Forming A Quadrature Signal By Using A Raster Algorithm

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The purpose of algorithms for raster graphics is to service the visualization on raster devices. They are characteristics of good response time because of the simple arithmetic operations which they perform in order to make a decision for selection of the relevant point of the raster network.

Such an algorithm is used in this paper as a source of coordinates of points which later shall approximate sinusoidal and cosinusoidal signal. The advantage of using a raster algorithm for a circle is that both signals are simultaneously formed.

The algorithm for circle is Brenham's in which it is evaluated the error /m/ which is made at every step at the choice of one or another point of the raster /fig.1/. At a point chosen (x_i , y_i) the possible following one is : the horizontal (x_i+1 , y_i), vertical (x_i , y_i-1) or diagonal (x_i+1 , y_i-1) [1].

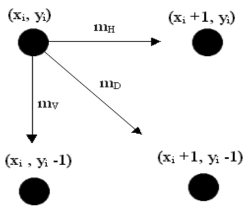


Fig.1

As a result of performance of the algorithm for a given radius, a file of integral values is obtained, which are the x and y coordinates of the separate pixels /points/. These selected points are discrete reports of a curve which approximates a central circle in an orthogonal system of coordinates. Each pair of coordinates of the i-th point (x_i,y_i) uniquely determine the angle formed between the radius-vector r_i and the ordinate axis (fig.1). The dependence is given with the formula (1).

(1) $$a = arctg(x_i / y_i)$$

Having in mind the parametrical circle equation (2) and the considerations stated above, it is ascertained that the algorithm provides digitalization by angles of signals determining curves which approximate sinusoidal or cosinusoidal signal (fig.2)[2].

(2) $$\left| \begin{array}{l} x(t) = R * \sin a(t) \\ y(t) = R * \cos a(t) \\ a \in [0 \div 2\pi] \end{array} \right.$$

After rasterization the generated values of the sinusoidal and cosinusoidal signal have quadrilateral symmetry regarding the coordinate axes. This makes it possible both to rasterize only one fourth of the circle, and to consider and analyze the signals which approximate it in one of the quadrants.

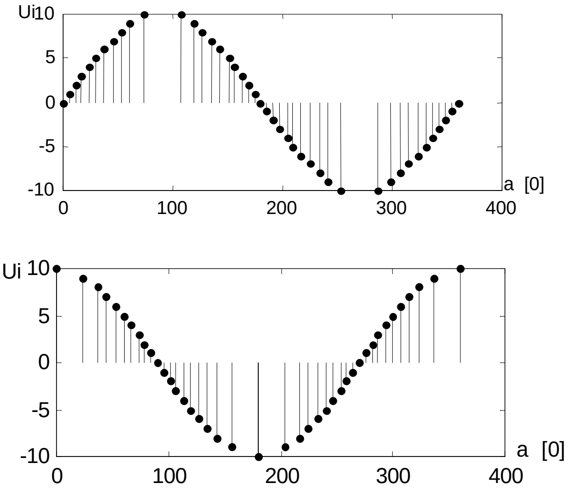


Fig. 2

Brenham's raster algorithm begins from a point having coordinates (0, R) , approximates a circle in the clockwise direction and calculates the pixels of the central circle for fourth quadrant. The obtained step-like approximating signals compared to the ideal calculated ones using (3) are given in (fig.3).

(3) $u(a)_{\sin} = R * \sin(a)$ and $u(a)_{\cos} = R * \cos(a)$, where $a \in [0 \div \pi / 2]$

The approximation of the two signals after rasterization of the circle is a step-like approximation with even quantization by level and uneven quantization by angle /time/.

The ideal sinusoid $u(a)_{sin}$ and the rasterizing curve $u^{\sim}(a)_{sin}$ change within the range $[0 \div R]$. For the cosinusoid $u(a)_{cos}$ and its rasterizing curve $u^{\sim}(a)_{cos}$ the interval is $[R \div 0]$. Both ranges are broken into *n* equal increases. In this particular case *n* = *R*. At this quantization the height of each step is one and its length is different and is determined according to its position in the interval of forming. The ordinate of the separate steps is [1 ; 2 ;...i ; i+1 ;.....R-1 ; R] for the sinusoid and [R ; R-1 ;...i ; i+1 ;....2 ; 1] for the cosinusoid. The abscissa of the points of the approximation is determined by formulation (1).

