Bond Graph Modelling and Simulation of Stochastic Systems using Bondsim – Simulink Tools

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***Abstract* – The extension of the bond graph theory for the modelling of nonlinear stochastic systems is given in this paper. For that purpose the new Bondsim elements are constructed. The efficiency of application of the proposed approach for modelling stochastic dynamic systems is illustrated using an example.**

***Keywords* – Stochastic system, Bond graph, Bondsim library**

I. INTRODUCTION

There are different approaches for deriving mathematical models. The one of them is using the bond graphs [1-4]. The fundamental advantage of this modelling is that it is based on the central physics concept-energy (bond graph consists of components which exchange energy or power using connections which connected them; these connections are bonds). The power is product of two variables: the effort e and the flow f. The effort (for example: voltage, force, pressure, etc.) and the flow (for example: current, velocity, volume flow rate, etc.) are generalization of similar phenomenon of physics. Therefore, the second advantage is that bond graphs can be used for the different types of systems (electrical, mechanical, hydraulic systems, etc.) and for their combinations (electro-mechanical, mechanical-hydraulic systems, etc.). The third advantage is that complex systems can be divided into simple elements using the bond graphs. The bond graphs give the complete description of dynamical systems and the state space equations can be derived easily. Also, stochastic dynamic systems can be modeled using this modeling technique.

Causes for the stochastic behaviour of a system may be either random excitations from the environment, or statistical variations in the material properties and geometrical configuration of the system itself, or both [5].

Calculation with stochastic dynamic is possible using new stochastic (random) Bondsim elements, which are presented in this paper.

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II. STOCHASTIC SYSTEMS MODELLING

In order to model stochastic systems, bond graph theory must be extended so the stochastic behaviour of a parameter can be incorporated in the model. For such an extension every parameter of a model is assumed to be composed of two parts – the deterministic part and the stochastic part. Thus parameter values for scalar (λ) as well as field $([\lambda])$ bond graph elements, can be approximated to the following form:

$$\lambda = \lambda_d\left(t, \bar{X}(t)\right) + \lambda_s\left(t, \bar{X}(t)\right)V(t) \qquad (1)$$

$$[\lambda] = \left[\lambda_d\left(t, \bar{X}(t)\right)\right] + \left[\lambda_s\left(t, \bar{X}(t)\right)\right]\bar{V}(t) \qquad (2)$$

where $\lambda_d\left(t, \bar{X}(t)\right)$ and $\lambda_s\left(t, \bar{X}(t)\right)$ represent deterministic and stochastic behaviour of a parameter, respectively, and $V(t)$ is the physical white noise.

The equations of stochastic process motions obtained from a bond graph model can be written in the following form:

$$\frac{d}{dt}\bar{X}(t) = \bar{f}\left(t, \bar{X}(t)\right) + \left[\sigma\left(t, \bar{X}(t)\right)\right]\bar{V}(t) \qquad (3)$$

where $\bar{X}(t)$ is a n – dimensional state vector of the response coordinates, $\bar{f}\left(t, \bar{X}(t)\right)$ is a vector function of the state variables, $\left[\sigma\left(t, \bar{X}(t)\right)\right]$ is an $n \times \mu_S$ matrix function and $\bar{V}(t)$ is a μ_S - dimensional independent random processes which influence the model through matrix $\left[\sigma\left(t, \bar{X}(t)\right)\right]$. Eq.(3) is known as *Langevin* equation. The vector $\bar{V}(t)$ is a vector of white noises. The white noise is conceived as a random process with mean value zero and a constant spectral density on the entire real axis. Such a process does not exist in the conventional sense, since it must have the Dirac function as covariance, and hence an infinite variance (and independent values at all points). The white noise is a useful mathematical idealization for describing the random influence that fluctuate rapidly and hence are virtually uncorrelated for different instants at time.

For a system with deterministic parameters, there is no need for any modification of the graph as long as there is no occurrence of differential causality. If some parameters of the system fluctuate randomly with time, then the scheme of causality needs a modification [5]. In bond graph modelling reciprocal of white noise is strictly not allowed. If any of the system parameters exhibits stochastic behaviour and if in the bond graph model the parameter gets inverted, the alternative measure has to be taken to avoid such reciprocation. The causality assignment of a bond graph model should be such that the multiplications of white noises in the mathematical model do not occur.

