Nonlinear State Observation in a Didactic Magnetic Levitation System

Milica B. Naumović

Abstract - **The magnetic levitation control system of a metallic sphere is an interesting and visual impressive device successful for demonstration many intricate problems for control engineering research. The dynamics of magnetic levitation system is characterized by its open-loop instability, highly nonlinearity and complexity. In this paper an approach to the nonlinear velocity estimation in the control system for positioning of the levitating sphere is addressed. Results of several simulation runs are given to verify the analytical investigations.**

Keywords - **Nonlinear observer, Magnetic levitation system, Control engineering education.**

I. Introduction

Magnetic levitators not only present intricate problems for control engineering research, but also have many important applications, such as high-speed transportation systems and precision frictionless magnetic bearings. From an educational viewpoint, this process is highly motivating and suitable for laboratory experiments and classroom demonstrations, as reported in the engineering education literature [1]-[9].

The classic magnetic levitation control experiment is presented in the form of laboratory equipment given in Fig. 1. The complete purchase of the Feedback Instruments Ltd. Maglev System 33-006 [10] was supported by WUS (**W**orld **U**niversity **S**ervice [11]) – Austria under Grant CEP (**C**enter of **E**xcellence **P**rojects).

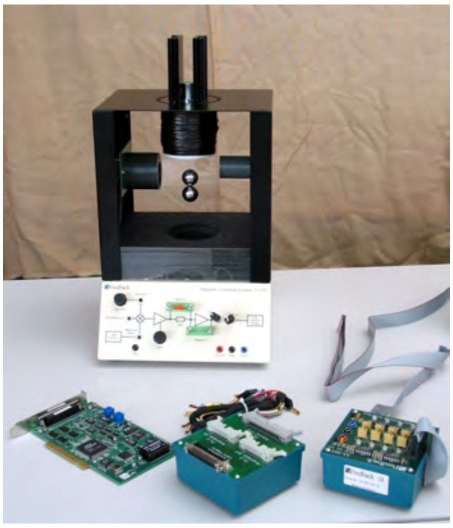

Fig. 1. Photograph of magnetic levitation system

Milica B. Naumović is with the Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš, Serbia and Montenegro, E-mail: nmilica@elfak.ni.ac.yu

This attraction-type levitator system is a challenging plant because of its nonlinear and unstable nature. The suspended body is a hollow steel ball of 25 mm diameter and 20 g mass. This results in a visually appealing system with convenient time constants. Both analogue and digital control solutions could be implemented. In addition, the system is simple and relatively small, that is portable.

In order to obtain smooth and sufficiently accurate position and speed signals, an observer structure is often implemented. The observer processes the voltage command and position signal from transducer as it is shown in Fig. 2.

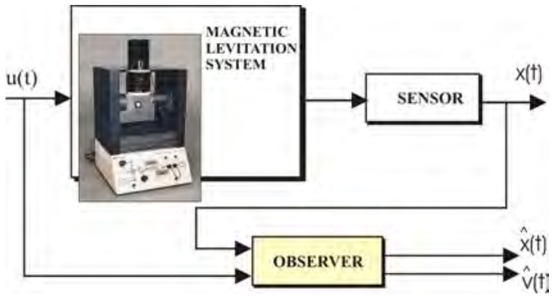

Fig. 2. Block diagram of the plant with observer

In this paper, referring to the problem of ball positioning beneath an electromagnet, the possibility of employing the Lie-algebraic method [12] in nonlinear velocity estimator design is considered. In the proposed method a nonlinear transformation is found, that brings the system into a canonical form, from where observer design is facilitated.

II. System Description

The Magnetic Levitation System (Maglev System 33-006 given in Fig. 1) is a relatively new and effective laboratory set-up very helpful for control experiments. The basic control goal is to suspend a steel sphere by means of a magnetic field counteracting the force of gravity. The Maglev System consists of a magnetic levitation mechanical unit (an enclosed magnet system, sensors and drivers) with a computer interface card, a signal conditioning unit, connecting cables and a laboratory manual.

In the analogue mode, the equipment is self-contained with inbuilt power supply. Convenient sockets on the enclosure panel allow for quick changes of analogue controller gain and structure. The bandwidth of lead compensation may be changed in order to investigate system stability and time response. Moreover, user-defined analogue controllers may be easily tested. Note, that the position of the sphere may be adjusted using the set-point control and the stability may be varied using the gain control.