The raster algorithm, the figures provided and the analysis below are implemented in the MATLAB 5.X media [3].

The deviation of the approximating curve from the ideal one is evaluated by a quadratic mean deviation. For its evaluation a programme is created by which it is checked the position of the step-like curve to the calculated one (3), i.e. a check is made of the position of the abscissa of the intersection point of the two curves $\boldsymbol{a^o_i}$ as to the abscissa of the transition point $\boldsymbol{a_i}$ (fig.3).

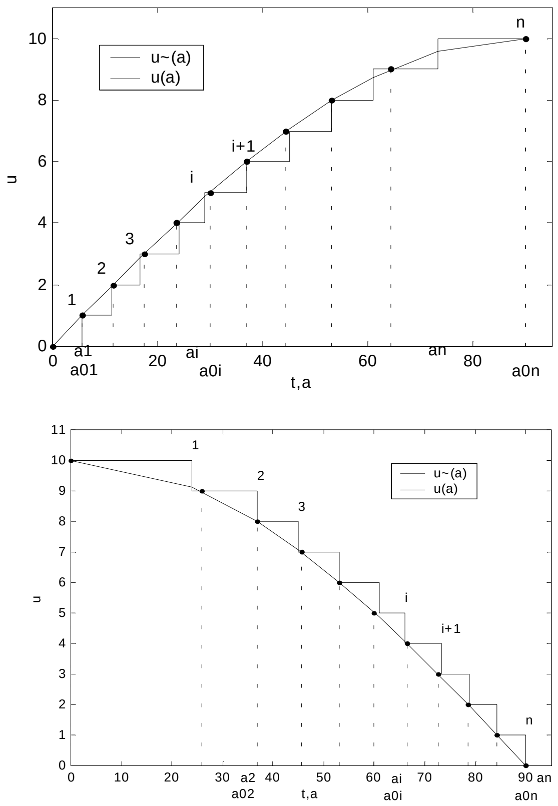
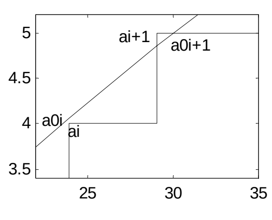


Fig.3

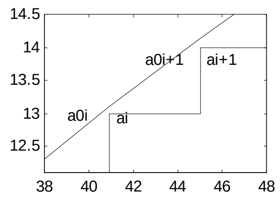
In order to calculate the quadratic mean deviation / QMD / of the approximating curve of the sinusoid it is necessary to consider its position as to the ideal one. In order to make a decision regarding the formulation for calculating QMD for each step (g_i), it is necessary to distinguish between four cases determined by the type of intersection of the two curves:

- $\boldsymbol{a^o_i < a_i}$ и $\boldsymbol{a_{i+1} < a^o_{i+1}}$



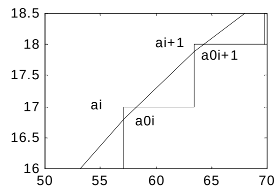
$$g_i = \int_{a_i^0}^{a_{i+1}} (R.\sin a - i)^2 da + \int_{a_{i+1}}^{a_{i+1}^0} (R.\sin a - (i+1))^2 da$$

- $\boldsymbol{a^o_i < a_i}$ и $\boldsymbol{a^o_{i+1} < a_{i+1}}$



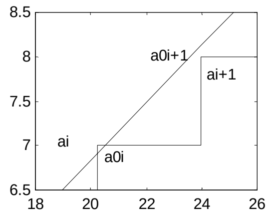
$$g_i = \int_{a_i^0}^{a_{i+1}} (R.\sin a - i)^2 da - \int_{a_{i+1}^0}^{a_{i+1}} (R.\sin a - (i+1))^2 da$$

