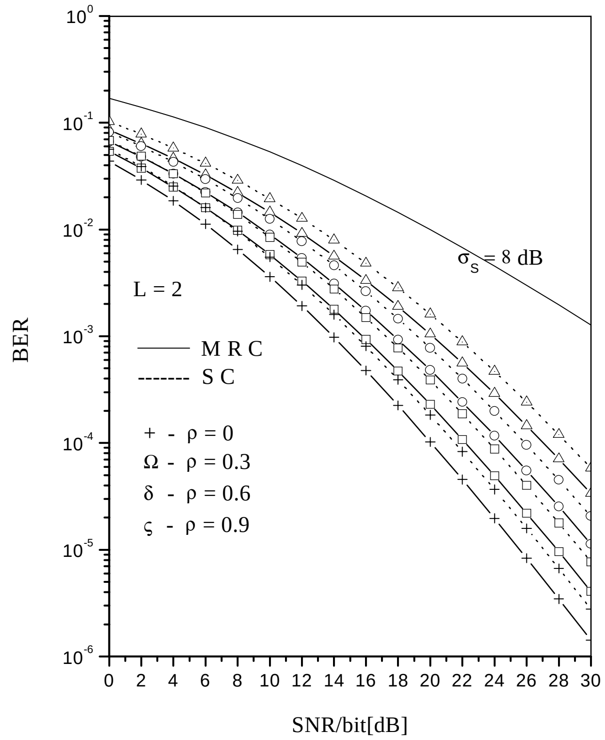

Fig. 1 Dependence of the BER of the SNR/bit for the diversity system with $L = 2$ SC and MRC combined micro branches with different values of correlation coefficient.

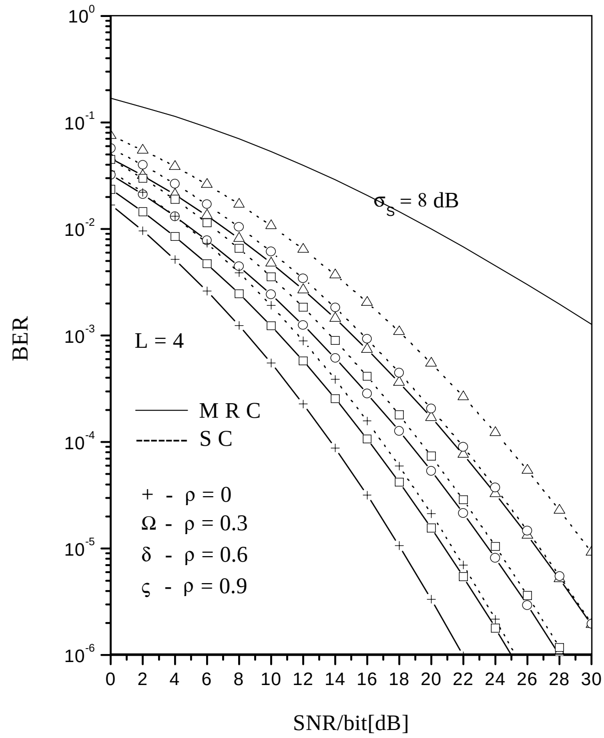

Fig. 2 Dependence of the BER of the SNR/bit for the diversity system with $L = 4$ SC and MRC combined micro branches with different values of correlation coefficient.

