Flux and Torque Ripple Minimization in DTC of Induction Motor

Nebojsa Mitrovic[1], Vojkan Kostic[2], Milutin Petronijevic[3] and Borislav Jeftenic[4]

***Abstract* – In this paper the torque ripple in Direct Torque Control schemes (DTC) is analyzed in detail. It is found that the torque ripple can be divided into two parts, one is only related to the motor parameter and the other is related to the applied voltage and the rotor angular velocity. Based on the analysis, a duty ratio modulation method is proposed. Simulation results indicate the effectiveness of the proposed torque ripple minimizing method.**

***Keywords* – induction motor drive, direct torque control, duty ratio modulation.**

I. INTRODUCTION

DTC schemes to become an alternative for the classic variable speed AC drives. A fast dynamic can be achieved by calculating the instantaneous torque and flux. Using switching table not only simplifies the control system, but also decrease computing time. With a three phase voltage source inverter, there are six non-zero voltage vectors and two zero voltage vectors which can be applied to the machine terminals. The stator flux can be estimated by integrating emf using measuring current and voltage vectors or DC link voltage. The torque can be calculated using *d-q* components of the estimated flux and measured currents [1, 2].

Besides its advantages in application, the conventional DTC system has its drawback. First, its switching frequency varies according to the motor speed and the hysteresis bands of torque and flux. Second, large torque ripple is generated especially in a low speed region. Third, in DTC the stator current contains much more harmonics than that fed with sinusoidal voltage [3–5].

In order to reduce the torque ripple and the current harmonics, high switching frequency is needed but this will bring high sampling frequency and increase the system cost. Multilevel inverter is another choice [6], which reduces the torque ripple and current harmonics by providing more selective voltage vectors. But this kind of topology needs more power devices and the control scheme becomes more complex, that make it only suitable for high power applications.

[1]Nebojsa Mitrovic is with the Faculty of Electronic Engineering, Beogradska 14, 18000 Nis, Yugoslavia, E-mail: nesa@elfak.ni.ac.yu

[2]Vojkan Kostic is with the Faculty of Electronic Engineering, Beogradska 14, 18000 Nis, Yugoslavia, E-mail: vkostic@elfak.ni.ac.yu

[3]Milutin Petronijevic is with the Faculty of Electronic Engineering, Beogradska 14, 18000 Nis, Yugoslavia, E-mail: milutin@elfak.ni.ac.yu

[4]Borislav Jeftenic is with the Faculty of Electrical Engineering, Bulevar Revolucije 73, 11000 Belgrade, Yugoslavia, E-mail: jeftenic@etf.bg.ac.yu

In this paper analysis work is done on the relationship between applied voltage and corresponding torque and stator flux variations. Based on the analysis, a new DTC strategy is introduced with torque ripple reduction by modulating the duty ratio of the applied voltage. Simulation results are presented to verify the effectiveness of the control algorithm.

II. PROPOSED DUTY RATIO MODULATION METHOD

The dynamic behavior of an induction machine is described by the equations written in a *d-q* stationary reference frame [7] as follows:

$$\begin{aligned}
v_{ds} &= R_s i_{ds} + \frac{\mathrm{d}\varphi_{ds}}{\mathrm{d}t} \\
v_{qs} &= R_s i_{qs} + \frac{\mathrm{d}\varphi_{qs}}{\mathrm{d}t} \\
0 &= R_r i_{dr} + \frac{\mathrm{d}\varphi_{dr}}{\mathrm{d}t} + \omega_r \varphi_{qr} \\
0 &= R_r i_{qr} + \frac{\mathrm{d}\varphi_{qr}}{\mathrm{d}t} - \omega_r \varphi_{dr} \\
\varphi_{ds} &= L_s i_{ds} + L_m i_{dr} \\
\varphi_{qs} &= L_s i_{qs} + L_m i_{qr} \\
\varphi_{dr} &= L_r i_{dr} + L_m i_{ds} \\
\varphi_{qr} &= L_r i_{qr} + L_m i_{qs}
\end{aligned} \tag{1}$$

where: v_{ds}, v_{qs}, i_{ds}, i_{qs}, i_{dr}, i_{qr}, φ_{ds}, φ_{qs}, φ_{dr}, φ_{qr}, R_s, R_r, L_s, L_r, L_m, ω_r *d-q* components of stator voltage vector, $\overline{v_s}$, *d-q* components of stator and rotor current vector, $\overline{i_s}$ and $\overline{i_r}$, *d-q* components of stator and rotor flux vector, $\overline{\varphi_s}$ and $\overline{\varphi_r}$, stator and rotor resistance and self inductance, mutual inductance and rotor angular speed expressed in electrical radians, respectively.

