

# Calculation of Energy Losses in Low Voltage Distribution Systems Using a Fuzzy Clustering Technique

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**Abstract** – A method for calculation of energy losses in low voltage distribution networks, based on the fuzzy clustering technique, is developed in this paper. The method required input data about the networks that can be collected easily: lengths of lines, number of line segments, number of customers, conductor's cross section area, conductor's material, electrical energy delivered by distribution transformers etc. The method is tested on the example of ninety low voltage networks. It is analyzed the optimal number of clusters as well as the optimal value of the parameter that defines fuzziness of fuzzy clustering.

**Keywords** – Low voltage networks, Energy losses, Fuzzy clustering.

## I. INTRODUCTION

Determination of electrical energy losses in low voltage distribution networks is very important for any electrical utility. For exact calculation of energy losses, the data about network parameters as well as the data about loads for any moment of the observed time interval are needed.

The reliability of data of network parameters, for low voltage distribution networks, is very small. Problem of the unreliability can be solved by further involvement on reviewing data base. Other, much difficult problem, when energy losses in low voltage distribution networks are calculated, lies is the fact that a small number of measuring exist in these networks. Therefore, the data bases about low voltage loads do not exist. The electric energy, delivered by distribution transformers, is usually the only available data about the loads.

In [1,2] the problem of low voltage losses calculation is simplified by classifying low voltage networks in two groups: town's and village's. In [2] is shown that regression method is suitable for calculation of losses in low voltage networks.

A method for calculation of energy losses in low voltage distribution networks, based on the fuzzy clustering technique, is developed in this paper. Fuzzy clustering is made on a spice that make objects with following attributes: lengths of lines, number of line segments, number of customers, conductor's cross section area, conductor's material, electrical energy delivered by distribution transformers etc.

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## II. FUZZY CLUSTERING ALGORITHM

Clustering is one of the most fundamental issues in pattern recognition. Given a finite set of data  $X$ , the problem of clustering is to find several cluster centers that can properly characterize relevant classes of  $X$ . In classical cluster analysis, these classes are required to form a partition of  $X$  such that degree of association is strong for data within blocks of the partition. When the requirement of a crisp partition of  $X$  is replaced with a weaker requirement of a fuzzy partition or a fuzzy pseudopartition of  $X$ , we refer to the emerging problem area as fuzzy clustering. Fuzzy pseudopartitions are often called fuzzy  $c$ -partition, where  $c$  designates the number of fuzzy classes in the partition. There are two basic methods of fuzzy clustering. One of them, which is based on fuzzy  $c$ -partitions, is called a fuzzy  $c$ -means clustering method.

Let  $X = \{x_1, x_2, \dots, x_n\}$  be a set of given data. A fuzzy pseudopartition or fuzzy  $c$ -partition of  $X$  is a family of fuzzy subsets of  $X$ , denoted by  $P = \{A_1, A_2, \dots, A_c\}$  witch satisfies:

$$\sum_{i=1}^c A_i(x_k) = 1, \quad k \in N_n, \quad (1)$$

for  $N_n = \{1, 2, \dots, n\}$ , and

$$0 < \sum_{k=1}^n A_i(x_k) < n \quad i \in N_c \quad (2)$$

where  $c$  is positive integer and  $N_c$  set of integers  $N_c = \{1, 2, \dots, c\}$ . Given a set of data  $X = \{x_1, x_2, \dots, x_n\}$ , where  $x_k$  in general is a vector:

$$x_k = [x_{k1}, x_{k2}, \dots, x_{ka}] \in R^a \quad (3)$$

for all  $k \in N_n$ , the problem of fuzzy clustering is to find a fuzzy pseudopartition and associated cluster centers by which the structure of the data is represented as best as posible.

The  $c$ -means algorithm is based on the assumption that the desired number of clusters  $c$  is given and, in addition, a particular distance, a real number  $m \in (1, \infty)$ , and a small positive number  $\epsilon$ , serving as a stopping criterion, are chosen.

*Step 1.*

Let  $t = 0$ . Select an initial fuzzy pseudopartition  $P^{(0)}$ .