- $\boldsymbol{a_i < a^o_i}$ и $\boldsymbol{a_{i+1} < a^o_{i+1}}$



$$g_i = \int_{a_i^0}^{a_{i+1}} (R.\sin a - i)^2 da + \int_{a_{i+1}}^{a_{i+1}^0} (R.\sin a - (i+1))^2 da$$

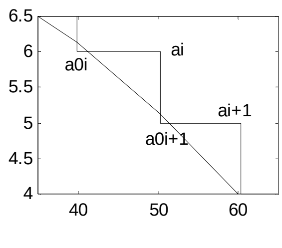
- $\boldsymbol{a_i < a^o_i}$ и $\boldsymbol{a^o_{i+1} < a_{i+1}}$



$$g_i = \int_{a_i^0}^{a_{i+1}} (R.\sin a - i)^2 da - \int_{a_{i+1}^0}^{a_{i+1}} (R.\sin a - (i+1))^2 da$$

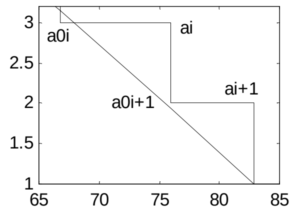
At approximation of the cosinusoid, the step-like curve is always above the ideal one. This reduces the calculations of QMD to two cases because the condition $\boldsymbol{a^o_i < a_i}$ is always true.

- $\boldsymbol{a^o_i < a_i}$ и $\boldsymbol{a^o_{i+1} < a_{i+1}}$



$$g_i = \int_{a_i^0}^{a_i} (R.\cos a - (R-i))^2 da + \int_{a_i}^{a_{i+1}^0} (R.\cos a - (R-i))^2 da$$

- $a^o_i < a_i$ и $a^o_{i+1} < a_i$



The result of the programme which calculates QMD for

$$g_i = \int_{a_i^0}^{a_i} (R.\cos a - (R-i))^2 da - \int_{a_{i+1}^0}^{a_i} (R.\cos a - (R-i))^2 da$$

each step is represented in the form of a table. The source of approximating signals is a raster algorithm of B for circle, applied for radius R=10 [the pixel].

C, D, E), in which the considered part of the one fourth of the circle may be positioned as to the points of the new raster. The step of increase of the more precise algorithm is defined according to formula (5).

(5) st = 1/ p, where p is an integer

The initial evaluation with which rasterization is commenced is calculation of the error at potential selection of a diagonally positioned point.

$\mathbf{\Delta_i} = D(\mathbf{D}_i) = (x_i + st)^2 + (y_i - st)^2 - R^2$
(6)

Case 1. Lets $\mathbf{\Delta_i < 0}$. Then the circle is positioned as in the cases A and B in the figure above and the reasonable selection is $H_i = (x_i + st, y_i)$ or $D_i = (x_i + st, y_i - st1)$. Another evaluation should be introduced in order to differentiate between these two options:

$\mathbf{\delta_i} = D(\mathbf{H}_i) + D(\mathbf{D}_i) = 2.(x_i + st)^2 + (y_i - st)^2 + y_i^2 - 2.R^2$ (7)

Step [i]	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**G**
QMD $u^{\sim}(a)_{sin}$	1.95	2.17	2.82	1.67	3.2	2.26	1.93	2.79	7.43	**26.67**	**52.89**
QMD $u^{\sim}(a)_{cos}$	**13.72**	6.15	3.04	3.84	2.35	2.18	2.48	2	1.95	1.93	**39.62**

Table.1

The highest QMD, i.e. the biggest inaccuracy of approximation for the sinusoid is on the last step and for the cosinusoid – on the first step. These are the intervals in which there are approximated the maximum values of the sinusoidal $u(a)_{sin}$ and the cosinusoidal $u(a)_{cos}$ signals.

The sum of QMD for each step gives the deviation of the whole approximating curve G (4).

(4) $$G = \sum_{i=1}^{n} g_i$$

In order to reduce the total QMD (G) it is necessary to process the steps which lead to the greatest inaccuracy. The new algorithm for these steps is based on Brenham's algorithm but provides less increase in level.