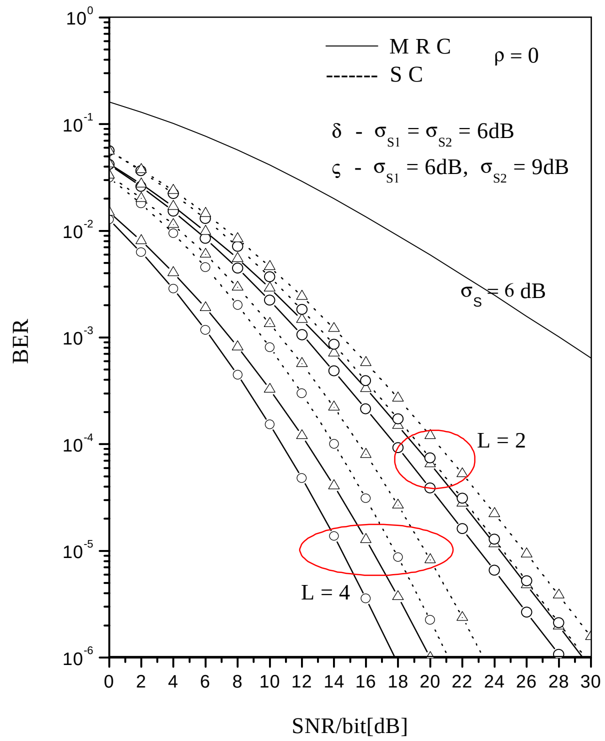

Fig. 3 Dependence of the BER of the SNR/bit for the diversity system with SC and MRC combined micro branches with equal and unequal power in uncorrelated macro branches ($\rho = 0$).

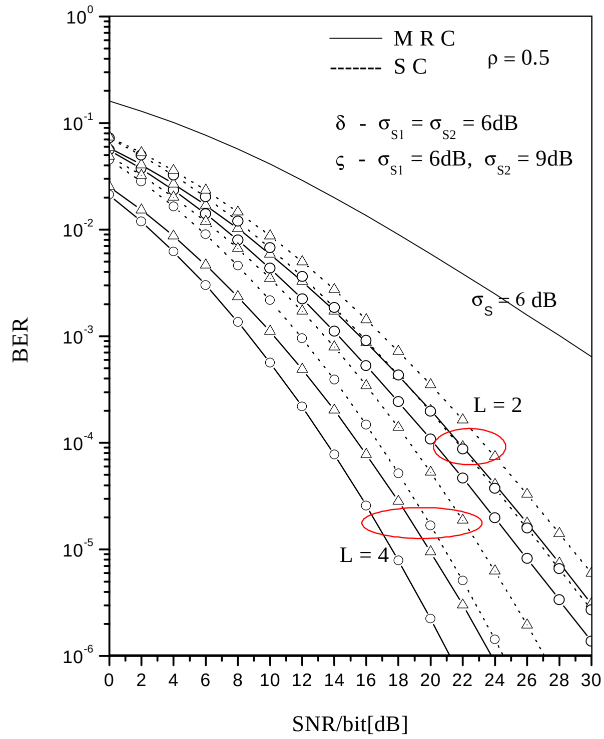

Fig. 4 Dependence of the BER of the SNR/bit for the diversity system with SC and MRC combined micro branches with equal and unequal power in correlated macro branches ($\rho = 0.5$).

Making the changes of variables $x_k = \frac{10\log_{10} z_{\max}}{\sqrt{2}\sigma_k}$, $k = 1, 2$ and using a Gauss-Hermite quadrature integration [6], $M_{\gamma_b}(s)$ will have a form

$$M_{\gamma_b}(s) = \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} A_i \cdot \left\{ \left(1 - s \cdot 10^{\frac{\sqrt{2} x_i \sigma_{S_1}}{10}} \right)^{-L} \times \left(1 - Q\left(\left(\frac{1}{\sqrt{1-\rho^2}\sigma_{S_2}} - \frac{\rho}{\sqrt{1-\rho^2}\sigma_{S_1}} \right) \cdot x_i \sqrt{2}\sigma_1 \right) \right) + \left(1 - s \cdot 10^{\frac{\sqrt{2} x_i \sigma_{S_2}}{10}} \right)^{-L} \times \left(1 - Q\left(\left(\frac{1}{\sqrt{1-\rho^2}\sigma_{S_1}} - \frac{\rho}{\sqrt{1-\rho^2}\sigma_{S_2}} \right) \cdot x_i \sqrt{2}\sigma_2 \right) \right) \right\} \quad (18)$$

Where $\{x_i\}, i = 1, 2, \ldots, n$ are the zeros of the nth-order Hermite polynomial, and $\{A_i\}, i = 1, 2, \ldots, n$ are corresponding weight factors. The sufficient value of n is 20.

