Noise Modeling and Analysis of a BJT Common – Emitter Stage

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***Abstract* - Accurate noise modeling is a prerequisite for circuit noise simulation. In this paper, a noise model for Bipolar Junction Transistor (BJT) common – emitter stage using input voltage and current equivalent generators is presented. Noise analysis of a model is performed. Analytical equations for input equivalent generators of a stage, as well as, for optimum noise factor and for optimum source resistance are worked out. MATLAB simulation results are presented.**

***Keywords* – noise modeling, noise analysis, noise factor, BJT common – emitter stage, MATLAB**

I. INTRODUCTION

Noise behavior is an important characteristic of electronic circuits, as it usually determines the fundamental limit of the performance of circuits. Circuit noise is generated within resistors, devices, and interconnections. Mathematically, noise is characterized in terms of mean and autocorrelation in the time domain, or the power spectral density in the frequency domain. The most important noise sources in circuit devices are thermal noise, shot noise and flicker noise [1].

The spectral density function of thermal noise, $i_{NT}^2/\Delta f$, can be given by

$$\frac{i_{NT}^2}{\Delta f}=\frac{4kT}{R} \quad , \tag{1}$$

where k is Boltzman's constant ($k=1.374x10^{-23}\ J/K$), T is the absolute temperature in degrees Kelvin, R is the resistance in ohms, and Δf is a circuit bandwidth in Herts.

The noise spectral density of shot noise, $i_{NS}^2/\Delta f$, can be described by

$$i_{Ss}^2/\Delta f=2qI \tag{2}$$

where q is the electron charge ($q=1.6x10^{-19}\ C$), and I is the average junction current.

The spectral density of flicker noise, $i_{Nf}^2/\Delta f$, can be determined by

$$\frac{i_{Nf}^2}{\Delta f}=KF\frac{I^{AF}}{f} \tag{3}$$

where KF is a constant for a particular device and AF is a constant in the range from 0.5 to 2.

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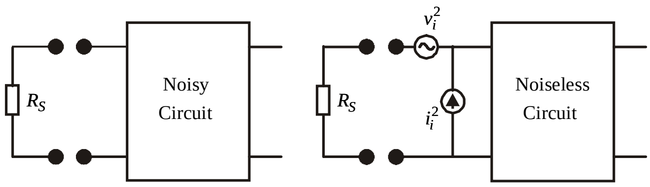
II. INPUT NOISE GENERATORS AND NOISE FACTOR OF ELECTRONIC CIRCUITS

A general noise model of an electronic circuit can be obtained by reflecting all internal noise sources to the input. In order for the reflected sources to be independent of the source impedance, two noise sources can be used – a series voltage source and a shunt current source. Furthermore, the noisy circuit can be presented as shown in Fig. 1(a), and the noiseless one can be presented as shown in Fig. 1(b). The representation in Fig. 1(b) is valid for any impedance, if correlation between the noise generators is considered.



(a) (b)

Fig. 1. A noisy (a) and a noiseless (b) circuit.

The equivalent noise input voltage v_{iN} can be considered as the voltage in series with v_S (see Fig. 3) that generates the same noise voltage at the output as all noise sources in the circuit [2].

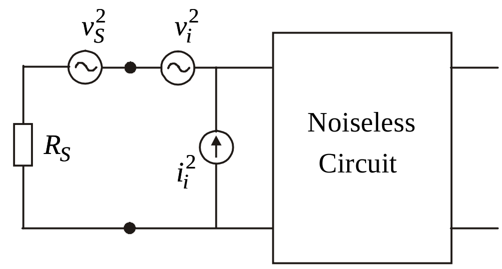


Fig. 3. A circuit with equivalent input noise generators.

The mean-square value of the total input equivalent noise can be found by

$$v_{iN}^2=v_S^2+v_i^2+i_i^2R_S^2 \quad , \tag{4}$$

where v_S^2 is the thermal noise of R_S.

In most practical circuits the correlation between v_i and i_i is small and may be neglected. If either v_i^2 or i_i^2 dominates, the correlation may be neglected in any case.

The value of v_i^2 can be found by shorting the input ports and equating the output noise in each case. In a similar manner, the value of $\overline{i_i^2}$ can be found by opening the input ports and equating the output noise in each case.

