A Unified Analysis and Characteristics of a DC-DC Converter Operating above or below the Resonance

Evgeniy I. Popov[1] and Michail H. Antchev[2]

Abstract - **A DC-DC converter with a serial resonant inverter is analyzed. The uniform expressions of the main parameters are given. The generalized transcendent equation for the devices conduction angles is numerically solved. The controlling characteristics are found. Computer simulations and experiments confirm the results. The study shows the advantages of the mode above the resonance frequency.**

Keywords - **Unified analysis, DC-DC converter, Resonant inverter, Conduction angles, Controlling characteristics.**

I. INTRODUCTION

The application of resonant power conversion implemented with a serial transistor inverter improves the efficiency of the DC-DC converters due to the zero current switching (ZCS) or/and zero voltage switching (ZVS) [1], [2], [3], [4], [5], [6]. Such a converter is analyzed separately in [1], [2] when operating at a frequency higher than the resonance one and in [4], [5] when operating at a frequency lower than the resonance one. But the converter controlling characteristics are not given in these publications because the transistor and diode conduction time intervals (and angles) are not determined for different values of the controlling frequency and the output voltage.

This paper is aimed at carrying out a unified analysis of a DC-DC converter implemented with a serial resonant transistor inverter operating at a frequency higher or lower than the resonance one. It is also aimed at obtaining the controlling characteristics related to the parameters of electrical energy conversion and component stress that will allow an adequate design of the power circuit.

II. ANALYSIS MAIN ASSUMPTIONS

A half-bridge (Fig.1) or a full-bridge converter are considered. The resonance frequency is $\omega_0 = 1/\sqrt{LC}$ and the characteristic impedance is $Z_0 = \sqrt{L/C}$. The circuit is analyzed in a normalized form: all voltages are divided by $V_d/2$ or by V_d for the half- or full-bridge configurations respectively (V_d is the supplying voltage); all currents are divided by $V_d/(2Z_0)$ or by V_d/Z_0 for the half- or full-bridge connections respectively.

At obtaining the controlling characteristics the independent variables are the normalized controlling frequency ω/ω_0 and the normalized output voltage $q = V_0/(V_d/2)$ or $q = V_0/V_d$ for the half- or full-bridge circuits respectively where $\omega=2\pi f$ is the controlling frequency and V_0 is the output voltage. The efficiency of the power circuit is considered close to unity. If $\alpha=\omega_0 t_D$ is the diode conduction angle and $\beta=\omega_0 t_Q$ is the transistor conduction angle then $\chi=\alpha+\beta=\pi\omega_0/\omega$.

For continious inverter current mode and $\omega>\omega_0$ we put $x=\beta$, $y=\alpha$, $x_{min}=0$, and the following constants are introduced $c=1$, $k=0$. When $\omega<\omega_0$ we put $y=\beta$, $x=\alpha$, $x_{max}=\pi$ and the constants are $c=-1$, $k=1$. In discontinuous inverter current mode ($\omega<0.5\omega_0$) $\alpha=\beta=\pi$, $\chi>2\pi$.

The main relations describing the converter operation at a frequency higher [1] or lower [4] than the resonance one taken into account in the paper are symmetrical in fact.

III. DETERMINATION OF TRANSISTOR AND DIODE CONDUCTION ANGLES

The conduction angles for continuous inverter current mode are determined according to the procedure described below (for discontinuous inverter current mode they are given above). The conduction angle x that has to fulfil the inequalities

$$cx < \cos^{-1}(-cq) \tag{1}$$

$$x > x_{min} \quad \text{for } \omega>\omega_0 \tag{2}$$

$$x < x_{max} \quad \text{for } \omega<\omega_0 \tag{3}$$

is found after solving the following transcendent equation when ω/ω_0 and q are known

$$x + k\pi + \tan^{-1}\frac{c(1-q^2)\sin x}{2q + c(1+q^2)\cos x} = \pi\frac{\omega_0}{\omega} = \chi \tag{4}$$