The steps for the causality assignment procedure for stochastic models are:

1. Causality is assigned to the port of a source element.
2. Causality may be propagated using elements 0, 1, TF, GY connected to the ports to which causality has been assigned. If any conflict arises at this stage, the model would be declared to be morphologically incompatible and should be modified.
3. Go back to step 1 until all source ports are assigned appropriate causality.
4. Integral causality is assigned to all storage ports.
5. The causality of a stochastic element ports should be assigned so that its parameter does not get inverted.
6. To the remaining element ports and internal bonds the causality should be assigned in order to minimize causal violations at the 0 and 1 junctions.

If any bond graph element has inverted parameter exhibiting stochastic behaviour, new bond graph elements should be added to avoid such reciprocation. Thus the bond graph model becomes more complex.

Further in the paper, the stochastic (random) Bondsim elements are presented. The use of these elements avoids the division with zero which can be appeared because of the stochastic behaviour of parameters.

III. Stochastic Bondsim – Simulink Elements

When the bond graph model is known for a dynamic system, Bondsim Simulink library may be used for directly obtaining of Simulink simulation models from bond graph models, without using the state-space equations. This library was realized for deterministic systems and contains elements (blocks) which were derived from bond graph elements based on the knowledge of the causality and the appropriate functional relations between inputs and outputs. The elements of this library and their application are described in detail in [4], while the method of the direct transformation of causal bond graph models into the block diagrams (Fakri transformation) is described in [3]. In order to get a simpler and more effective direct transformation using the Bondsim tool, a connection with Fakri transformation was proposed [6].

Calculation with stochastic dynamic is possible using new stochastic (random) Bondsim elements given in Fig. 1. On the example of Random Inductance (Ir) Bondsim element, the way for stochastic Bondsim elements obtaining, is described. On Fig.2 the block diagram of Ir element is given. This block diagram realizes the next functional relation between its input and output:

$$f = f(0) + \frac{1}{I_S}\int e dt \qquad (4)$$

where I_S is stochastic parameter:

$$I_S = (1 \pm tol) \cdot I \qquad (5)$$

Parameter I has deterministic value and tol is random number, $0 < tol < 1$, generated by the Uniform Random Number block (Fig.3). This block generates uniformly distributed random numbers over a specifiable interval with a specifiable starting seed. The seed is reset each time a simulation starts. The generated sequence is repeatable and can be produced by any Uniform Random Number block with the same seed and parameters.

