Fast Method for Asymmetrical Load-Flow Solution in Sequence Domain

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Abstract **– In this paper a new fast method for asymmetrical load-flow solution in sequence domain is presented. The entire power system is modelled with three decoupled positive-, negative- and zero-sequence circuits. The proposed method is consisted of a system of non-linear equations which represents the positive- and two systems of linear equations which represent the negative- and zero-sequence circuits. The solution of non-linear equations is by Newton-Raphson procedure and solution of linear systems of equations is by Gauss's method.**

Keywords **– Asymmetrical load-flow, sequence domain, decoupled sequence circuits.**

I. INTRODUCTION

The load-flow studies are the most frequent calculations in the power system analysis. The steady state symmetrical load-flows studies (SLF) are performed in the more efficient and comfortable sequence domain instead of in the phase domain. Usually, power system states deviate more or less from the symmetrical states, therefore symmetrical states are only approximations of the actual states of three-phase power systems. Always, the three-phase electrical power system states are asymmetrical. The presence of long unbalanced (untransposed) transmission lines and asymmetrical or single-phase loads (as induction furnaces and electrical railways substations) cause: negative-sequence currents at generator terminals rise heating in their rotors; malfunctions of protective relays; zero-sequence currents increase greatly the effect of inductive coupling between parallel transmission lines; higher power system loss etc. Because of these reasons, for more precise analysis of three-phase power system states, the asymmetrical load-flow (ALF) analysis are required. Also, ALF calculations are required in the process of transmission line designing to study the effects of various phase arrangements, or in operating situation with single pole switching, etc.

Usually, the methods for ALF solutions are in phase domain [1-3]. The main reasons for avoiding the sequence domain in the ALF methods are: (1) presence of phase shifts of the three-phase transformers (ideal transformers with complex turns ratios in their sequence circuits); (2) mutually couplings among sequence circuits in the points of power system unbalances and (3) asymmetrical phase loads, which cannot be specified in the sequence domain. Applying new scaling concept [4], unbalanced line decoupled model in sequence domain and asymmetrical phase loads model specified in the sequence domain [5-7], the entire power system can be modeled with three decoupled positive, negative and zero-sequence circuits. For proper definition of the ALF methods in sequence domain an enhanced bus classification is proposed in [8], [9].

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II. ENHANCED BUS CLASSIFICATION

Usual bus classifications are performed in accordance with the specification of values of quantities associated with power system buses. Twelve real-valued quantities are associated with each three-phase bus: three pairs of voltage magnitudes and angles, as well as three pairs of active and reactive injected powers. Three pairs of active and reactive power balance equations, for each three-phase bus, describe these quantities. These equations are extended with relations representing control laws associated with buses in which the three-phase active powers and reactive powers or voltage magnitudes are controlled. To provide a correct treatment of reactive power limits enforcement at the generators, as well as to simplify the ALF method, the standard three types buses classification is enhanced to a new four types buses classification by introducing a new or $P_\Sigma Q_\Sigma$ type of buses (Table I). The new classification is independent of the domain which the ALF problem is stated in:

1. $P_\Sigma V$ bus is a bus in which the value of three-phase injected active power (P_Σ) and the control law of the automatic voltage regulator (AVR) are specified. Values of three pairs of magnitudes and angles of voltages, as well as values of three pairs of injected active and reactive powers are unknown. Applying the synthesizing procedure [8], [9], this bus is suppressed in the high voltage bus of the step-up transformer.
2. θV bus (slack bus) is a bus in which the angle of a voltage and the control law of the generator AVR are specified. Values of three magnitudes and two angles of voltages and values of three pairs of injected active and reactive powers are unknown. Applying the synthesizing procedure, this bus is suppressed in the high voltage bus of the step-up transformer.
3. $P_\Sigma Q_\Sigma$ bus is a bus in which values of three-phase injected active and reactive powers (sums of phase powers P_Σ and Q_Σ) are specified. Values of three pairs of magnitudes and angles of voltages and values of three pairs of injected active and reactive powers are unknown. It is newly introduced type of buses, which is necessary to provide the correct treatment of Q limits enforcement at $P_\Sigma V$ buses. Also, this type of bus is suppressed in the high voltage bus of the step-up transformer.