Substituting (20), using $-1/\sin^2\phi$ as argument of $M_{\gamma_b}(s)$, in (14), we get expression for average BER in system with employed MRC micro- and dual, correlated, power unbalanced macrodiversity system with BPSK signalling.

Similar, when SC is applied, corresponding form of $M_{\gamma_b}(s/z_{\max})$ has a form

$$M_{\gamma_b}(s/z_{\max}) = L \sum_{j=0}^{L-1} \binom{L-1}{j} (-1)^j \left(1 - s\gamma_{\max} + j \right)^{-1}. \quad (19)$$

Substituting this expression in (15) we can get

$$M_{\gamma_b}(s) = \frac{L}{\sqrt{\pi}} \sum_{i=1}^{n} A_i \cdot \left\{ \sum_{j=0}^{L-1} \left[\binom{L-1}{j} (-1)^j \left(1 - s \cdot 10^{\frac{\sqrt{2} x_i \sigma_{S_1}}{10}} + j \right)^{-1} \right] \times \left(1 - Q\left(\left(\frac{1}{\sqrt{1-\rho^2}\sigma_{S_2}} - \frac{\rho}{\sqrt{1-\rho^2}\sigma_{S_1}} \right) \cdot x_i \sqrt{2}\sigma_1 \right) \right) + \sum_{j=0}^{L-1} \left[\binom{L-1}{j} (-1)^j \left(1 - s \cdot 10^{\frac{\sqrt{2} x_i \sigma_{S_2}}{10}} + j \right)^{-1} \right] \times \left(1 - Q\left(\left(\frac{1}{\sqrt{1-\rho^2}\sigma_{S_1}} - \frac{\rho}{\sqrt{1-\rho^2}\sigma_{S_2}} \right) \cdot x_i \sqrt{2}\sigma_2 \right) \right) \right\} \quad (20)$$

where $\{x_i\}$ and $\{A_i\}$ are defined as in (18).

Again, substituting (22) in (14), we get expression for average BER in system with employed SC micro- and dual, correlated, power unbalanced macrodiversity system with BPSK signalling.

III. Numerical Results

Figs. 1 and 2 illustrate performances of diversity receiver employing MRC and SC microdiversity reception, and dual macrodiversity reception with different values of correlation coefficient ρ, and equal branch powers ($\sigma_{S_1} = \sigma_{S_2}$). The performances decrease as the ρ gets bigger value for both of microdiversity cases, as it is expected. Degradation of performance due to correlation of macrodiversity branches becomes more evident when the number of microdiversity branches is greater. Furthermore, Figs. 3 and 4 depict dependence of the BER of the SNR/bit for the diversity system with SC and MRC combined microdiversity branches with equal and unequal power in uncorrelated and correlated macrodiversity branches, respectively. It is noticed that performances decrease when the power in macrodiversity branches is unbalanced, which is more noticeable when ρ has biger value.

IV. Conclusion

In this paper we analyze systems with implemented MRC and SC microdiversity and dual macrodiversity selection techniques in an additive white Gaussian noise channel in the presence of Rayleigh fading with dual correlated, power unbalanced, log-normal shadowing and compare different microscopic selection methods where the dependence to BER of the average SNR/bit is used as the measure of the performances. Numerical evaluations of performances are done using MGF (moment-generating function) approach [2], which represent a power tool that simplifies numerical calculations. It is noticed that performances decrease as the ρ gets bigger value for both of microdiversity cases, especially for greater number of microdiversity branches. Also, the effect of power unbalancing is is more evident ρ has biger value.

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