# Controllability of a Small Hydro Power Plant

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*Abstract* – **The paper presents the analysis of controllability theory for hydroenergetic installation of the Small Hydro Power Plant "Strezevo" using mathematical algebra. The analysis is based on the linear mathematical model of this plant treated as an object of automatic control.**

*Keywords* – **Controllability, Linear mathematical model, Small hydro power plant.**

## I. INTRODUCTION

Kalman introduced the notion of controllability in 1960. **Controllability** is an important property of a control system. This property plays a crucial role in many control problems, such as stabilization of unstable systems by feedback, or optimal control.

At the power industrial objects the primar task, before the task of controller synthesis, is the analysis of the object's controllability. If we suppose that the mathematical model which represents the object's dynamic behavior is known, then the question is: Is it possible for every state of the system to find a define action (control vector) that brings it into the desired state? Roughly, the concept of controllability denotes the ability to move a system around its entire configuration space using only certain admissible manipulations.

An example of its applicability is a water turbine. The electrical network loading changes disturb the current steady state of the turbine and this results in decreasing or increasing the turbine angular speed, which is not plausable. Therefore, the turbine controller should provide a control action which will annulate these disturbances.

The above example is about the state controllability, but in most cases we are interested in the output controllability. The *state controllability* is usually taken to mean that it is possible, by admissible inputs, to steer the states from any initial value to any final value within some time window. A linear controllable system may be defined as a system which can be steered to any state from the zero initial state. *Output controllability* means the ability to manipulate the outputs of a system by admissible inputs. For a system with several outputs, it might not be possible to manipulate these outputs independently by the admissible inputs, in which case the system is not output controllable.

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## II. MATHEMATICAL MODEL

A linear mathematical model of a hydro energetic installation (Fig. 1) consists of an accumulation, a supply tunnel, a compression pipeline, a water chamber, a hydroelectric generating set and electric net is derived.

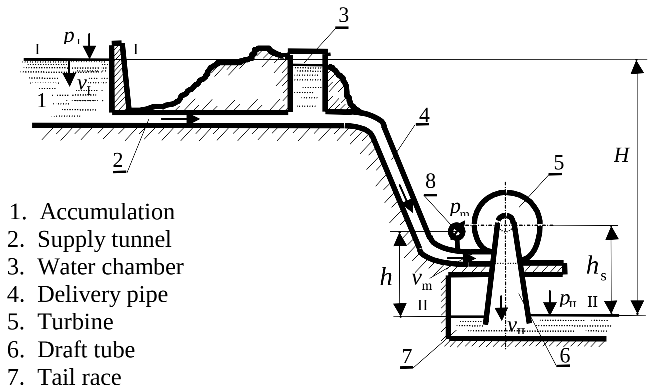
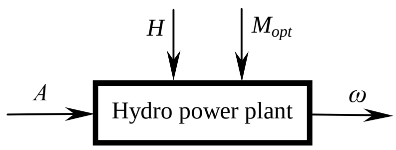



Fig. 1. Hydro electrical power plant

The block diagram of the hydro power plant as an object of automatic control is given in Fig. 2.




$A$ [m$^2$] – opening of the inlet runner
$M_{opt}$ [Nm] – momentum of the loading
$H$ [m] – disposal (total) head
$\omega$ [s$^{-1}$] – angular velocity of the turbine

Fig. 2 Block diagram of HPP

Analyzing the dynamics of each constructive part (subsystem) of the general system, the mathematical model is developed. The model is linearised based on realistically adopted and critically studied assumptions and the following equations are obtained:

- Equation of the supply tunnel

$$\Delta \dot{\bar{q}}_{t} = -\frac{2\left(C_{t}+C_{v}\right)}{T_{t}}\cdot\Delta\bar{q}_{t} - \frac{1}{T_{t}}\cdot\bar{h}_{v} + \frac{1+C_{t}}{T_{t}}\cdot\Delta\bar{H}_{br} \qquad (1)$$

- Equation of the surge tank

$$\dot{\bar{h}}_{v} = \frac{1}{T_{v}}\cdot\Delta\bar{q}_{t} - \frac{1}{T_{v}}\cdot\Delta\bar{q}_{c} \qquad (2)$$