TABLE I. THE ENHANCED BUS CLASSIFICATION FOR THE ALF PROBLEM (c.l. – CONTROL LAW, PHASE DOMAIN *abc*, SEQUENCE DOMAIN *dio*).

BUS TYPE	$P_\Sigma V$		θV		$P_\Sigma Q_\Sigma$		PQ	
	abc	*dio*	*abc*	*dio*	*abc*	*dio*	*abc*	*dio*
SPECIFIED VALUES	P_Σ c.l. of AVR	P_Σ c.l. of AVR	θ_a c.l. of AVR	θ^d c.l. of AVR	P_Σ Q_Σ	P_Σ Q_Σ	P_a, P_b, P_c Q_a, Q_b, Q_c	P^d, P^i, P^o Q^d, Q^i, Q^o
UNKNOWN VALUES	U_a, U_b, U_c $\theta_a, \theta_b, \theta_c$ P_a, P_b, P_c Q_a, Q_b, Q_c	U^d, U^i, U^o $\theta^d, \theta^i, \theta^o$ P^d, P^i, P^o Q^d, Q^i, Q^o	U_a, U_b, U_c θ_b, θ_c P_a, P_b, P_c Q_a, Q_b, Q_c	U^d, U^i, U^o θ^i, θ^o P^d, P^i, P^o Q^d, Q^i, Q^o	U_a, U_b, U_c $\theta_a, \theta_b, \theta_c$ P_a, P_b, P_c Q_a, Q_b, Q_c	U^d, U^i, U^o $\theta^d, \theta^i, \theta^o$ P^d, P^i, P^o Q^d, Q^i, Q^o	U_a, U_b, U_c $\theta_a, \theta_b, \theta_c$	U^d, U^i, U^o $\theta^d, \theta^i, \theta^o$

4. PQ bus is a standard type of buses in which values of three pairs of injected active and reactive powers are specified. Values of three pairs of magnitudes and angles of voltages are unknown.

Three complex or six real equations, representing the current or power balances for each three-phase bus, are on disposal to solve the unknown values of quantities presented in Table 1. Six unknown values are fully covered by these equations for PQ buses only. Twelve unknown values associated with other buses are covered by eight equations only–six previously noted balanced equations and two relations corresponding to specified (controlled) values. Thus four equations have to be established to cover the remaining four unknown values. This problem is solved by applying new scaling concept [4] and synthesizing procedure [8], [9]. With the new scaling concept the transformer complex turn ratios are eliminated from the power system sequence circuits. The synthesizing procedure enables to suppress the equivalent parameters (of the generator and it's corresponding step-up transformer) of the negative and zero-sequence circuits in the transmission network. This suppression enables zero-valued injected currents and powers in the corresponding negative and zero-sequence nodes. Now, the issue of shortage of four equations corresponding to each $P_\Sigma V$, θV and $P_\Sigma Q_\Sigma$ bus in the sequence domain can be solved.

III. POWER SYSTEM SEQUENCE CIRCUITS DECOUPLING

When the power system elements (balanced or unbalanced) are modeled in phase domain there are mutual inductive and capacitive couplings between phases. But, if balanced elements (practically all generators, transformers and transposed lines) are modeled in sequence domain all mutual couplings between phases and sequence circuits are eliminated [7].

When the unbalanced lines are considered in sequence domain, there are couplings among positive-, negative- and zero-sequence and the line model cannot be presented with lumped-π decoupled sequence circuits. In this case, 6x6 node-admittance matrix representatives of the line is full with non-zero elements, just like the 6x6 node-admittance matrix in the phase domain. Thus, the power system model in sequence domain cannot be presented with three linear decoupled sequence circuits. The points of mutually coupling among positive-, negative- and zero-sequence power system circuits are just these unbalanced lines. Inductive and capacitive mutual couplings among positive-, negative- and zero-sequence are expressed with non-zero off-diagonal elements in 6x6 node admittance matrix. Instead of mutually admittances, the couplings can be expressed by compensation current sources [5], [7]. Thus, the unbalanced line model can be presented with three decoupled sequence circuits as it is depicted in Fig. 1a-c. The mutual couplings are replaced by corresponding controlled sources – current sources.