From (1), the induction machine can be modeled with stator and rotor flux as the state variables as:

$$\begin{aligned}
\frac{\mathrm{d}\varphi_{ds}}{\mathrm{d}t} &= -\frac{1}{\sigma T_s}\varphi_{ds} + \frac{L_m}{\sigma T_s L_r}\varphi_{dr} + v_{ds} \\
\frac{\mathrm{d}\varphi_{qs}}{\mathrm{d}t} &= -\frac{1}{\sigma T_s}\varphi_{qs} + \frac{L_m}{\sigma T_s L_r}\varphi_{qr} + v_{qs} \\
\frac{\mathrm{d}\varphi_{dr}}{\mathrm{d}t} &= \frac{L_m}{\sigma T_r L_s}\varphi_{ds} - \frac{1}{\sigma T_r}\varphi_{dr} - \omega_r \varphi_{qr} \\
\frac{\mathrm{d}\varphi_{qr}}{\mathrm{d}t} &= \frac{L_m}{\sigma T_r L_s}\varphi_{qs} + \omega_r \varphi_{dr} - \frac{1}{\sigma T_r}\varphi_{qr}
\end{aligned} \tag{2}$$

where: $\sigma = 1 - \frac{L_m{}^2}{L_s L_r}$, $T_s = \frac{L_s}{R_s}$ and $T_r = \frac{L_r}{R_r}$ leakage coefficient, stator and rotor time constant, respectively.

The electromagnetic torque can be written in the terms of stator and rotor flux as:

$$T_e = \frac{3}{2} p \frac{L_m}{\sigma L_s L_r} \left(\varphi_{qs} \varphi_{dr} - \varphi_{ds} \varphi_{qr} \right) \quad (3)$$

where p is number of pole pairs.

From (1) stator flux can be written in the terms of stator voltage and stator current as:

$$\begin{aligned} \varphi_{ds} &= \int \left(v_{ds} - R_s i_{ds} \right) \mathrm{d}t \\ \varphi_{qs} &= \int \left(v_{qs} - R_s i_{qs} \right) \mathrm{d}t \end{aligned} \quad (4)$$

From (2), it can be seen that there is an inertia part between stator flux and rotor flux. The stator flux responses faster to the control action than the rotor flux does, and it can be assumed that the rotor flux does not change when the time interval is very small. So from (3) we can get that turning the stator flux can easily change the electromagnetic torque. Equation (4) indicates that the stator flux follows the stator voltage very quickly.

Because the value of sampling period, Δt, is very small, the stator and rotor flux state function in (2) can be expressed in discrete forms as (5).

$$\begin{aligned} \varphi_{ds_{k+1}} &= \varphi_{ds_k} + \left(-\frac{1}{\sigma T_s} \varphi_{ds_k} + \frac{L_m}{\sigma T_s L_r} \varphi_{dr_k} + v_{ds_k} \right) \Delta t \\ \varphi_{qs_{k+1}} &= \varphi_{qs_k} + \left(-\frac{1}{\sigma T_s} \varphi_{qs_k} + \frac{L_m}{\sigma T_s L_r} \varphi_{qr_k} + v_{qs_k} \right) \Delta t \\ \varphi_{dr_{k+1}} &= \varphi_{dr_k} + \left(\frac{L_m}{\sigma T_r L_s} \varphi_{ds_k} - \frac{1}{\sigma T_r} \varphi_{dr_k} - \omega_{rk} \varphi_{qr_k} \right) \Delta t \\ \varphi_{qr_{k+1}} &= \varphi_{qr_k} + \left(\frac{L_m}{\sigma T_r L_s} \varphi_{qs_k} + \omega_{r_k} \varphi_{dr_k} - \frac{1}{\sigma T_r} \varphi_{qr_k} \right) \Delta t \end{aligned} \quad (5)$$