*Step 2.*

Calculate the  $c$  cluster centers  $v_1^{(t)}, \dots, v_c^{(t)}$  by relation

$$v_i = \frac{\sum_{k=1}^n [A_i(x_k)]^m x_k}{\sum_{k=1}^n [A_i(x_k)]^m} \quad (4)$$

For  $P^{(t)}$  and the chosen value of  $m$ .

*Step 3.*

Update  $P^{(t+1)}$  by the following procedure: For each  $x_k \in X$ , if  $\|x_k - v_i^{(t)}\|^2 > 0$  for all  $i \in N_c$ , then define

$$A_i^{(t+1)}(x_k) = \left[ \sum_{j=1}^c \left( \frac{\|x_k - v_j^{(t)}\|^2}{\|x_k - v_i^{(t)}\|^2} \right)^{\frac{1}{m-1}} \right]^{-1} \quad (5)$$

if  $\|x_k - v_i^{(t)}\|^2 = 0$  for some  $i \in I \subseteq N_c$ , then define

$A_i^{(t+1)}(x_k)$  for  $i \in I$  by any nonnegative real numbers satisfying

$$\sum_{i \in I} A_i^{(t+1)}(x_k) = 1 \quad (6)$$

and define  $A_i^{(t+1)}(x_k) = 0$  for  $i \in N_c - I$ .

*Step 4.*

Compare  $P^{(t)}$  and  $P^{(t+1)}$ . If  $|P^{(t+1)} - P^{(t)}| \leq \varepsilon$ , then stop; otherwise, increase  $t$  by one and return to Step 2.

In Step 4,  $|P^{(t+1)} - P^{(t)}|$  denotes a distance between  $P^{(t+1)}$  and  $P^{(t)}$  in the space  $R^{n \times c}$ . An example of this distance is

$$|P^{(t+1)} - P^{(t)}| = \max_{i \in N_c, k \in N_n} |A_i^{(t+1)}(x_k) - A_i^{(t)}(x_k)|. \quad (7)$$

In the algorithm, the parameter is selected according to the problem under consideration. When  $m \rightarrow 1$ , the fuzzy  $c$ -means converges to a "generalized" classical  $c$ -means. When  $m \rightarrow \infty$ , all cluster centers tend towards the centroid of data set  $X$ . That is, the partition becomes fuzzier with increasing. Currently, there is no theoretical basis for an optimal choice for the value of  $m$ . However, it is established that the algorithm converges for any  $m \in (1, \infty)$ .

### III. PROCEDURE OF CALCULATION OF ENERGY LOSSES

The presented fuzzy clustering algorithm can be applied for calculation of electrical losses in low voltage distribution networks. Following steps should be made:

*Step 1*

From the set of  $N_n$  low voltage networks,  $N_t$  test networks are selected. Additional measurements, that enable losses calculation by classical methods, are made on the test networks.

*Step 2*

A spice that consists of  $N_t$  objects is now formed. Any object has  $N_a$  attributes, whereat value of energy losses is one attribute (absolute or percentage).

*Step 3*

For chosen number of clusters  $c$  and real number  $m \in (1, \infty)$ , fuzzy clustering is made, and objects that represent cluster centers are stored in memory.

*Step 4*

A spice that consists of  $N_n - N_t$  objects with  $N_a - 1$  attributes is formed. Energy losses are not considered as attribute of objects.

*Step 5*

Using the modified  $c$ -means method, fuzzy clustering is made for cluster centers determined in step 3 without the energy attributes included. The modification of  $c$ -means method is following: on the step 2 of  $c$ -means algorithm shown in II, the centers of clusters are not determined then centers of the test networks are retained.

*Step 6*

For any object (low voltage networks), absolute or percentage energy losses are calculated summing the memberships values multiplied with attributes of corresponding center of cluster that represent energy losses.

The following questions required answer:

- what is optimal cluster number,
- what value of real number  $m$  is optimal,
- what attributes shall we consider.

These questions have not unique solution, and as will be seen from the following example, answers depend on number of disposed data.