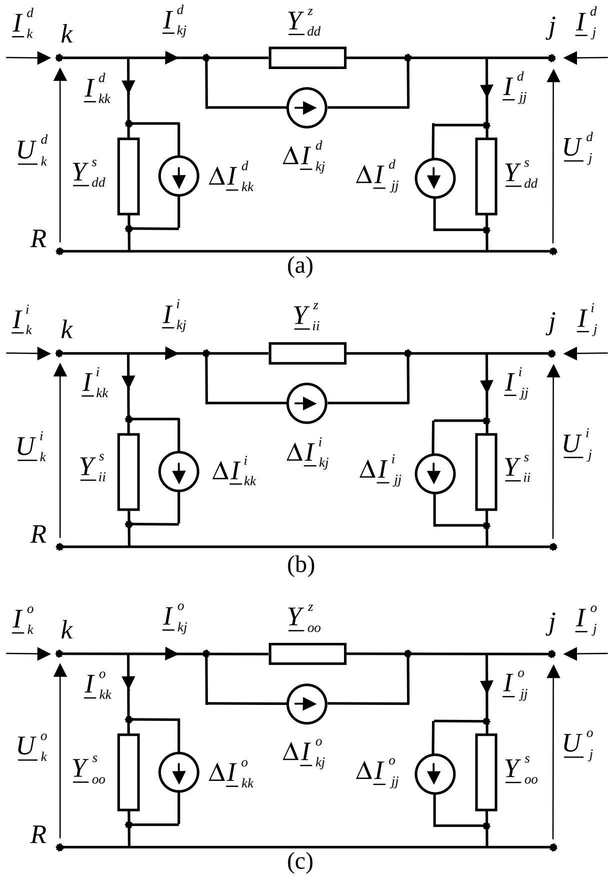

Fig. 1. Unbalanced line decoupled positive- (a), negative- (b) and zero-sequence circuit (c) in absolute value domain.

The current controlled sources in series and shunt branches of each sequence lumped-π circuit include the coupling influences from the other sequences. The self-admittances and the current sources currents in series and shunt branches of any sequence from the Fig. 1 can be calculated very easy as it is shown in [7], [9].

The injected currents in the ends *k* and *j*, of any sequence lumped-π circuit can be corrected by the compensation currents [7], [9]. These corrections enable the omit ion of the current sources from the sequence circuits in Fig. 1a-c and obtaining the final decoupled, compensated, scaled, unbalanced line model in sequence domain, depicted in Fig. 2.

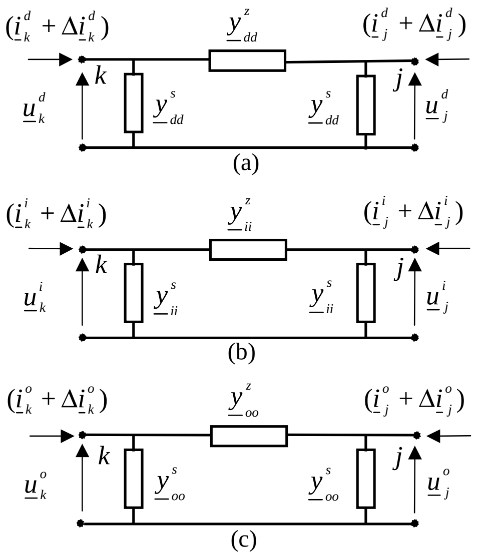

Fig. 2. Unbalanced line decoupled, compensated, scaled positive- (a), negative- (b) and zero-sequence (c) lumped-π circuits.

Applying above described unbalanced line model, new scaling concept, and synthesizing procedure the entire power system can be represented with three decoupled positive-, negative- and zero-sequence circuits.