In the digital mode, the Maglev System operates with MATLAB® /SIMULINK® software. Feedback Software for SIMULINK® is provided for the control models and interfacing between the PC and the Maglev system hardware.

III. MAGNETIC LEVITATION DYNAMICS

The modeling of the electromagnetic levitation system is based on its electrical, mechanical, and electromechanical equations [6]-[9] , as follows

$$\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= v \\
\frac{\mathrm{d}^2x}{\mathrm{d}t^2} &= g - \frac{C}{m}\frac{i^2}{x^2} \\
\frac{\mathrm{d}i}{\mathrm{d}t} &= -\frac{R}{L}i + \frac{2C}{L}\frac{i}{x^2}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{e}{L}
\end{aligned} \qquad (1)$$

where m, x and v denote the ball's mass, position and velocity, i is the current in the coil of the electromagnet, e is the applied voltage, R is the coil's resistance, $L(x)$ is the coil's inductance, g is the gravitational constant and C is the magnetic force constant. Note that the inductance $L(x)$ is a nonlinear function of the ball's position with some typical approximation that can be found in the literature [9].

Adopting $x(t)$, $v(t)$ and $i(t)$ as state variables, and voltage e as control signal, the equation of motion for the magnetic levitation system can be written as

$$\begin{aligned}
\frac{\mathrm{d}x_1}{\mathrm{d}t} &= x_2 \\
\frac{\mathrm{d}x_2}{\mathrm{d}t} &= g - \frac{C}{m}\frac{x_3^2}{x_1^2} \\
\frac{\mathrm{d}x_3}{\mathrm{d}t} &= -\frac{R}{L}x_3 + \frac{2C}{L}\frac{x_2x_3}{x_1^2} + \frac{u}{L}
\end{aligned} \qquad (2)$$

IV. NONLINEAR OBSERVER DESIGN

Nonlinear observer form is defined to be a canonical form for which an observer can be constructed with a linear error dynamics. Such a concept has emerged as a dual to the feedback linearization problem known in nonlinear control systems theory [13]. The system can be transformed into a nonlinear observer form via a coordinate change. The Lie-algebraic method has be proved to be an effective means of coordinate transformation map obtaining.

Consider the vector-valued functions

$$\mathbf{f}: \Re^n \to \Re^n \quad \text{and} \quad \mathbf{g}: \Re^n \to \Re^n .$$

Definition 1. The Lie bracket is defined by

$$[\mathbf{f},\mathbf{g}](\mathbf{x}) = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x})\mathbf{g}(\mathbf{x}) - \frac{\partial \mathbf{g}}{\partial \mathbf{x}}(\mathbf{x})\mathbf{f}(\mathbf{x}) , \qquad (3)$$

where $\partial\mathbf{f}/\partial\mathbf{x}$ and $\partial\mathbf{g}/\partial\mathbf{x}$ are the Jacobian matrices of $\mathbf{f}$ and $\mathbf{g}$, respectively.

Using an alternative notation, we can represent the Lie bracket as

$$[\mathbf{f},\mathbf{g}] = \left(\mathrm{ad}^1\,\mathbf{f},\mathbf{g}\right) . \qquad (4)$$

We also define

$$\left(\mathrm{ad}^i\,\mathbf{f},\mathbf{g}\right) = \left[\mathbf{f},\left(\mathrm{ad}^{i-1}\,\mathbf{f},\mathbf{g}\right)\right] \qquad (5)$$

with $\left(\mathrm{ad}^0\,\mathbf{f},\mathbf{g}\right) = \mathbf{g}$.

Let $\mathrm{d}h$ denotes the gradient of scalar function h with respect to $\mathbf{x}$, that is $\mathrm{d}h = \nabla^{\mathrm{T}}h$. Let $\langle\cdot,\cdot\rangle$ denotes the inner product on $\Re^n$.

Definition 2. $\mathsf{L}_{\mathbf{f}}h$ represents the Lie derivative of h with respect to $\mathbf{f}$ and is defined by

$$\mathsf{L}_{\mathbf{f}}h = \langle \mathrm{d}h,\mathbf{f}\rangle . \qquad (6)$$

We shall employ the following notation

$$\mathsf{L}_{\mathbf{f}}^{\,i}h = \mathsf{L}_{\mathbf{f}}\left(\mathsf{L}_{\mathbf{f}}^{\,i-1}h\right) \qquad (7)$$

where $\mathsf{L}_{\mathbf{f}}^{\,0}h = h$.