Then the other conduction angle y is

$$y = \chi - x \tag{5}$$

IV. INVERTER PARAMETERS

The main parameters of the inverter circuit can be expressed in a generalized and normalized form as follows:

The peak capacitor voltage is

$$V_{CPEAK} = \frac{(1-cq)(1-\cos x)}{q + c\cos x} \tag{6}$$

The peak output current (at primary of transformer) is

$$I_{PEAK} = \frac{1 - cq^2 - k2q\cos x}{q + c\cos x} \tag{7}$$

The average supplying current is

---

[1] Evgeniy I. Popov is with the Faculty of Electronic Technique and Technologies, Technical University, Sofia – 1000, Bul. Kl. Ohridski 8, Bulgaria, email: epopov@tu-sofia.bg.

[2] Michail H. Antchev is with the Faculty of Electronic Technique and Technologies, Technical University, Sofia – 1000, Bul. Kl. Ohridski 8, Bulgaria, email: antchev@tu-sofia.bg.

$$I_d = \frac{2q(1-cq)(1-\cos x)}{\chi(q+c\cos x)} \tag{8}$$

(in the half-bridge circuit this value has to be divided by 2).

The average output current (at primary of transformer) is

$$I_{AVG} = \frac{2(1-cq)(1-\cos x)}{\chi(q+c\cos x)} \tag{9}$$

The RMS output current (at primary of transformer) is

$$I_{RMS} = \{\frac{1}{\chi}[I_0^2(\frac{y}{2}+\frac{\sin 2y}{4})+(1+cq+V_{C0})^2(\frac{y}{2}-\frac{\sin 2y}{4})-$$

$$-cI_0(1+cq+V_{C0})\sin^2 y+(1-cq+V_{C1})^2(\frac{x}{2}-\frac{\sin 2x}{4})]\}^{\frac{1}{2}} \tag{10}$$

where I_0 is the initial output current (in the beginning of conduction of diodes at $\omega > \omega_0$ or transistors at $\omega < \omega_0$)

$$I_0 = \frac{(1-q^2)\sin x}{q+c\cos x} \tag{11}$$

The quantity V_{C0} is the initial capacitor voltage (in the beginning of conduction of diodes at $\omega > \omega_0$ or transistors at $\omega < \omega_0$)

$$V_{C0} = \frac{q(1-cq)(1-\cos x)}{q+c\cos x} \tag{12}$$

The quantity V_{C1} is the capacitor voltage in the beginning of conduction of transistors at $\omega > \omega_0$ or diodes at $\omega < \omega_0$

$$V_{C1} = \frac{c(1-cq)(1-\cos x)}{q+c\cos x} \tag{13}$$

The values determining the average transistor and diode currents can be calculated from

$$I_1 = \frac{(1-q^2)(1-\cos x)}{2\chi(q+c\cos x)} \tag{14}$$

$$I_2 = \frac{(1-cq)^2(1-\cos x)}{2\chi(q+c\cos x)} \tag{15}$$

When the converter operates at a controlling frequency higher than the resonance one ($\omega > \omega_0$) the average transistor current is $I_{QAVG} = I_1$ and the average diode current is $I_{DAVG} = I_2$. When the converter operates at a controlling frequency lower than the resonance one ($\omega < \omega_0$) the average transistor and diode currents are $I_{QAVG} = I_2$, $I_{DAVG} = I_1$.