IV. Fast Method Definition

Each load-flow model is based on a power system linear model, which is stated in terms of complex voltages and currents. The most widely used linear power system model is that of the nodal voltage equation. For the power system with *n* three-phase buses, i.e. 3*n* phase nodes, this model in sequence domain (*dio*) says:

$$\underline{\mathbf{Y}}_{3n\times 3n}^{dio}\underline{\mathbf{U}}_{3n\times 1}^{dio} = \underline{\mathbf{I}}_{3n\times 1}^{dio} . \quad (1)$$

In the Eq. 1, $\mathbf{Y}_{3n\times 3n}^{dio}$ is node-admittance matrix of the power system equivalent sequence circuits. The complex voltages and injected currents in all power system buses are elements of vectors $\underline{\mathbf{U}}_{3n\times 1}^{dio}$ and $\underline{\mathbf{I}}_{3n\times 1}^{dio}$. They consist of sub vectors of dimensions 3×1, containing the sequence complex quantities. The node-admittance matrix is formed for a sequence circuits without ideal transformers with complex turn ratios. If the number of generators in the power system is n_g, the application of the synthesizing procedure enables power system buses reduction for $2n_g$ buses. Now, the power system can be treated as a system with $r = n - 2n_g$ buses or $3r$ nodes. Therefore, the power system model given by Eq. (1) gets new form with reduced dimensions:

$$\underline{\mathbf{Y}}_{3r\times 3r}^{dio}\underline{\mathbf{U}}_{3r\times 1}^{dio} = \underline{\mathbf{I}}_{3r\times 1}^{dio} . \quad (2)$$

Finally, if the decoupled, compensated, scaled, unbalanced line model in sequence domain is applied, the entire power system can be represented by new model with three systems of linear equations:

$$\mathbf{Y}_{r\times r}^{d}\mathbf{U}_{r}^{d} = \mathbf{I}_{rc}^{d} , \quad (3)$$

$$\mathbf{Y}_{r\times r}^{i}\mathbf{U}_{r}^{i} = \mathbf{I}_{rc}^{i} , \quad (4)$$

$$\mathbf{Y}_{r\times r}^{o}\mathbf{U}_{r}^{o} = \mathbf{I}_{rc}^{o} . \quad (5)$$

Each of these systems of equations represent the nodal voltage equations for the power system positive-, negative- and zero-sequence decoupled circuits. The matrices of injected currents, corrected by compensation currents (as result of circuits decoupling) $\mathbf{I}_{rc}^{d}$, $\mathbf{I}_{rc}^{i}$ and $\mathbf{I}_{rc}^{o}$ for positive-, negative- and zero-sequence decoupled circuits respectively, are consisted of node injected complex currents. At first, the Eq. (3) can be conjugated and after that multiplied from the left by a diagonal matrix containing the complex positive-sequence voltages. As the result of this procedure, a new nonlinear system of equations representing the power system positive-sequence is obtained:

$$\mathbf{U}_{r,dij}^{d}\left(\mathbf{Y}_{r,r}^{d}\right)^{*}\left(\mathbf{U}_{r}^{d}\right)^{*} = \mathbf{S}_{rc}^{d} . \quad (6)$$

In Eq. (6), matrix $\mathbf{S}_{rc}^{d}$ represents complex, compensated injected powers in the positive sequence circuit nodes [9]. Applying the Taylor's procedure, the nonlinear system of equations given by matrix Eq. (6), can be transformed in the new linear system of equations. This new system is consisted of equations for differences between the injected specified and calculated powers $-\Delta\mathbf{S}_{kor}^{d}$ in the power system positive-sequence circuit nodes, represented by the Jacobian $\mathbf{J}^{d}$ (for this sequence circuit) and unknown differences of voltage magnitudes and angles given by the matrix $\Delta\mathbf{X}^{d}$:

$$\mathbf{J}^{d}\Delta\mathbf{X}^{d} = \Delta\mathbf{S}_{kor}^{d} . \quad (7)$$

or in the well known form with sub matrices:

$$\begin{bmatrix} \mathbf{H}^{d} & \mathbf{N}^{d} \\ \mathbf{M}^{d} & \mathbf{L}^{d} \end{bmatrix} \begin{bmatrix} \Delta\theta^{d} \\ \Delta\mathbf{U}^{d}/\mathbf{U}^{d} \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{P}_{kor}^{d} \\ \Delta\mathbf{Q}_{kor}^{d} \end{bmatrix} . \quad (8)$$

Actually, the matrix Eq. (8) has the same form as the equations which represents the symmetrical load-flow model [10]. The Eqs. (8), (4) and (5) together represent the model of the new fast method for asymmetrical load-flow solution.