Assuming no correlation between v_i^2 and i_i^2, is obtained

$$\frac{N_a}{N_i}=\frac{v_i^2+i_i^2R_S^2}{v_S^2} \quad . \tag{5}$$

Thus, the noise factor fop the two – port circuit is

$$F=1+\frac{N_a}{N_i}=1+\frac{v_i^2}{4kTR_S\Delta f}+\frac{i_i^2R_S}{4kT\Delta f} \quad . \tag{6}$$

It is obvious, that for small R_S, v_i^2 dominates, whereas for large R_S, i_i^2 dominates.

There is an optimal R_S for minimum F. Solving Eq. (7), as a result $R_{S,opt}$ is found.

$$\frac{dF}{dR_S}=-\frac{v_i^2}{4kTR_S^2\Delta f}+\frac{i_i^2R_S^2}{4kTR_S^2\Delta f}=0 \quad . \tag{7}$$

$$R_{S,opt}=\frac{v_i^2}{i_i^2} \quad . \tag{8}$$

Substituting Eq. (8) into Eq. (6) gives

$$F_{opt}=1+\frac{i_i^2R_S}{2kT\Delta f} \quad . \tag{9}$$

IV. Noise Modeling and Analysis of a BJT Common – Emitter Stage

A. BJT Noise Model

A key element in most analog circuits is a bipolar junction transistor. From various BJT models [3], [4], [5] for noise modeling and analysis of analog circuits a model shown in Fig. 4 is the most suitable.

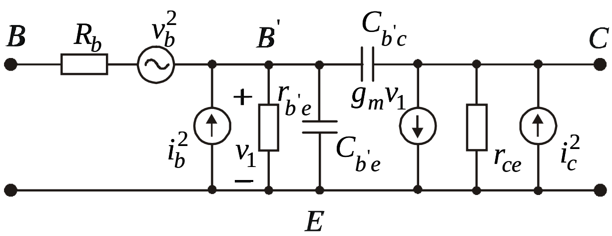


Fig. 4. BJT noise model.

Thermal noise du to the resistance R_b can be modeled by a thermal noise source

$$v_b^2=4kTR_b\Delta f \quad . \tag{10}$$

Base and collector currents are considered to be independent. Therefore, the shot noise can be described by

$$i_b^{'2}=2qI_B\Delta f \tag{11}$$

at the base and

$$i_c^2=2qI_C\Delta f \tag{12}$$

at the collector.

The total *1/f* noise of the transistor is described by a noise source in parallel with base – emitter contact. This leads to the noise source at the base following

$$i_b^2=2qI_B\Delta f+KF\frac{I_B^{AF}}{f}\Delta f \quad . \tag{13}$$

B. Noise Model of BJT a Common – Emitter Stage

Using BJT model in Fig. 4, a noise model for common – emitter stage, shown in Fig. 5, is synthesized.

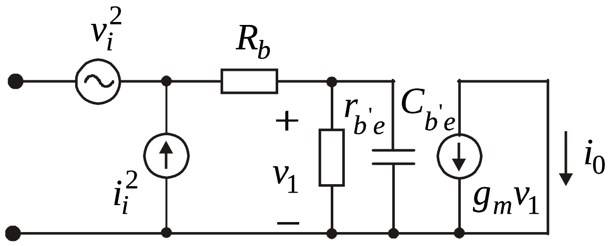


Fig. 5. BJT common-emitter stage noise model

A noise voltage generator v_i^2 and a noise current generator i_i^2 model the noise in a common - emitter stage.

By shorting the input ports, is obtained

$$i_o=g_mv_b+i_c=g_mv_i \quad . \tag{14}$$

Since r_b is small, i_b is neglected. Thus, from Eq. (14) follows that

$$v_i^2=v_b^2+\frac{i_c^2}{g_m^2} \quad , \tag{15}$$

and

$$\frac{v_i^2}{\Delta f}=4kT\left(r_b+\frac{1}{2g_m}\right)=4kTR_{eq} \quad . \tag{16}$$

In Eq. (16) R_{eq} is the equivalent input resistance and

$$R_{eq}=r_b+\frac{1}{2g_m} \quad .$$

Similarly, by opening the input ports, as can be seen from Fig. 5,

$$i_o=\beta(j\omega)i_b+i_c=\beta(j\omega)i_i \tag{17}$$

from where the current of the input noise generator can be expressed by

$$i_i^2=i_b^2+\frac{i_c^2}{\left|\beta(j\omega)\right|^2} \quad . \tag{18}$$

Thus

$$\frac{i_i^2}{\Delta f}=2q\left[I_B+KF'\frac{I_B^{AF}}{f}+\frac{I_C}{\left|\beta(j\omega)\right|^2}\right]=2qI_{eq} \tag{19}$$

where

$$KF'=\frac{KF}{2q}, \quad \beta(j\omega)=\frac{\beta_0}{1+\beta_0\dfrac{C_{b'e}+C_{b'c}}{g_m}j\omega} \quad .$$