V. INVERTER CHARACTERISTICS

The independent variables namely the normalized controlling frequency ω/ω_0 and the normalized output voltage are varied (for instance $\omega/\omega_0 = 0.1\text{-}1.9$ with a step of *0.1* and *q=0.1-0.9* with a step of *0.2*). In the continuous inverter current mode the transcendent equation (4) is numerically solved by a computer program when the restrictions expressed by (1), (2), (3) are obeyed and the conduction angles *x*, and *y* (5) are determined. Then the main parameters of the inverter circuit expressed by (6), (7), (8), (9), (10), (14), (15) applying also (11), (12), (13) are calculated. The main converter characteristics after the calculation are graphically displayed in Fig. 2 – Fig. 10. These characteristics show the diode and transistor conduction angles, the peak capacitor voltage, the peak output current, the average supplying current, the average output current, the RMS output current, the average transistor and diode currents as functions from the independent variables ω/ω_0 and *q* respectively. They are applied for assessment of the appropriate mode of operation and adequate design of the converter elements. Of course the controlling characteristics can be very easily obtained for different values of ω/ω_0 and *q*. Discontinuity of the graphics is observed around the resonance frequency. There the operation is not recommended due to the larger stresses of the converter elements.

VI. COMPUTER SIMULATION

PSPICE computer simulations of the half-bridge converter are carried out with the following data $V_0=60$ V, $V_d=305$ V, $L=205\ \mu H$, $C=33$ nF, $q=0.3934$ in three cases: 1) $\omega/\omega_0 = 1{,}362$ (period $T=2\pi/\omega=12$ μS); 2) $\omega/\omega_0 = 0{,}6537$ ($T=2\pi/\omega=25$ μS); 3) $\omega/\omega_0 = 0{,}3632$ ($T=2\pi/\omega=45$ μS). The main results from the proposed method of calculation and from the PSPICE simulations in a power circuit with ideal switching devices are summarized in Table 1. The graphical results from the PSPICE simulations with real device models (power transistors IRF450) are shown in Fig. 11, Fig. 12 and Fig. 13 for cases 1, 2 and 3 respectively. The results from the proposed method of calculation and from the PSPICE simulations are in good agreement that confirms the correctness of the unified analysis and the characteristics. Fig. 12 and Fig. 13 also show the spikes of the currents through the semiconductor devices due to the simultaneous conduction of a diode and a transistor from opposite shoulders of the circuit and due to the discharge of the snubber capacitor right away after the transistor turns on. These effects are avoided when the converter operates at a controlling frequency higher than the resonance one.

VII. EXPERIMENTAL STUDY

A number of experiments of the half-bridge circuit having the same data as stated above are carried out. The experimental results are close to these given in Table. 1. For instance Fig. 14 shows the waveforms of the transistor drain-source voltage and the output current at case 1 of the computer simulation. The measured values of the conduction angles of diodes and transistors, peak capacitor voltage, and peak output current are 44°, 88°, 188 V, 3.4 A respectively. They compare well to the calculated results from the proposed method and the PSPICE simulation.

VIII. CONCLUSIONS

A unified analysis of a DC-DC converter implemented with a half-bridge or a full-bridge serial resonant transistor inverter operating at a frequency higher or lower than the resonance one is carried out. Uniform mathematical expressions for the main parameters of the power circuit are proposed. The conduction angles of diodes and transistors are determined by numerically solving a transcendent equation. The converter characteristics when the controlling frequency and the output voltage vary are calculated, and graphically displayed. They are applied for assessment of the appropriate mode of operation and adequate design of the converter elements. It can be concluded that the operation at a frequency higher than the resonance one should be preferred due to the smaller peak capacitor voltage and absence of current spikes through the semiconductor devices (smaller switching power losses at turn on). The converter is also studied by computer simulations and experimentally. There is a good agreement between the theoretical expressions and characteristics, the computer simulation and experimental results that shows the correctness of the proposed analysis.

REFERENCES

[1] M. H. Antchev, G. J. Maleev "Analysis of a transistor inverter operating at a frequency higher than the resonance one", Journal "Electrical Engineering and Electronics", vol. 5-6, 2000, pp. 12-17 (In Bulgarian).

[2] S. Valtchev, J. B. Klaasens "Efficient resonant power conversion", IEEE Transactions on Industrial Electronics, vol. 37, No. 6, December 1990, pp. 490-495.

[3] A. F. Witulski, R. W. Erickson "Design of the series resonant converter for minimum component stress", IEEE, Transactions on Aerospace and Electronic Systems, vol. AES-22, No. 4, July 1986, pp. 356-363.