With these definitions, the Lie derivative of $\mathrm{d}h$ with respect to the vector field $\mathbf{f}$ takes the form

$$\mathsf{L}_{\mathbf{f}}(\mathrm{d}h) = \left(\frac{\partial(\mathrm{d}h)^{\mathrm{T}}}{\partial\mathbf{x}}\mathbf{f}\right)^{\mathrm{T}} + (\mathrm{d}h)\frac{\partial\mathbf{f}}{\partial\mathbf{x}} . \qquad (8)$$

One may easy verify that so defined Lie derivatives are realated by the following so-called Leibnitz formula

$$\mathsf{L}_{[\mathbf{f},\mathbf{g}]}h = \langle \mathrm{d}h,[\mathbf{f},\mathbf{g}]\rangle = \mathsf{L}_{\mathbf{g}}\mathsf{L}_{\mathbf{f}}h - \mathsf{L}_{\mathbf{f}}\mathsf{L}_{\mathbf{g}}h . \qquad (9)$$

Note that the following relation is valid

$$\mathrm{d}\mathsf{L}_{\mathbf{f}}h = \mathsf{L}_{\mathbf{f}}(\mathrm{d}h). \qquad (10)$$

Consider the class of time-invariant nonlinear single-input/single-output systems described by

$$\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) \\
y &= h(\mathbf{x}) ,
\end{aligned} \qquad (11)$$

where $\mathbf{x} \in \Re^n$, $y \in \Re$, and $\mathbf{f}$ and h are n-dimensional vector and scalar function, respectively. We desire to find a nonlinear transformation $\mathbf{P}: \Re^n \to \Re^n$, where

$$\mathbf{x} = \mathbf{P}(\mathbf{x}^*) , \qquad (12)$$

such that, in the new coordinate, system (Eqs. (11)) is represented as

$$\dot{\mathbf{x}}^* = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix} \mathbf{x}^* - \begin{bmatrix} f_0^*(x_n^*) \\ f_1^*(x_n^*) \\ \vdots \\ f_{n-1}^*(x_n^*) \end{bmatrix} = \mathbf{f}^*(\mathbf{x}^*) \qquad (13)$$

$$y = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} \mathbf{x}^* \quad .$$

It is convenient to utilize Lie-algebraic notation [12], [13], as follows

$$\dot{\mathbf{x}} = \frac{\partial \mathbf{P}}{\partial \mathbf{x}^*} \mathbf{f}^*(\mathbf{x}^*) \ , \qquad (14)$$

where
$$\frac{\partial \mathbf{P}}{\partial \mathbf{x}^*} = \begin{bmatrix} \dfrac{\partial \mathbf{P}}{\partial x_1^*} & \cdots & \dfrac{\partial \mathbf{P}}{\partial x_n^*} \end{bmatrix} . \qquad (15)$$

It is straightforward to show that all the columns of $\partial \mathbf{P} / \partial \mathbf{x}^*$ is possible to express in terms of the single "starting vector" $\partial \mathbf{P} / \partial x_1^*$ as

$$\frac{\partial \mathbf{P}}{\partial \mathbf{x}^*} = \left[\left(\mathrm{ad}^0 \, \mathbf{f}, \frac{\partial \mathbf{P}}{\partial x_1^*} \right) \ \left(\mathrm{ad}^1 \, \mathbf{f}, \frac{\partial \mathbf{P}}{\partial x_1^*} \right) \ \cdots \ \left(\mathrm{ad}^{n-1} \, \mathbf{f}, \frac{\partial \mathbf{P}}{\partial x_1^*} \right) \right] . \qquad (16)$$

To obtain an expression for the starting vector, the equation $y = h(\mathbf{x}) = x_n^*$ is used, and by repeated application of Leibnitz's formula given by Eq. (9), one may show that

$$\begin{bmatrix} \mathrm{L}_{\mathbf{f}}^0 (\mathrm{d}h)(\mathbf{x}) \\ \mathrm{L}_{\mathbf{f}}^1 (\mathrm{d}h)(\mathbf{x}) \\ \vdots \\ \mathrm{L}_{\mathbf{f}}^{n-1} (\mathrm{d}h)(\mathbf{x}) \end{bmatrix} \frac{\partial \mathbf{P}}{\partial x_1^*} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \quad . \qquad (17)$$