Because for the *r*-th, θV bus (or slack bus), the complex voltage is specified ($\underline{U}_{r,sp}^{d} = U_{r,sp}^{d}\angle\theta_{r,sp}^{d}$), the equations representing the power differences for this bus are not included in the system given by Eq. (8).

For all *g* buses, type $P_{\Sigma}V$, the three-phase injected power in phase domain $P_{g,sp}^{\Sigma}$ are specified. This power expressed through sequence voltages and currents is given by equation:

$$P_{g,sp}^{\Sigma} = 3\mathrm{Re}\left[\underline{U}_g^d \left(\underline{I}_g^d\right)^* + \underline{U}_g^i \left(\underline{I}_g^i\right)^* + \underline{U}_g^o \left(\underline{I}_g^o\right)^*\right], \; g \in \{P_\Sigma V\}. \quad (9)$$

As it was mentioned above, the synthesizing procedure enables to account with zero injected currents in this type of nodes in the negative- and zero-sequence circuits ($\underline{I}_g^i = 0$ and $\underline{I}_g^o = 0$). Taking into account this fact, the three-phase injected power given by Eq. (9) can be expressed as:

$$P_{g,sp}^{\Sigma} = 3\mathrm{Re}\left[\underline{U}_g^d \left(\underline{I}_g^d\right)^*\right] = 3\mathrm{Re}\left[\underline{S}_g^d\right] = 3P_g^d, \quad g \in \{P_\Sigma V\}. \quad (10)$$

From Eq. (10), the injected specified power in the buses type $P_\Sigma V$, in positive-sequence circuit is calculated very easy:

$$P_g^d = P_{g,sp}^d = \frac{1}{3} P_{g,sp}^{\Sigma}, \quad g \in \{P_\Sigma V\}. \quad (11)$$

Also, for this type of buses, the positive-sequence voltage magnitude $-U_{g,sp}^d$, is specified. Because, the value of the positive-sequence voltage angle $-\theta_{g,sp}^d$ is unknown, there is only one equation (for differences between the injected specified and calculated active powers) in matrix Eq. (8), for each bus of this type.

If the number of the $P_\Sigma Q_\Sigma$ type of buses is *q*, then for each bus there are two equations (in matrix Eq. (8)) for differences between the injected specified and calculated three-phase active and reactive powers. As specified powers in the nodes of the positive sequence circuit is taken one third of the three-phase active power and one third of the maximum or minimum possible injected reactive power (depending which limit of reactive power is achieved).

For all of *p* buses type PQ, there are two equations in the matrix Eq. (8). Because, for these buses the phase active and reactive powers are specified, it is necessary to express specified injected active power $-P_{p,sp}^d$ and reactive power $-Q_{p,sp}^d$ in the nodes of the positive-sequence circuit. This procedure is explained in [7], [9].

Taking into account the above explanations, the conclusion is that the whole number of equations in the system given by Eq. (8) is $k = 2p + 2q + g$. The solution of systems of equations Eq. (8), (4) and (5) is by iterative procedure.

With the above proposed fast method, the problem of ALF solution is transformed in the easy solution of SLF problem and solution of two systems of linear equations representatives of negative- and zero-sequence power system circuits.

V. METHOD VERIFICATION

The fast method is tested on the entire power system of the Republic of Macedonia consisting of 63 buses of 400, 220 and 110 kV voltage level, 53 lines, 5 interconecting transformers and 9 equivalent generators and step-up transformers. Eight states (variants) are considered. The comparison of the results obtained by Newton–Rapson's method for ALF in phase domain [1] and the proposed fast method in this paper is performed. The proposed fast method in sequence domain is very efficient and robust, because the amount of iterations/CPU time required for calculations and memory storage are significantly smaller then the Newton-Rapson's method for ALF in phase domain.

VI. CONCLUSION

In this paper the efficient fast method for asymmetrical load-flow solution is given. The efficiency is achieved by applying several advancements as: enhanced bus classification, sequence circuits decoupling, new scaling concept and synthesizing procedure. The form of the decoupled positive-sequence part of the presented ALF model is reduced to the form of the classical SLF problem. Thus, the same SLF Newton–Rapson procedure can be applied inside the ALF solution procedure. The negative- and zero-sequence parts of the presented ALF model are represented by two systems of linear equations and solved by Gauss's method of coefficient elimination.

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