[4] R. King, T. A. Stuart. "Modeling the full-bridge series-resonant power converter" IEEE, Transactions on Aerospace and Electronic Systems, vol. AES-18, No. 4, July 1982, pp. 449-459.

[5] R. King, T. A. Stuart. "Inherent overload protection for the series resonant converter", IEEE Transactions on Aerospace and Electronic Systems, vol. AES-19, No. 6, November 1983, pp. 820-829.

[6] M. Mohan, T. M. Undeland, W. P. Robbins. "Power electronics, converters, applications and design", 2^{nd} ed., New York, Wiley, 1995.

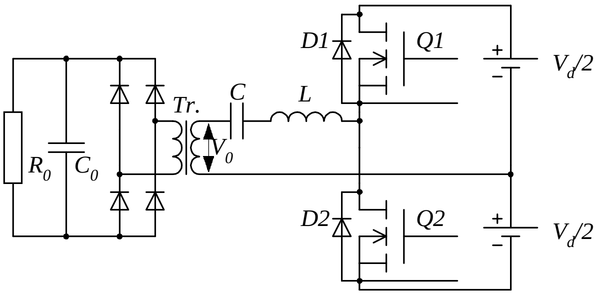

Fig. 1. Half-bridge resonant DC-DC converter.

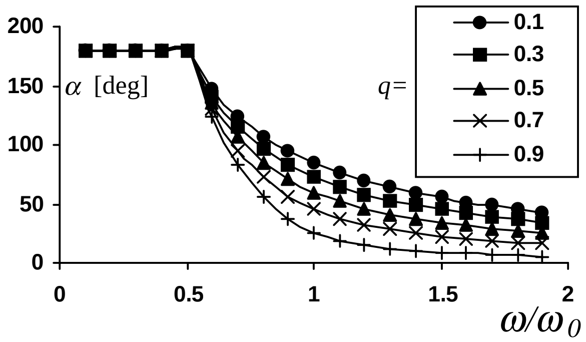

Fig. 2. Diode conduction angle.

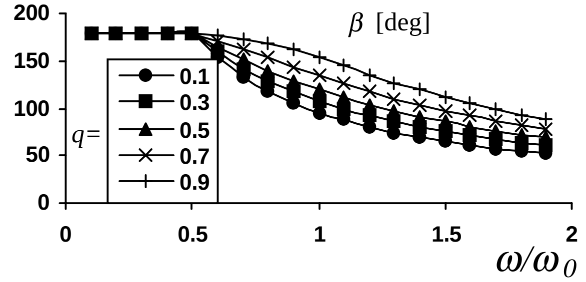

Fig. 3. Transistor conduction angle.

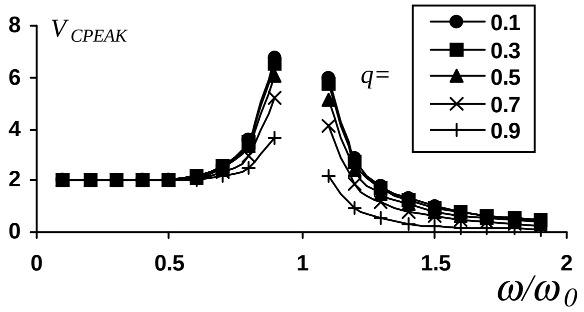

Fig. 4. Peak capacitor voltage.

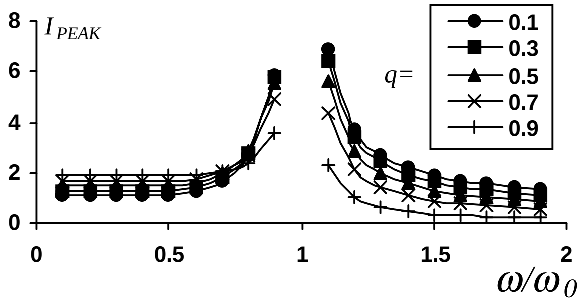

Fig. 5. Peak output current.