- Equation of the delivery pipe

$$\Delta\dot{\bar{q}}_c = \frac{2C_{\rm v}(1+C_{\rm c})}{T_{\rm c}}\cdot\Delta\bar{q}_{\rm t} + \frac{(1+C_{\rm c})}{T_{\rm c}}\cdot\bar{h}_{\rm v} - \frac{2C_{\rm c}e_1+1}{T_{\rm c}e_1}\cdot\Delta\bar{q} + $$

$$+\frac{e_7}{T_{\rm c}e_1}\cdot\Delta\bar{f} + \frac{e_2}{T_{\rm c}e_1}\cdot\Delta\bar{A} \tag{3}$$

- Equation of frequency of the electricity
  - For isolated SHPP

$$\Delta\dot{\bar{f}} = -\frac{K_{\rm v}}{T}\cdot\Delta\bar{A} + \frac{K_q}{T}\cdot\Delta\bar{q} - \frac{e}{T}\cdot\Delta\bar{f} - \frac{1}{T}\cdot\Delta\bar{P}_L \tag{4}$$

  - For SHPP in parallel electrical network

$$\Delta\dot{\bar{f}} = -\frac{K_{\rm v}}{T_{sa}}\cdot\Delta\bar{A} + \frac{K_q}{T_{sa}}\cdot\Delta\bar{q} - \frac{e_a}{T_{sa}}\cdot\Delta\bar{f} - \frac{1}{T_{sa}}\cdot\Delta\bar{P}_{\kappa a} \tag{5}$$

This system of equations is obtained in normal form, which is most convenient for transfer into state space form and allows direct choice of the state values, control value, controlled and disturbance values. Choosing:

- State vector $\mathbf{x} = \begin{bmatrix}\Delta\bar{q}_{\rm t} & \bar{h}_{\rm v} & \Delta\bar{q}_{\rm c} & \Delta\bar{f}\end{bmatrix}^T$
- Input vector $\mathbf{x}_u = \begin{bmatrix}\Delta\bar{H}_{\rm br} & \Delta\bar{A} & \Delta\bar{P}_L\end{bmatrix}^T$
- Control vector $\mathbf{u} = u = \Delta\bar{A}$
- Disturbance vector $\mathbf{z} = \begin{bmatrix}\Delta\bar{H}_{\rm br} & \Delta\bar{P}_L\end{bmatrix}^T$
- Output vector $\mathbf{x}_i = x_i = \Delta\bar{f}$

the mathematical model into state space form is obtained:

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$$
$$\mathbf{x}_i = \mathbf{C}\,\mathbf{x} \tag{6}$$

and the particular matrices are given below:

- For isolated SHPP

$$\mathbf{A} = \begin{bmatrix} -\frac{2(C_{\rm t}+C_{\rm v})}{T_{\rm t}} & -\frac{1}{T_{\rm t}} & 0 & 0 \\ \frac{1}{T_{\rm v}} & 0 & -\frac{1}{T_{\rm v}} & 0 \\ \frac{2C_{\rm v}(1+C_{\rm c})}{T_{\rm c}} & \frac{1+C_{\rm c}}{T_{\rm c}} & -\frac{2C_{\rm c}e_1+1}{T_{\rm c}e_1} & \frac{e_7}{T_{\rm c}e_1} \\ 0 & 0 & \frac{K_q}{T} & -\frac{e}{T} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & \frac{e_2}{T_{\rm c}\cdot e_1} & -\frac{K_{\rm v}}{T}\end{bmatrix}^{\rm T}$$

$$\mathbf{C} = \begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix} \tag{7}$$

- For SHPP in parallel electrical network

$$\mathbf{A} = \begin{bmatrix} -\frac{2(C_{\rm t}+C_{\rm v})}{T_{\rm t}} & -\frac{1}{T_{\rm t}} & 0 & 0 \\ \frac{1}{T_{\rm v}} & 0 & -\frac{1}{T_{\rm v}} & 0 \\ \frac{2C_{\rm v}(1+C_{\rm c})}{T_{\rm c}} & \frac{1+C_{\rm c}}{T_{\rm c}} & -\frac{2C_{\rm c}e_1+1}{T_{\rm c}e_1} & \frac{e_7}{T_{\rm c}e_1} \\ 0 & 0 & \frac{K_q}{T_{sa}} & -\frac{e_a}{T_{sa}} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & \frac{e_2}{T_{\rm c}\cdot e_1} & -\frac{K_{\rm v}}{T_{sa}}\end{bmatrix}^T$$

$$\mathbf{C} = \begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix} \tag{8}$$

## III. THE CONTROLLABILITY CRITERIA

**Definition 1**. A linear system is said to be *completely controllable* if, for all initial times $t_0$ and all initial states $\mathbf{x}(t_0)$, there exists some input function that drives the state vector to any final state $\mathbf{x}(t_1)$ at some finite time $t_1 > t_0$.