Equation (5) clearly shows the variation of the stator flux due to the applied voltage vector, for given operating conditions. Neglecting the stator resistance effects, stator flux equation from (5) can be written as (6).

$$\begin{aligned} \varphi_{ds_{k+1}} &\cong \varphi_{ds_k} + v_{ds_k} \Delta t \\ \varphi_{qs_{k+1}} &\cong \varphi_{qs_k} + v_{qs_k} \Delta t \end{aligned} \quad (6)$$

From (6), it appears that the stator flux variation has the same direction of the applied voltage vector and an amplitude which is proportional to $\left| \overline{v_{s_k}} \right|$ and Δt.

The electromagnetic torque at (k+1) sampling time can be written as:

$$T_{e_{k+1}} = \frac{3}{2} p \frac{L_m}{\sigma L_s L_r} \left(\varphi_{qs_{k+1}} \varphi_{dr_{k+1}} - \varphi_{ds_{k+1}} \varphi_{qr_{k+1}} \right) \quad (7)$$

Substitute (5) into (7) and neglect the term proportional to the square of Δt, the torque at time of t_{k+1} is written as below:

$$T_{e_{k+1}} = T_{e_k} + \Delta T_{e1_k} + \Delta T_{e2_k} \quad (8)$$

where:

$$\Delta T_{e1_k} = -T_{e_k} \left(\frac{1}{T_s} + \frac{1}{T_r} \right) \frac{\Delta t}{\sigma} \quad (9)$$

$$\begin{aligned} \Delta T_{e2_k} = \frac{3}{2} p \frac{L_m}{\sigma L_s L_r} [& (v_{qs_k} - \omega_{r_k} \varphi_{ds_k}) \varphi_{dr_k} - \\ & (v_{ds_k} + \omega_{r_k} \varphi_{qs_k}) \varphi_{qr_k}] \Delta t \end{aligned} \quad (10)$$

Equation (8) indicates that the torque ripple can be composed into two parts, ΔT_{e1_k} i ΔT_{e2_k}. It is show in (9) that the applied voltage vector and rotor speed will not influence to ΔT_{e1_k}. It is only related to the motor parameters and its effect is to reduce the torque. The second part, ΔT_{e2_k}, represents the effect of the applied voltage vector on torque variation and is dependent on operating condition. For a given voltage vector this part is mainly affected by the rotor speed.

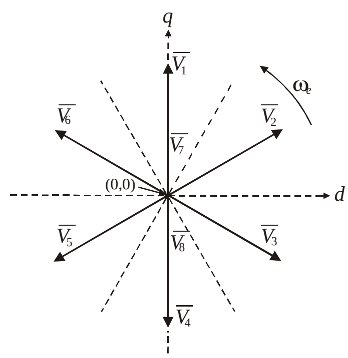

Fig. 1. The inverter output voltage vectors

In general DTC scheme, only one of eight voltage vectors is applied during each sampling period as shown in Fig. 1. And the torque ripple is unable to control during this time interval. If we can divide the sampling period into two parts and send different voltage vectors in the two parts, then the torque ripple can be controlled and reduced. From (8) it is easy to know that if we can make $\Delta T_{e2_k} = -\Delta T_{e1_k}$ then the average torque ripple of one sampling period is null. The torque ripple itself will be reduced too.

Assumung the sampling period is divided into two parts, $\Delta t = \Delta t_1 + \Delta t_2$. Voltage vector $\overline{v_{s1}}$ is applied during Δt_1, and $\overline{v_{s2}}$ during Δt_2, then:

$$\begin{aligned} \Delta T_{e2_k} = \frac{3}{2} p \frac{L_m}{\sigma L_s L_r} [& (v_{qs1_k} M_1 + v_{qs2_k} M_2 - \omega_{r_k} \varphi_{ds_k}) \varphi_{dr_k} - \\ & (v_{ds1_k} M_1 + v_{ds2_k} M_2 + \omega_{r_k} \varphi_{qs_k}) \varphi_{qr_k}] \Delta t \end{aligned} \quad (12)$$

where: $M_1 = \frac{\Delta t_1}{\Delta t}$ and $M_2 = \frac{\Delta t_2}{\Delta t}$ modulation ratio of $\overline{v_{s1_k}}$ and $\overline{v_{s2_k}}$, respectively.