The system form given by Eq. (13) is called nonlinear opserver form and the matrix

$$\mathrm{O}(\mathbf{x}) = \begin{bmatrix} \mathrm{L}_{\mathbf{f}}^0 (\mathrm{d}h)(\mathbf{x}) \\ \mathrm{L}_{\mathbf{f}}^1 (\mathrm{d}h)(\mathbf{x}) \\ \vdots \\ \mathrm{L}_{\mathbf{f}}^{n-1} (\mathrm{d}h)(\mathbf{x}) \end{bmatrix} \qquad (18)$$

is termed the observability matrix of the system defined by Eq. (11). The starting vector $\partial \mathbf{P} / \partial x_1^*$ is equal to the last column of the matrix $\mathrm{O}^{-1}(\mathbf{x})$.

Note that it is not easy to obtain in general the coordinate transformation map since it requires one to solve a set of partial differential equations. However, the advantage of employing the preceding technique of putting the system into observer form, is that observer design in the new coordinate system is extremely simplified. Namely, for the case of a single-output system consider now the observer state equations

$$\dot{\hat{\mathbf{x}}}^* = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix} \hat{\mathbf{x}}^* - \begin{bmatrix} f_0^*(x_n^*) \\ f_1^*(x_n^*) \\ \vdots \\ f_{n-1}^*(x_n^*) \end{bmatrix} - \mathbf{K}(\hat{y} - y), \qquad (19)$$

where $\hat{y}$ and the gain matrix $\mathbf{K}$ are defined as $\hat{y} = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} \hat{\mathbf{x}}^*$ and $\mathbf{K} = \begin{bmatrix} k_0 & \cdots & k_{n-1} \end{bmatrix}^{\mathrm{T}}$.

If the error vector is defined as

$$\mathbf{e}^* = \hat{\mathbf{x}}^* - \mathbf{x}^* , \qquad (20)$$

it follows that the homogeneous differential equation becomes

$$\dot{\mathbf{e}}^* = \begin{bmatrix} 0 & \cdots & 0 & -k_0 \\ 1 & \cdots & 0 & -k_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & -k_{n-1} \end{bmatrix} \mathbf{e}^* \quad . \qquad (21)$$

Hence, if $\mathbf{K}$ is chosen so that the system in Eq. (21) is asymptotically stable, the reconstruction error will always converge to zero. Via an appropriate selection of $\mathbf{K}$, it is easy to assign the spectrum of the characteristic polynomial of the Eq. (21) given by

$$\Delta_o(s) = k_0 + k_1 s + \cdots + k_{n-1} s^{n-1} + s^n \quad . \qquad (22)$$

V. Observer Design Problem Solution

Note, that in most cases some, but not all, of the state variables can be measured. In this case, we need to estimate the unmeasured state variables and the estimated values can be used to perform the adequate control. In the considered magnetic levitation system the ball's position and velocity will be estimated. Namely, we choose the state variables $x_1 = x$ and $x_2 = v$; for the plant output (y) we take ball's position which is controlled by the applying voltage (input u).

According to the consideration given in [5], we have for the plant parameters: the ball's mass $m = 2.12 \times 10^{-2}$ kg, the gravity acceleration $g = 9.81 \mathrm{m/s^2}$, the electromechanical conversion coefficient $k = 1.2 \times 10^{-4} \ \mathrm{Nm^2/A^2}$ as well as the current/voltage coefficient $\rho = 0.166 \ \mathrm{A/V}$ obtaining constant $C = k\rho^2 = 3.307 \times 10^{-6} \ \mathrm{Nm^2/V^2}$. These parameters lead to the elements of the vector function $\mathbf{f}(\mathbf{x})$ in Eq. (11) given by

$$f_1(\mathbf{x}) = x_2$$
$$f_2(\mathbf{x}) = 9.81 - 1.56 \times 10^{-4} \frac{u^2}{x_1^2} \quad . \qquad (23)$$