Matrices **A** and **B** determine controllability of the system.

According to the well known algebraic criteria the system

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u}$$
$$\mathbf{x}_i = \mathbf{C}\,\mathbf{x}$$

is state-controllable if and only if the controllability matrix

$$\mathbf{P} = \begin{bmatrix}\mathbf{B} & \mathbf{A}\,\mathbf{B} & \mathbf{A}^2\,\mathbf{B} & \ldots\ldots & \mathbf{A}^{n-1}\,\mathbf{B}\end{bmatrix} \tag{9}$$

has full rank.

This criterion represents the test for state controllability of a linear system.

Another kind of controllability may be useful from a practical perspective, namely, complete *output controllability*, which is defined as the ability to drive the output vector to the origin in finite time. This property involves all matrices **A**, **B**, **C.** The **output controllability** matrix of the system is

$$\mathbf{Q} = \mathbf{C}\cdot\mathbf{P} = \begin{bmatrix}\mathbf{C}\,\mathbf{B} & \mathbf{C}\,\mathbf{A}\,\mathbf{B} & \mathbf{C}\,\mathbf{A}^2\,\mathbf{B} & \ldots\ldots & \mathbf{C}\,\mathbf{A}^{n-1}\,\mathbf{B}\end{bmatrix} \tag{10}$$

The sufficient condition for complete output controllability of the system is that the rank(**Q**) is equal to the number of outputs.

## IV. CONTROLLABILITY ANALYSIS FOR SPECIFIC HPP

Controllability analysis is performed for the Small Hydro Power Plant "Strezevo" taken as a specific HPP in this paper. SHPP "Strezevo" belongs to the group of SHPP with the powerhouse located at the base of a dam. This SHPP is located on the main canal of the Strezevo dam and affords energy utilization of the hydrological potential of the water from the reservoir Strezevo. Namely, the artificial lake Strezevo is a part of the Hydro System "Strezevo", which provides required quantities of water for irrigation of 20.200 ha of agricultural land in Bitola region of Pelagonia Valley, Republic of Macedonia. Apart from irrigation, this hydro-system provides water to additionally supplement the required quantities of unprocessed water for water supply, supplying technological water for one part of the industry, energy utilization of the hydrological potential of the water, as well as flood control.

Using the project data for the SHPP "Strezevo":

| | |
|---|---|
| Accumulation | 112 000 000 $m^3$ |
| Installed discharge | 3 x 2,67 = 8 $m^3/s$ |
| Installed power | 3 x 0,8 = 2,4 MW |
| Turbine Francis | |
| Nominal net head | 30 m |
| Turbine speed | 750 $min^{-1}$ |
| Diameter of turbine wheel | 600 mm |

the numerical values for the time constants and other coefficients in the mathematical model are determined. Thus, the linear mathematical model for this specific SHPP is obtained in the form given by the Eq. (6), where the particular matrices have had the following values:

- For isolated SHPP

$$\mathbf{A}=\begin{bmatrix} -0{,}0083 & -3{,}5714 & 0 & 0 \\ 0{,}0026 & 0 & -0{,}0026 & 0 \\ 0{,}0017 & 1{,}7100 & -3{,}2918 & -0{,}1282 \\ 0 & 0 & 0{,}4578 & -0{,}0545 \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix} 0 \\ 0 \\ 2{,}5641 \\ -0{,}2644 \end{bmatrix} \qquad \mathbf{C}=\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \qquad (11)$$

- For SHPP in parallel electrical network

$$\mathbf{A}=\begin{bmatrix} -0{,}0083 & -3{,}5714 & 0 & 0 \\ 0{,}0026 & 0 & -0{,}0026 & 0 \\ 0{,}0017 & 1{,}7100 & -3{,}2918 & -0{,}1282 \\ 0 & 0 & 0{,}0223 & -1{,}109 \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix} 0 \\ 0 \\ 2{,}5641 \\ -0{,}0129 \end{bmatrix} \qquad \mathbf{C}=\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \qquad (12)$$

The state space dimension $n$ for this mathematical model is 4, so that, the system is state controllable if the rank of matrix **P** (Eq. 9) is equal to 4. According to its mathematical model, the number of outputs for this specific system is equal to 1; thus, the output controllability test means that the system is output controllable if the matrix **Q** (Eq. 10) has rank equal to1.

The controllability test for the considered SHPP is performed using the mathematical software package MATLAB. The calculation results are given below.