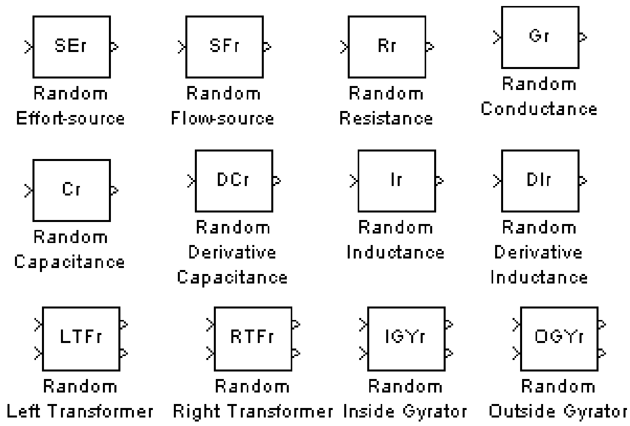

Fig.1. Random Bondsim elements

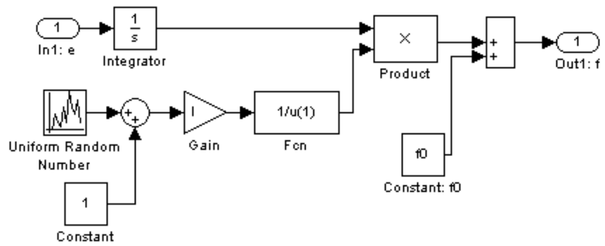

Fig.2. Random Inductance Bondsim element

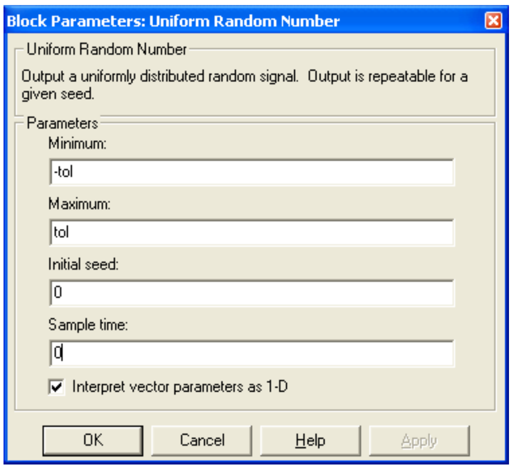

Fig.3. Dialog box of Uniform Random Number

Dialog box of Random Inductance Bondsim element is given in Fig.4.

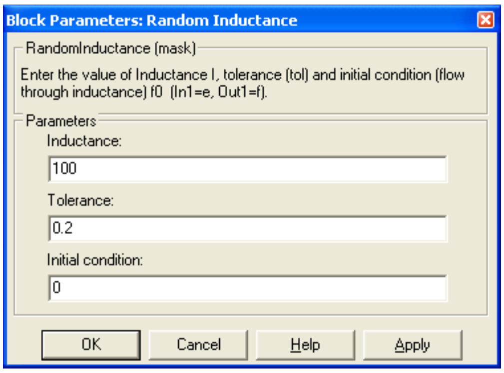

Fig.4. Dialog box of Random Inductance Bondsim element

IV. Simulation Results

The application of stochastic Bondsim elements will be illustrated using an electrical circuit example shown in Fig.5. The parameter values are: I=980A, C=0.351F, n=1963, L=100H, $R_1 = 1/11.6\Omega$, $R_2 = 5000\Omega$ and the initial conditions are: capacitor voltage equal to 0V and coil current equal to 0A.

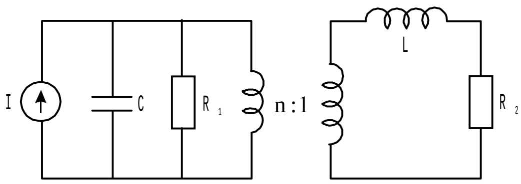

Figure 5. The electrical circuit.

The causal bond graph model of given example is shown in Fig. 6.

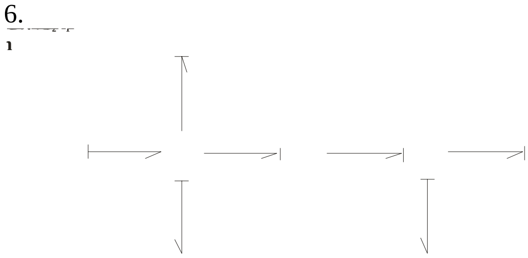
Figure 6. The causal bond graph model.

Two cases are considered: a) stochastic inductivity parameter with parameter tol =0.2 and b) stochastic flow-source with parameter tol=0.05. Bondsim simulation model with Ir element is given in Fig.7.

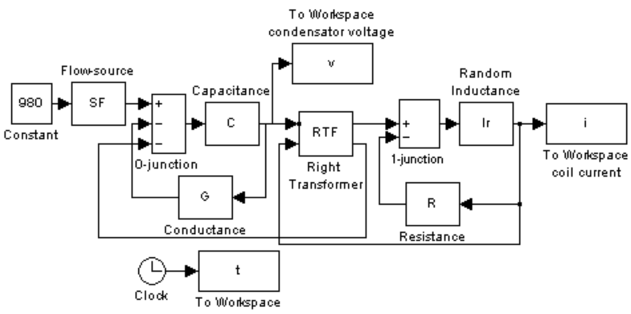

Figure 7. Bondsim simulation model with Ir element

On Figs. 8a) and 8b) the simulation results of coil current for deterministic case (without stochastic) and coil current for stochastic case (with stochastic Ir element) are given respectively.