From Eq. (19) it is obvious that the equivalent input shot noise current I_{eq} can be expressed as

$$I_{eq} = I_B + KF' \frac{I_B^{AF}}{f} + \frac{I_C}{|\beta(j\omega)|^2} \quad .$$

C. Total Equivalent Noise Voltage of a BJT Common - Emitter Stage

Substituting Eqs. (16) and (19) into Eq. (4) and taking into consideration that $v_S^2 = 4kTR_S$, the total equivalent noise voltage with a source resistance R_S is found as

$$\begin{aligned} v_{iN}^2 &= \left(4kT(R_S + r_b + \frac{1}{2g_m}\right)\Delta f \\ &+ R_S^2 2q\left(I_B + KF' \frac{I_B^{AF}}{f} + \frac{I_C}{|\beta(j\omega)|^2}\right)\Delta f \\ &= 2qR_S^2(A+B)\Delta f \end{aligned} \qquad (20)$$

where

$$A = \frac{2\varphi_T}{R_S^2}\left(R_S + r_b + \frac{1}{2g_m}\right)$$

$$B = I_B + KF' \frac{I_B^{AF}}{f} + \frac{I_C}{|\beta(j\omega)|^2}$$

and $\varphi_T = kT/q$.

D. Noise Factor of a BJT Common - Emitter Stage

Neglecting flicker noise and substituting $I_C = I_B/\beta_F$ into Eq. (19) gives

$$\frac{i_i^2}{\Delta f} = 2q\left(\frac{I_C}{\beta_F} + \frac{I_C}{|\beta(j\omega)|^2}\right) \quad . \qquad (21)$$

Using Eqs. (6), (16) and (21) the noise factor of a BJT common – emitter stage is obtained as

$$\begin{aligned} F &= 1 + \frac{1}{R_S}\left(r_b + \frac{1}{2g_m}\right) \\ &+ R_S\left[\frac{g_m}{2\beta_F} + \frac{g_m}{2\beta_o^2}\left(1 + \beta_o^2\left(\frac{\omega}{\omega_T}\right)^2\right)\right] \end{aligned} \qquad (22)$$

where $\omega_T = 1/f_T$ (f_T - transition frequency).

For high - frequency circuits, if $\omega/\omega_T >> 1/\beta_o$ and $\omega/\omega_T >> 1/\beta_F$

$$F \approx 1 + \frac{1}{R_S}\left(r_b + \frac{1}{2g_m}\right) + R_S\frac{g_m}{2}\left(\frac{\omega}{\omega_T}\right)^2 \quad . \qquad (23)$$

Solving equation $dF/dg_m = 0$ for fixed R_S and ω_T gives

$$g_{m,opt} = \frac{1}{R_S}\frac{\omega_T}{\omega} \quad . \qquad (24)$$

and

$$F_{opt} = 1 + \frac{r_b}{R_S} + \frac{\omega}{\omega_T} \quad . \qquad (25)$$

Similarly, from equation $dF/dR_S = 0$, for fixed g_m and ω_T, are determined

$$R_{S,opt} = \sqrt{\frac{2r_b}{g_m} + \frac{1}{g_m^2}}\ \frac{\omega}{\omega_T} \quad , \qquad (26)$$

and

$$F_{opt} = 1 + \sqrt{2r_b g_m + 1}\ \frac{\omega}{\omega_T} \quad . \qquad (27)$$

For low – frequency circuits, if $\omega/\omega_T << 1/\beta_o$ and $\omega/\omega_T << 1/\beta_F$

$$F \approx 1 + \frac{1}{R_S}\left(r_b + \frac{1}{2g_m}\right) + R_S\frac{g_m}{2}\frac{1}{\beta_F} \quad . \qquad (28)$$

Applying optimization procedure to Eq. (28), the following equations for $g_{m,opt}$, $R_{S,opt}$ and F_{opt} are obtained:

$$g_{m,opt} = \frac{1}{R_S}\sqrt{\beta_F} \quad , \qquad (29)$$

$$F_{opt} = 1 + \frac{r_b}{R_S} + \frac{1}{\sqrt{\beta_F}} \qquad (30)$$

for fixed R_S and β_F,

and

$$R_{S,opt} = \sqrt{\frac{2r_b}{g_m} + \frac{1}{g_m^2}}\ \sqrt{\beta_F} \quad , \qquad (31)$$

$$F_{opt} = 1 + \sqrt{2r_b g_m + 1}\ \frac{1}{\sqrt{\beta_F}} \qquad (32)$$

for fixed g_m and β_F.