### IV. EXAMPLE OF ENERGY LOSSES CALCULATION

In order to analyse proposed method, calculation of energy losses are made for ninety low voltage networks. The authors made calculations of losses, using modified  $c$ -means method, for 23 different combinations of parameter  $m$  (1.05 to 1.75), number of clusters and chosen data set comprehended in analysis. Obtained values of percentage losses are shown in Table I. The following data are comprehended in analysis:

1. number of line sections,
2. median cross section area of first line section  $(3 \cdot S_f + S_0) / 4$ ,
3. the smallest median cross section area of line  $(3 \cdot S_{f \min} + S_{0 \min}) / 4$ ,
4. resistivity of conductor,
5. electrical energy delivered by distribution transformers,
6. load factor,

where  $S_p$  and  $S_n$  are cross section areas of phase and neutral conductors respectively.

For selection 1, shown in Table I, all available data are comprehended. Selection 2 does not comprehend resistivity of conductors, and selection 3 does not comprehend load factors.



$m$	selection 1												selection 2						selection 3					
	1.05				1.2				1.4				1.75			1.05			1.2			1.4		
num. of clu.	6	12	20	10	20	6	12	20	6	12	20	6	12	20	10	15	20	10	20	12	20	12	20	
69	3.88	3.46	3.34	2.87	4.28	1.25	5.13	4.06	1.47	5.88	5.07	3.74	3.52	0.93	0.93	0.93	0.93	0.93	1.89	1.25	2.92	1.25	2.92	2.64
70	0.493	3.46	3.33	9.01	4.5	3.84	5.81	4.83	4.74	7.34	5.81	1.65	6.36	1.93	0.64	4.13	2.45	0.65	3.29	3.3	3.57	1.9	3.6	1.97
71	1.11	1	0.84	1.05	0.85	0.71	1.23	0.89	0.73	1.45	1.13	1.22	3.52	0.93	0.93	0.94	0.94	0.97	1.19	1.06	1.19	1.08	1.19	1.25
72	1.40	1	0.84	1.06	0.85	1.04	1.31	0.89	1.04	1.75	1.75	2.74	4.79	1.93	2.78	3.99	2.31	9.47	1.19	1.06	1.19	1.06	1.22	1.05
73	10.86	15.4	13.6	17.9	10.7	3.64	6.13	10.9	5.84	6.96	7.88	8.46	6.36	6.71	4.37	8.76	6.86	6.1	15.6	18.1	16.1	6.52	16.2	7.12
74	2.593	3.46	3.33	3.15	3.89	3.09	6	5.35	3.97	6.09	5.53	5.78	6.33	1.93	2.78	4.23	2.31	2.93	3.29	7.52	3.58	1.91	3.79	2.3
75	3.32	0.29	0.27	0.27	0.3	0.24	0.62	0.72	0.58	2.06	2.29	2.81	0.52	0.32	0.24	0.25	0.25	0.25	0.05	0.24	0.24	0.24	0.35	0.35
76	11.54	18.5	15.4	11	13.6	4.35	6.22	6.16	5.28	10.9	6.69	9.02	6.36	14.3	0.98	5.13	5.91	14.4	8.02	12.2	3.84	14	5.04	13.5
77	4.794	3.46	3.36	2.86	3.75	3.09	5.58	3.32	3.61	5.75	4.66	4.98	3.52	2.63	2.78	3.91	2.72	2.9	4.45	6.73	4.17	6.46	5.19	6.65
78	0.40	0.29	6.18	0.45	4.72	1.66	2.51	4.2	3.89	4.21	4.2	4.45	2.01	0.36	0.4	0.86	0.54	0.55	0.05	0.24	0.25	0.26	0.55	0.11
79	2.06	0.29	0.27	0.27	0.28	0.24	0.54	0.33	0.29	1.82	1.64	2.53	0.47	0.32	0.24	0.24	0.24	0.05	0.24	0.24	0.24	0.28	0.28	0.06
80	4.49	3.46	3.33	3.15	3.72	3.1	6.08	5.33	3.9	5.97	5.31	5.63	6.35	1.93	2.78	4.05	2.2	2.91	3.29	12.2	3.57	1.95	3.66	5.47
81	4.631	3.46	3.36	3.2	3.56	3.77	3.99	4.1	4.71	5.91	4.87	4.91	3.52	2.89	3.19	3.56	4.5	3.13	4.45	5.25	3.29	3.68	3.51	3.7
82	0.77	0.29	0.27	0.4	0.63	0.4	1.24	2.18	2.11	3.43	3.59	3.89	2	0.32	0.4	0.37	0.44	0.45	0.05	0.24	0.26	0.28	0.76	0.18
83	7.54	18.5	16.2	11.8	16.1	12.1	7.04	12	9.94	13	10.5	9.73	6.36	14.4	17.9	15.7	17	11.2	15.6	18.1	16.1	12.8	16.1	8.84
84	0.34	1	0.84	0.59	0.86	0.82	1.07	1.16	1.4	1.94	2.61	3.6	3.52	4.54	9.23	3.54	4.33	7.72	1.19	0.52	1.19	1.23	1.24	1.38
85	0.92	1	0.84	0.52	0.85	0.71	1.03	0.99	0.77	1.45	1.78	2.16	0.47	0.93	0.93	0.93	0.93	0.94	1.19	2.91	1.19	1.31	1.23	1.26
86	8.80	18.5	16.2	18.1	15.5	17	6.63	13.3	15.8	8.72	12.3	19.4	6.36	32.1	25.5	15.4	9.58	17.5	29	34.6	21	25.6	18.1	23.1
87	7.99	18.5	29.5	38.8	25.5	38.1	7.24	18.5	28.7	7.69	9.12	12.8	29.3	32.1	37.3	24.6	15.9	34.5	29.6	38.8	29.6	37.4	29.4	29.1
88	0.28	1	0.84	0.52	0.85	0.71	1.05	0.91	0.71	1.27	1.27	1.18	0.47	0.93	0.93	0.93	0.93	1.19	0.52	1.19	1.31	1.21	1.26	
89	6.20	18.5	16.2	17.7	15.6	16.1	6.2	14.7	14.8	7.27	8.37	7.89	6.36	14.4	4.38	14.5	18.8	4.09	15.6	18	16.1	5.99	16.1	3.92
90	3.40	3.46	3.33	3.79	3.55	3.14	6.13	5.06	3.94	5.53	4.86	5.6	6.31	2.58	2.78	3.89	3.45	2.9	3.3	12.2	4.38	12	5.52	13