Only the first fourth of the central circle, located in the first quadrant was considered. Lets assume that at the i-th step of this algorithm there was selected point $P_i = (x_i, y_i)$. This is the point after which it is necessary to reduce the

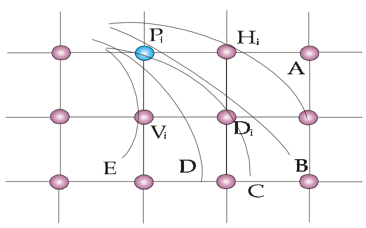


Fig.4

increase in level from 1 to a smaller step (st). The next choice should be only one of the points: $H_i = (x_i + st, y_i)$, $V_i = (x_i, y_i - st)$ or $D_i = (x_i + st, y_i - st)$, shown in fig.4. In the same figure there are shown the possible ways (A, B,

At $\mathbf{\delta_i < 0}$ / case A / **H** is always selected$_i$, because $D(\mathbf{H}_i) \le 0$ and $D(\mathbf{D}_i) < 0$, hence and $\delta_i < 0$.

At $\mathbf{\delta_i > 0}$ / case B / **D** is selected$_i$. Both terms of the sum $\mathbf{\delta_i}$ (7) have different signs, at $D(\mathbf{H}_i) > D(\mathbf{D}_i)$ the selected point is $\mathbf{D}_i$, i.e. the curve B.

At $\mathbf{\delta_i = 0}$, **D** is selected$_i$.

After selecting one of the two points, the new initial evaluation is:

$\mathbf{\Delta_{i+1}} = D(\mathbf{D}_{i+1}) = (x_{i+1} + st)^2 + (y_{i+1} - st)^2 - R^2$

Case 2. Now $\mathbf{\Delta_i > 0}$. This corresponds to positions D and E in fig.6 . As in the preceding case, a new evaluation is introduced in order to differentiate between these two options:

$\mathbf{\varepsilon_i} = D(\mathbf{V}_i) + D(\mathbf{D}_i) = (x_i + st)^2 + 2.(y_i - st)^2 + x_i^2 - 2.R^2$ (8)

The similar analysis of the options for selection between D and E , leads to the conclusion:

- at $\mathbf{\varepsilon_i} > 0$, **V** is selected$_i$
- at $\mathbf{\varepsilon_i} < 0$, $\mathbf{D}_i$ is elected
- at $\mathbf{\varepsilon_i} = 0$, $\mathbf{D}_i$ is selected.

After selecting one of the two points, the new initial evaluation is:

$\mathbf{\Delta_{i+1}} = D(\mathbf{D}_{i+1}) = (x_{i+1} + st)^2 + (y_{i+1} - st)^2 - R^2$

Case 3. As it is clear in fig.4 (position C), at $\mathbf{\Delta_i = 0}$, the circle passes exactly through the diagonally positioned point **D**i and it is the one to be selected.

The next initial evaluation is :

$\mathbf{\Delta_{i+1}} = D(\mathbf{D}_{i+1}) = (x_{i+1} + st)^2 + (y_{i+1} - st)^2 - R^2$

After performing the algorithm described above with step st = 0.2 applied for the last and first step relevantly for the sinusoid and the cosinusoid, there are obtained approximating curves shown in (fig.5) .

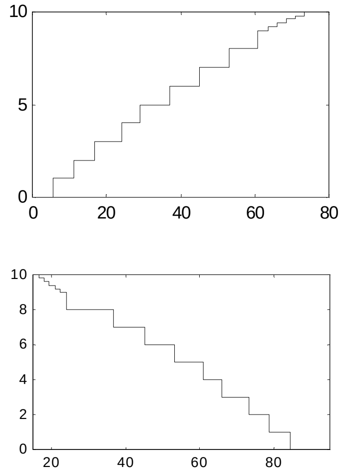


Fig. 5

After applying the new algorithm per one step of the approximating curves, the sum of QMD for the sinusoid is G_{sin}= 38.24, and for the cosinusoid G_{cos}= 29.68 . With decrease of the step through which a decision is made for the next point of the curve, its quality is improved, but the number of iterations of the raster algorithm is increased, which leads to decrease of its response time. The suggested algorithm may be applied for more than one, as well as for all steps of the curves.

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