In this DTC scheme, the voltage vector $\overline{v_{s1_k}}$ is decided by the conventional switching table to increase the torque. Voltage vector $\overline{v_{s2_k}}$ is selected to decrease the torque, and make $\Delta T_{e2_k} = -\Delta T_{e1_k}$. Because only the voltage vectors $\overline{V_7}$ (000) and $\overline{V_8}$ (111) will decrease flux and torque at any condition, so $\overline{V_7}$ is selected to full fill Δt_2 to make the scheme simple. And we can get:

$$M_1 = \frac{T_{e_k} \dfrac{K_1}{K_2} + \omega_{r_k} \left(\varphi_{ds_k} \varphi_{dr_k} + \varphi_{qs_k} \varphi_{qr_k} \right)}{v_{qs1_k} \varphi_{dr_k} - v_{ds1_k} \varphi_{qr_k}} \qquad (13)$$

where: $K_1 = \left(\frac{1}{T_s} + \frac{1}{T_r} \right) \frac{1}{\sigma}$ and $K_2 = \frac{3}{2} p \frac{L_m}{\sigma L_s L_r}$.

Equation (13) indicates that M_1 is dependent on T_{e_k}, ω_{r_k}, motor parameters and other components. But all quantities from (13) are necessary for realization speed sensorless DTC algorithm.

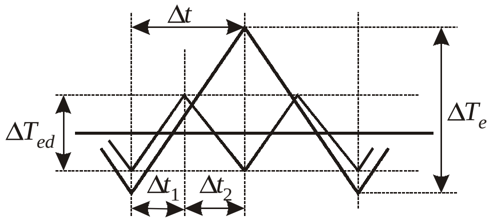

Fig. 2. The torque ripple with and without duti ratio

Fig. 2 present the graphic effectiveness of applying duty ratio modulation. Here ΔT_{ed} presents the torque ripple amplitude with duty ratio modulation, while ΔT_e presents the torque ripple without duty ratio modulation.

III. New DTC Scheme Configuration

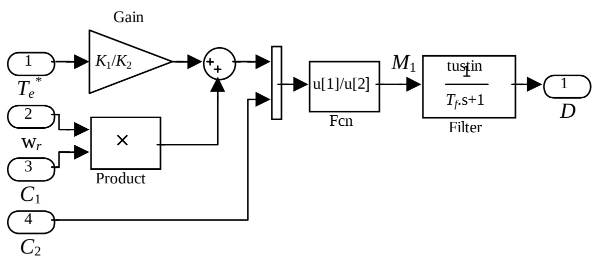

Fig. 3. Output voltage duty cycle ratio determination diagram

The strategy above gives the principle of a new DTC scheme with duty ratio modulation. In this scheme the selection of $\overline{v_{s1}}$ at k+1 sampling time is done by the conventional method as show in Fig. 1. But in practical system there must be some adjustments. Fig. 3 gives the diagram for determining the duty cycle of output voltage vector. In this figure D indicate the final output duty cycle ratio, T_f is a time constant for filtering (T_f =0.001-0.01), T_e^* is the given reference torque and ω_r is rotor angular speed expressed in electrical radians. Quantities C_1 i C_2 are parts of (13): $C_1 = \varphi_{ds_k} \varphi_{dr_k} + \varphi_{qs_k} \varphi_{qr_k}$, $C_2 = v_{qs1_k} \varphi_{dr_k} - v_{ds1_k} \varphi_{qr_k}$.