- For isolated SHPP

$$\mathbf{P}=\begin{bmatrix}\mathbf{B} & \mathbf{A\,B} & \mathbf{A}^2\,\mathbf{B} & \mathbf{A}^3\,\mathbf{B}\end{bmatrix}$$

$$\mathbf{P}=\begin{bmatrix} 0 & 0 & 0{,}0238 & -0{,}0783 \\ 0 & -0{,}0067 & 0{,}0219 & -0{,}0715 \\ 2{,}5641 & -8{,}4066 & 27{,}5091 & -90{,}0155 \\ -0{,}2644 & 1{,}1883 & -3{,}9133 & 12{,}8070 \end{bmatrix}$$

$$rank(\mathbf{P}) = 4$$

$$\mathbf{Q}=\begin{bmatrix}\mathbf{C\,B} & \mathbf{C\,A\,B} & \mathbf{C\,A}^2\,\mathbf{B} & \mathbf{C\,A}^3\,\mathbf{B}\end{bmatrix}$$

$$\mathbf{Q}=\begin{bmatrix}-0{,}2644 & 1{,}1883 & -3{,}9133 & 12{,}8070\end{bmatrix}$$

$$rank(\mathbf{Q}) = 1$$

- For SHPP in parallel electrical network

$$\mathbf{P}=\begin{bmatrix}\mathbf{B} & \mathbf{A\,B} & \mathbf{A}^2\,\mathbf{B} & \mathbf{A}^3\,\mathbf{B}\end{bmatrix}$$

$$\mathbf{P}=\begin{bmatrix} 0 & 0 & 0{,}0238 & -0{,}0786 \\ 0 & -0{,}0067 & 0{,}0219 & -0{,}0721 \\ 2{,}5641 & -8{,}4389 & 27{,}7584 & -91{,}3034 \\ -0{,}0129 & 0{,}0715 & -0{,}2675 & 0{,}9156 \end{bmatrix}$$

$$rank(\mathbf{P}) = 4$$

$$\mathbf{Q}=\begin{bmatrix}\mathbf{C\,B} & \mathbf{C\,A\,B} & \mathbf{C\,A}^2\,\mathbf{B} & \mathbf{C\,A}^3\,\mathbf{B}\end{bmatrix}$$

$$\mathbf{Q}=\begin{bmatrix}-0{,}0129 & 0{,}0715 & -0{,}2675 & 0{,}9156\end{bmatrix}$$

$$rank(\mathbf{Q}) = 1$$

According to the presented results, it can be concluded that the treated SHPP is state and output controllable.

## V. CONCLUSION

Based on the known controllability criteria, in this paper we perform a controllability test for specific SHPP. We introduce the mathematical model of SHPP as an object of automatic control given in state space form. For a specific SHPP "Strezevo" we use the mathematical model to explore the contolability of this SHPP for isolated work, as well as for work in parallel electrical network. The results gained from controllability analysis confirm that the SHPP "Strezevo" as an object of automatic control is complete state and output controllable.

## APPENDIX - LIST OF SYMBOLS

$\Delta\bar{q}_t$ - supply tunnel water flow in relative coordinates;

$\bar{h}_v$ - water level in the surge tank in relative coordinates;

$\Delta\bar{q}_c$ - delivery pipe water flow in relative coordinates;

$\Delta\bar{f}$ - electrical network frequency in relative coordinates;

$\Delta\bar{A}$ - opening of the inlet runner in relative coordinates;

$\Delta\bar{P}_L$ - generator loading disturbance in relative coordinates;

$\Delta\bar{H}_{br}$ - disposal (total) head in relative coordinates;

$e_1$ - relative change of flow $q$ as a function of the net head $H$ at $\omega = const$ and $A = const$;

$e_2$ - relative change of flow $q$ as a function of the inlet runner opening $A$ at $\omega = const$ and $H = const$;

$e_3$ - relative change of the efficiency coefficient $\eta$ as a function of the net head $H$ at $\omega = const$ and $A = const$;

$e_4$ - relative change of the efficiency coefficient $\eta$ as a function of the inlet runner opening $A$ at $\omega = const$ and $H = const$;

$e_p$ - self-regulation loading coefficient;

$C_t$ - loss coefficient for the supply tunnel;

$C_v$ - coefficient of the kinetic energy;

$C_c$ - loss coefficient for the delivery pipe;

$T_v$ - time constant of the surge tank;

$T_t$ - time constant of the supply tunnel;

$T_c$ - time constant of the delivery pipe;

$T_a$ - time constant of the turbine and generator mechanical inertia.

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