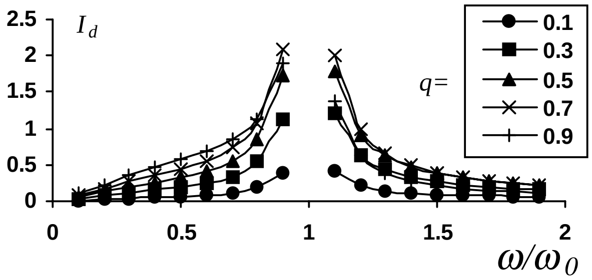

Fig. 6. Average supplying current.

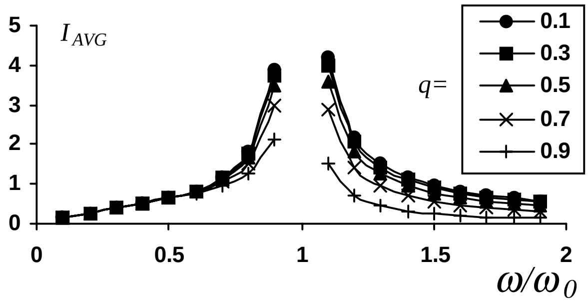

Fig. 7. Average output current.

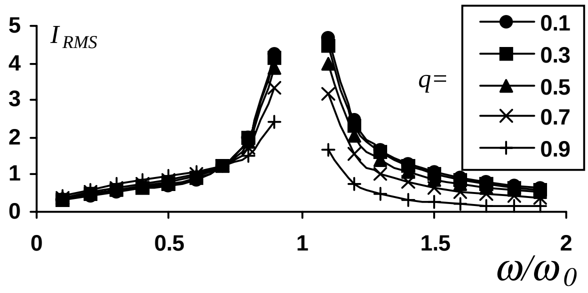

Fig. 8. RMS output current.

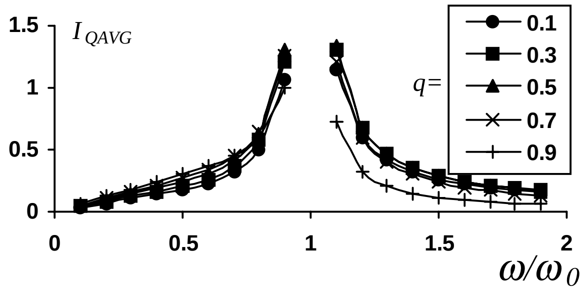

Fig. 9. Average transistor current.

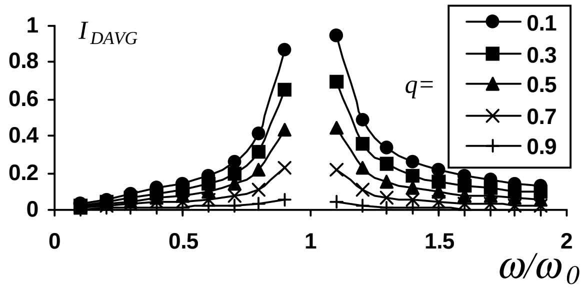

Fig. 10. Average diode current.

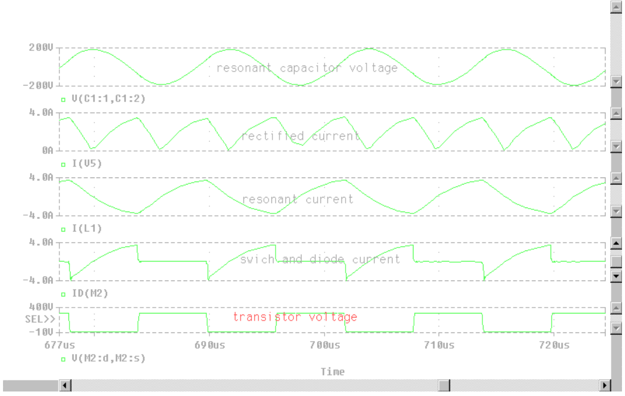

Fig. 11. Simulation results above the resonance.