IV. Simulation Results

Simulations have been carried out for the comparison of a conventional and proposed DTC schemes. The simulations were conducted using Matlab/Simulink simulation package [8]. The hysteresis loop bands in both schemes are the same, 2.5 Nm for torque and 0.01 Vs for flux. The system is run under load torque of 15 Nm with DC bus voltage of 540 V. Torque limit is set to 45 Nm. During speed change duty ratio is set to 0.9. The system sampling frequency is 15 kHz and switching frequency is 7.5 kHz. The motor's parameter is listed in Table I.

TABLE I
INDUCTION MOTOR PARAMETER

Rated power	4 kW
Rated line to line voltage	400 V
Rated speed	1440 rpm
Rated torque	26.7 Nm
Rated stator flux	0.95 Vs
Pole pairs	2
R_s	1.13 Ω
R_r	0.9 Ω
L_s	0.142 H
L_r	0.143 H
L_m	0.13 H
Load inertia	0.06 kgm^2

In both cases the motor starts at t=0 s with speed reference of 720 rpm, and the reference speed is changed to 1080 rpm at t=0.7 s. Fig. 4 show the duty cycle ratio variation. During speed acceleration the modulation ratio is at set value, providing the maximum electromagnetic torque and less ripple than conventional DTC. When the rotation speed reaches the reference, the modulation ratio decrease to a certain value to reduce torque ripple.

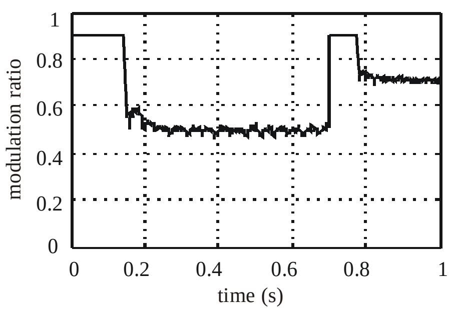

Fig. 4. Output voltage modulation ratio

Figs. 5 and 6 show the speed, torque, current and flux variance in the conventional and proposed DTC scheme. In steady state it can be found that with duty ratio modulation the torque ripple is reduced effectively but the ripple still will increase as speed raise up, according to (12). During speed change torque ripple is reduced because value of duty ratio modulation is set to 0.9. The stator flux ripple is reduced too, and ripple is less at lower speed. Another advantage of this DTC scheme is reduction of the current harmonics. So this kind of control scheme does not deteriorate the dynamic performance of DTC but at the same time the stable performance is improved.