Column 2 of Table I shows value of percentage energy losses given by additional measurement, while columns 3-25 show calculated percentage energy losses.

Meaning of different colours in Table I is following:

- networks with a large value of electrical losses that are not identified,
- networks with a large value of electrical losses that are identified properly,
- networks for which a large value of electrical losses is identified erroneously.

On the bases of results shown in Table I, we can conclude the best results are given in following two cases:

- $m=1.05$ , for 12 clusters and all data comprehended,
- $m=1.4$ , for 20 clusters and all data comprehended.

Table I shows only a small piece of results. The authors was made analysis for different number of networks, and concluded following:

- requested number of test networks is 50 to 100,
- all available data shall be comprehended,
- optimal number of clusters depend on number of test networks and shall be 15% to 30% of the number of test networks,
- parameter  $m$  gives the optimal value in the range of 1 to 1.5, thereat value of  $m$  increase with increasing the number of clusters,
- total percentage electrical losses of all networks are quite accurate if previous conclusions are satisfied.

## V. CONCLUSION

A new method for calculation of energy losses in low voltage distribution networks is developed in this paper. The method is based on the fuzzy clustering technique. Beside the data about parameters of networks, method requires electrical energies delivered by distribution transformers. Additionally,

measurements, that enable losses calculation by classical methods, are made for selected test networks.

Optimal number of clusters depends on number of test networks. The authors suggest that number of cluster gives the value in the range of 15% to 30% of number of test networks. Parameter  $m$  that defines fuzziness of clustering shall be selected from the range of 1 to 1.5. For proposed values of parameter  $m$  and number of clusters, the calculated value of total electrical losses of all networks is quite accurate.

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