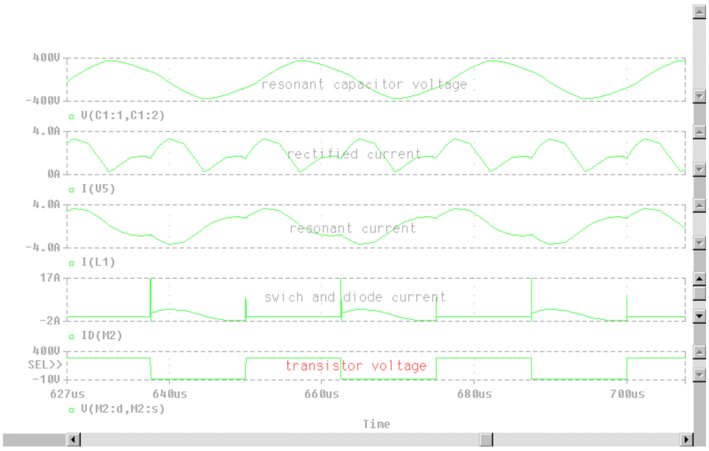

Fig. 12. PSPICE simulation results at a frequency lower than the resonance one and continuous output current.

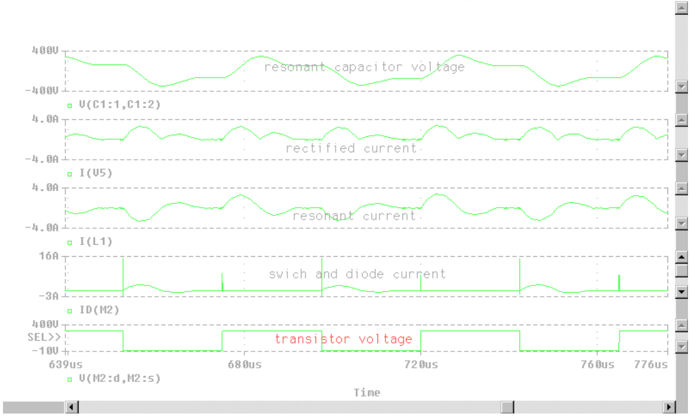

Fig. 13. PSPICE simulation results at a frequency lower than the resonance one and discontinuous output current.

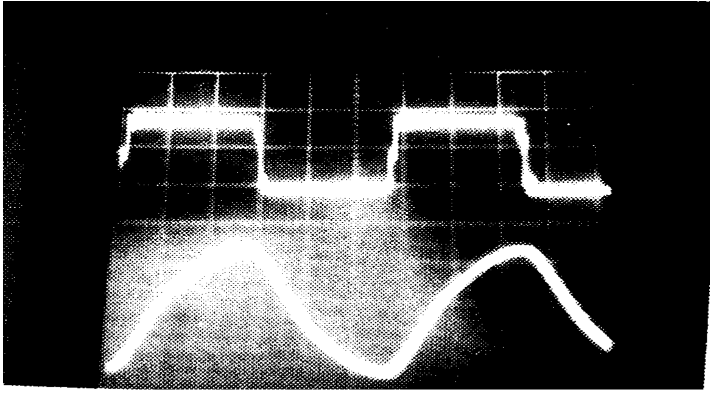

Fig. 14. Waveforms of the transistor drain-source voltage (150 V/div) and the output current (2 A/div). Time scale 2 μS/div.

TABLE 1. THEORETICAL AND SIMULATION RESULTS.

№	***q***	***ω/ω₀***	***α***	***β***	**Method**
	-	-	deg.	deg.	
1	0.3934	1.362	45 44.8	87.2 87.1	Proposed PSPICE
2	0.3934	0.6537	122 122	153 153	Proposed PSPICE
3	0.3934	0.3632	180 180	180 180	Proposed PSPICE