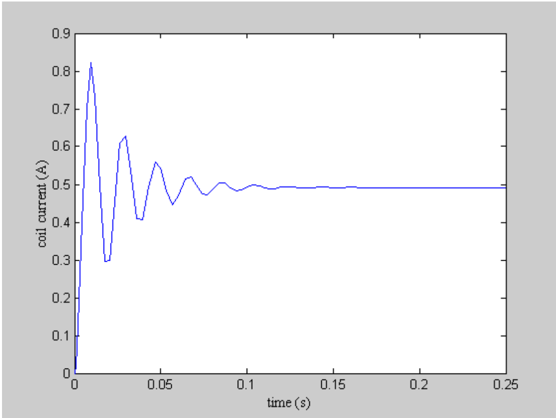

Fig.8a). Coil current for deterministic case (without stochastic)

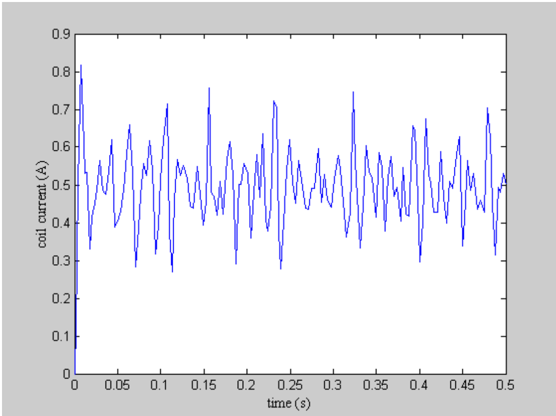

Fig.8b). Coil current for stochastic case (with stochastic Ir element)

The simulation results of capacitor voltage for deterministic case (without stochastic) and capacitor voltage for stochastic case (with stochastic Ir element) are given respectively on Figs. 9a) and 9b).

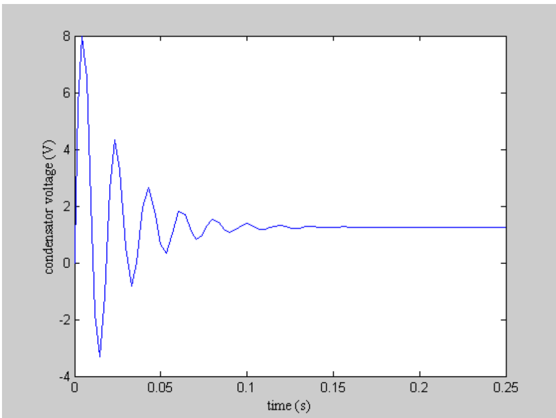

Fig.9a). Capacitor voltage for deterministic case (without stochastic)

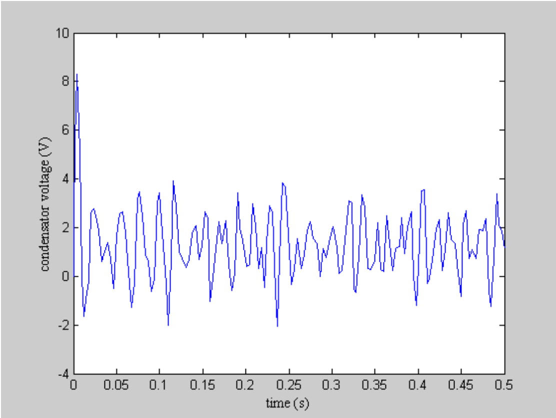

Fig.9b). Capacitor voltage for stochastic case (with stochastic Ir element)

In the second case the stochastic excitation (Random Flow-source) is considered. On Fig.10 the block diagram of SFr element is given.

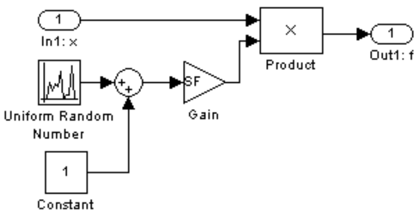

Fig.10. Random Flow-source Bondsim element

Bondsim simulation model with SFr element is given in Fig.11.