A straightforward calculation given in the previous section yields transformation $\mathbf{P}: \mathfrak{R}^2 \to \mathfrak{R}^2$, where

$$\mathbf{x} = \mathbf{P}(\mathbf{x}^*) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \mathbf{x}^* \qquad (24)$$

such, that the system given by Eqs. (11) and (23) may be transformed into observer form as

$$\dot{\mathbf{x}}^* = \begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}\mathbf{x}^* + \begin{bmatrix}9.81-1.56\times10^{-4}\dfrac{u^2}{x_2^{*2}}\\ 0\end{bmatrix} = \mathbf{f}^*(\mathbf{x}^*) \quad (25)$$

and $$y_1 = \begin{bmatrix}0 & 1\end{bmatrix}\mathbf{x}^* = x_2^* \ . \quad (26)$$

For the system given in the canonical coordinates by Eqs.(25) and (26), the observer can be constructed with a linear error dynamics, i.e.

$$\dot{\hat{\mathbf{x}}}^* = \begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}\hat{\mathbf{x}}^* + \begin{bmatrix}9.81-1.56\times10^{-4}\dfrac{u^2}{x_2^{*2}}\\ 0\end{bmatrix} - \begin{bmatrix}k_0\\ k_1\end{bmatrix}\left(\hat{y}_1 - y_1\right). \quad (27)$$

The continuous domain observer in the original coordinates is obtained as

$$\dot{\hat{\mathbf{x}}} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\hat{\mathbf{x}} - \begin{bmatrix}0\\ 9.81-1.56\times10^{-4}\dfrac{u^2}{x_1^2}\end{bmatrix} - \begin{bmatrix}k_1 & 1\\ k_0 & 0\end{bmatrix}\left(\hat{y} - y\right) \quad (28)$$

$$\hat{y} = \begin{bmatrix}1 & 0\end{bmatrix}\hat{\mathbf{x}} \ ,$$

with characteristic polynomial of linear error equation given by

$$\Delta_o(s) = k_0 + k_1 s + s^2 \ . \quad (29)$$

VI. Simulation Results

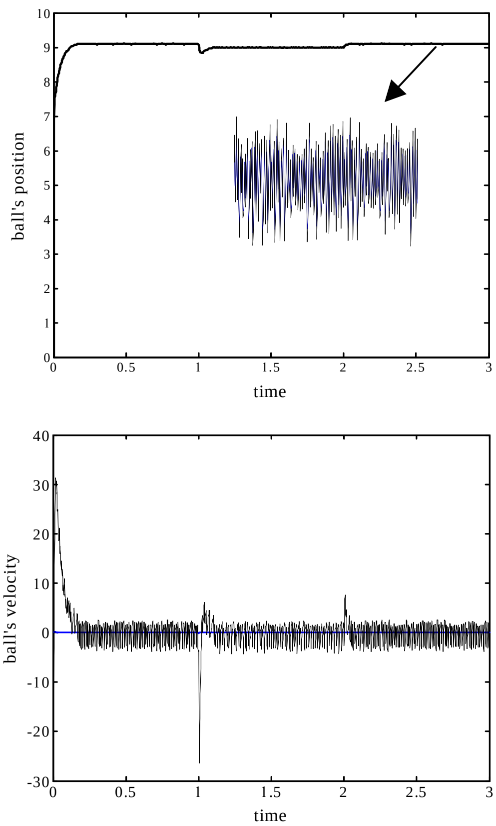

Fig. 3. Estimation results derived by observer (Eq. (28))

Results of analytical design given above are verified by several simulation runs. The system is exited by the pulse generator signal with period of $2\,\mathrm{s}$. The traces of position and velocity signals in Fig. 3 show the ability of observer given by Eq. (28) $\left(k_0 = 6.4\times10^5,\ \ k_1 = 1.6\times10^3\right)$ to estimate the state variables. Note that the system has been simulated in all details, taking into account the limited resolution (8-bit, as the worst case) of position sensor.

VII. Conclusion

Using the magnetic field to levitate a steel ball, the Magnetic Levitation System as a teaching aid enables the theoretical study and practical investigation of basic and advanced approaches to control of nonlinear unstable systems. The simple structure of nonlinear state estimator, proposed in this paper, is verified by the computer simulation. In the suggested observer the observation error vector is multiplied by the observer gain matrix which is obtained after a special coordinate transformation map. Note that finding a suitable nonlinear transformation requires the precise knowledge of the system nonlinear dynamics.

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