E. Demonstration example

In order to demonstrate the effectiveness and the accuracy of the above stated approach a MATLAB file for computation and simulation the noise of a common – emitter stage is written. Assume that a NPN BJT type 2N2222A is used in the stage. Computed results for optimum source resistance's, on condition that f = 1kHz and varying collector current I_C, are given in Table I.

TABLE I
OPTIMUM VALUES FOR SOURCE RESISTANCE

I_C, mA	1.00	0.50	0.10	0.05
$R_{S,\,opt}$, Ω	150	200	2000	4000

The results of the simulations are shown in Figs. 6 and 7. In Fig. 6 the frequency dependence of noise factor with varying the values of collector current and optimum source resistance, is presented. Fig. 7 is the result of noise factor versus source resistance simulation.

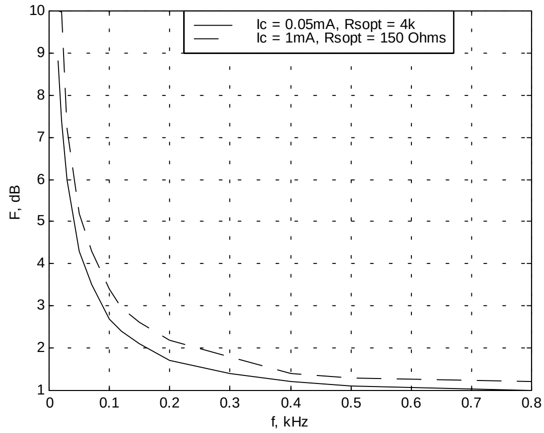


Fig. 6. Noise factor as a function of frequency.

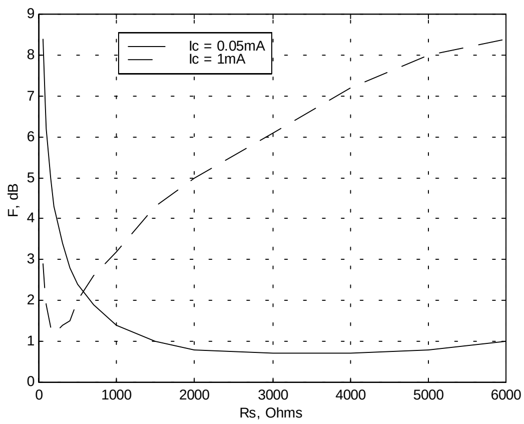


Fig. 7. Noise factor as a function of source resistance.

The calculation and simulation outcomes of the BJT common - emitter stage coincide with the values given in manufacturer' s data books [6].

IV. CONCLUSION

A simple but effective approach for noise modeling and analysis of a BJT common – emitter stage is developed. Noise generators are used to model the total input noise of a stage. Analytical equations for noise factor and source resistance of low and high–frequency circuits are obtained. The calculation and simulation outcomes of a common – emitter stage by implementing various BJT types support the efficiency and fidelity of the approach proposed in this paper. This approach can be applied successfully to the other BJT configurations. The equivalent input noise generators of a common – base stage or emitter follower are the same as those of a common – emitter stage. For the common – base configuration, since its current gain ≈ 1, any noise current at the output is referred directly back to the input without reduction. For the emitter follower, since its voltage gain ≈ 1, any noise voltage at the output, including noise due to Z_L, is transformed unchanged to the input.

REFERENCES

[1] F. Sischka, Noise Modeling for Semiconductors, Agilent Technologies GmbH, Munich, 2002.

[2] Jieh-Tsorng Wu, Noise Analysis and Modeling, National Chiao-Tung University, 2001.

[3] G. Massobrio, P. Antognietti, Semiconductor Device Modeling with SPICE, McGraw-Hill, New Jork, 1993.

[4] F. X. Sinnesbichler, M. Fischer, G. R. Olbrich, "Accurate Extraction Method for 1/f-Noise Parameters Used in Goomel-Poon Type Bipolar Transistor Models", IEEE MTT-S Symposium, Baltimore, 1998, pp. 1345-1348.

[5] Pesha D. Petrova, "Noise Modeling of Semiconductor Devices", TELSIKS'03, Proceedings of Papers, vol. 2, pp.577-580, Nis, Yugoslavia, 2003.

[6] Motorola Semiconductor Technical Data, Motorola, Inc, 1999-2003.