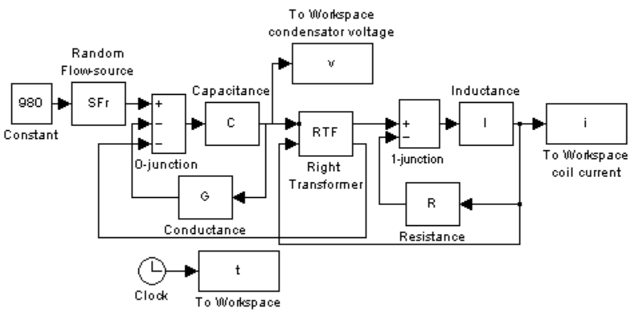

Fig. 11. Bondsim simulation model with SFr element

On Figs. 12a) and 12b) the simulation results of coil current and capacitor voltage for stochastic case (with stochastic SFr element) are given respectively.

V. CONCLUSION

Using the stochastic Bondsim library a visually more distinct simulation model is obtained and it enables simpler manipulation of stochastic elements of the simulation model. The application of stochastic Bondsim library retained the computational as well as the topological structure of the system. The simple application of proposed method is illustrated using a concrete example of modeling and simulation.

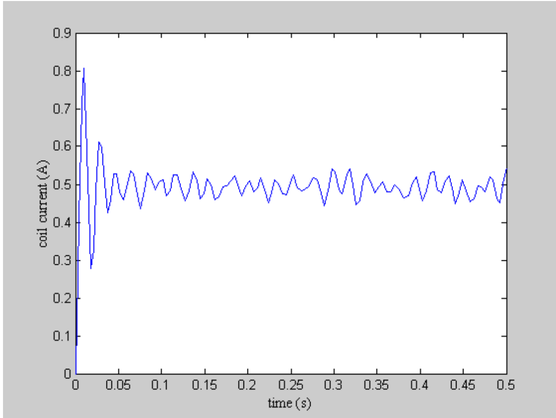

Fig.12a). Coil current for stochastic case (with stochastic SFr element)

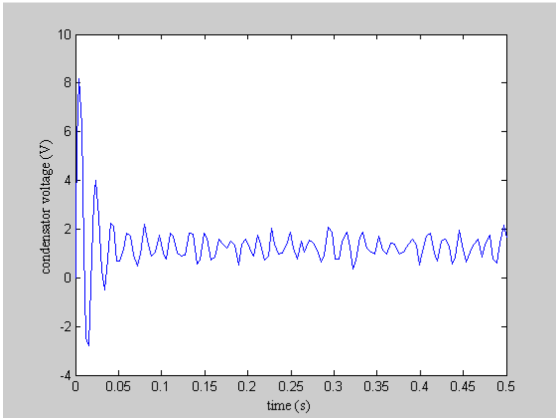

Fig.12b). Capacitor voltage for stochastic case (with stochastic SFr element)

REFERENCES

[1] P. J. Gawthrop, L. Smith, "Metamodelling: Bond graphs and dynamic systems", Prentice Hall International, London, 1996.

[2] J. U. Thoma, "Simulation by bond graphs: Introduction to a graphical method", Springer-Verlag, Berlin, 1990.

[3] A. Fakri, F. Rocaries, A. Carriere, "A simple method for the conversion of bond graph models in representation by block diagrams", *International Conference on Bond Graph Modeling and Simulation (ICBGM'97)*, pp.15-19, Phoenix, 1997.

[4] B. Vidojković, D. Antić, B. Danković, "Bondsim-Simulink tools for bond graph modelling and simulation", *Proc. 7th Symposium of Mathematics and its Applications*, pp.243-248, Timisoara, 1997.

[5] R. Nantu, "Structured Approach to Stochastical System Dynamics", Ph.D. Thesis, Department of Mechanical Engineering, Indian Institute of Technology, Kharagpur, India, July, 1998.

[6] D. Antić, B.Vidojković, "Obtaining System Block Diagrams based on Bond Graph Models and Application of Bondsim Tools", *International Journal of Modelling & Simulation*, Vol 21, Number 4, pp.257-262, 2001.