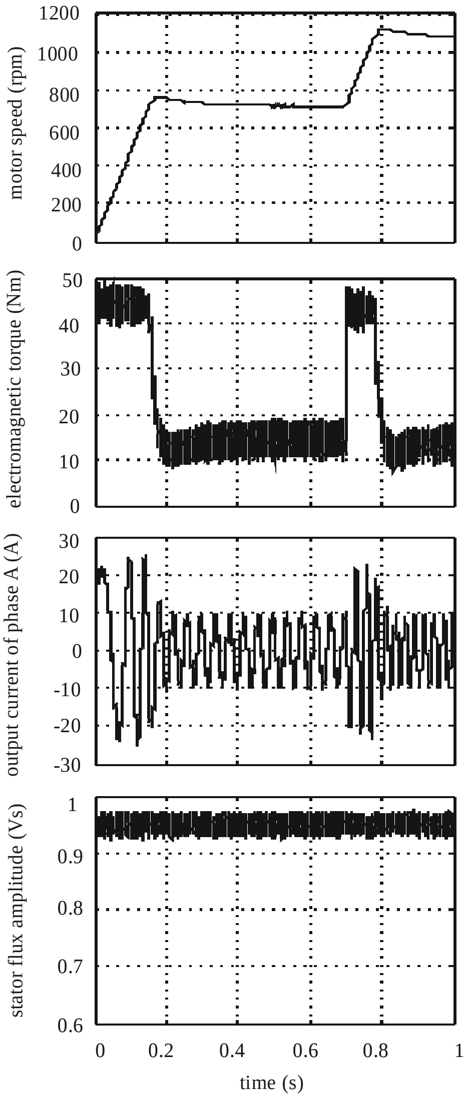

Fig. 5. The speed, torque, current and flux in conventional DTC scheme

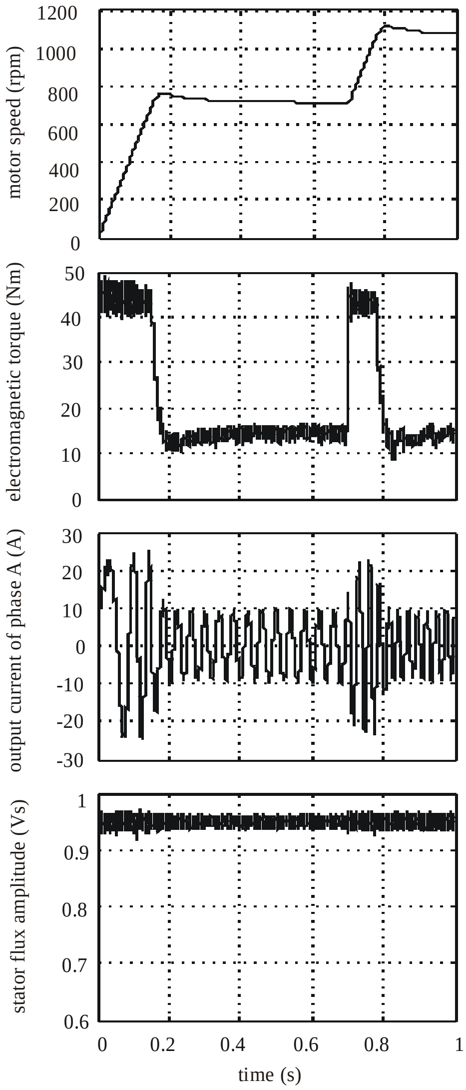

Fig. 6. The speed, torque, current and flux in the proposed DTC scheme

V. CONCLUSION

In this paper the torque ripple in DTC is analyzed in detail. The torque ripple minimization method based on duty ratio modulation of output voltage is described. This method does not deteriorate the dynamic performance of DTC but at the same time the stable performance is improved. The method is very easy to be embedded in conventional DTC scheme without increasing the system complexity greatly. Simulation results prove the advantages of this method.

REFERENCES

[1] R. Krishnan, *Electric Motor Drives, Modeling, Analysis, and Control*, Prentice Hall, New Jersey, 2001.

[2] Nebojsa Mitrovic, Vojkan Kostic, Milutin Petronijevic and Borislav Jeftenic, "Comparison by Simulation of Torque Control Schemes for Electric Drive Application", ICEST 2003, pp. 417-420, Sofia, Bulgaria, 16-18 October 2003.

[3] S. Kaboli, M. R. Zolghadri, S. Haghbin, A. Emadi, "Torque ripple minimization in DTC of induction motor based on optimized flux value determination", Proc. 29th Annual Conference of the IEEE Industrial Electronics Society, IECON03, Roanoke, VA, USA, pp. 431-435.

[4] D. Casadei, G. Serra, A. Tani, "Analytical investigation of torque and flux ripple in DTC schemes for induction motor", Industrial Electronics, Control and Instrumentation, IECON 97. 23rd International Conference, vol. 2, pp: 552-556, 1997.

[5] N.R.N. Idris and A.H.M. Yatim, "Reduced Torque Ripple And Constant Torque Switching Frequency Strategy For Direct Torque Control Of Induction Machine", In. Conf. Rec. IEEE-APEC, pp. 154-161, vol. 1, 2000.

[6] Kyo-Beum Lee, Joong-Ho Song, Ick Choy and Ji-Yoon Yoo, "Torque Ripple Reduction in DTC of Induction Motor Driven by Three-Level Inverter With Low Switching Frequency", IEEE Trans. on Power Electronics, Vol. 17, No. 2, March 2002.

[7] Paul C. Krause, Oleg Wasynczuk, Scott D. Sudhoff, *Analysis of Electric Machinery*, IEEE Press, New York, 1994.

[8] Tang L., Rahman M.F.,"A Matlab/Simulink Model based on Power System Blockset - A New Direct Torque Control Strategy for Permanent Magnet Synchronous Motor Drive System", Australasian Universities Power Engineering Conference, Perth, Australia, September, 2001, pp. 281-286.