On A Class of the Discrete Oscillators with Several Attractors

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Abstract – **In this paper a class of discrete nonlinear oscillators is presented, with several steady states. This oscillators has possibility to generate oscillations with different amplitudes and frequencies. Oscillators generate periodical oscillation which are invariant to external disturbances.**

Keywords – **Nonlinear oscillator, discrete attractor, amplitude, frequency.**

I. INTRODUCTION

Continous oscillators which generate harmonic signals with required frequency and amplitude is well known [2,3,4], as linear oscillators and Van der Pole's oscillator. However, these oscillators are sensitive to external disturbances, because each disturbance is a new initial condition. In the paper [1] is presented a class of the nonlinear oscillator with several steady states.

Nonlinear Van der Pole's oscillator has stable oscillations, but this oscillation is not simple periodical.

Oscillator, presented in this paper, can generate plain periodical discrete oscillations with several required amplitudes and frequencies. In other words, oscillator can generate several discrete oscillations, by setting up different initial conditions. These oscillators are robust with respect to external disturbances.

II. NECESSARY CONDITIONS FOR THE EXISTENCE OF OSCILLATIONS OF DISCRETE NONLINEAR SYSTEM

Let consider discrete nonlinear system:

$$\sum_{i=0}^{n} a_i\left(x(k)\right)x(k+i)=0,\quad a_n\left(x(k)\right)=1 \tag{1}$$

where coefficients $a_i(x)$ are functions of the $x(k)$ i.e.:

$$a_i(x)=\varphi_i\left(x(k)\right)=\varphi_i(x),\quad i=0,....,n-1 \tag{2}$$

are nonlinear function of state coordinates $x(k)$.

Let consider the linear equation instead equation (1):

$$\sum_{i=0}^{n} a_i x(k+i)=0,\quad a_n=1 \tag{3}$$

The characteristic polynomial of the (3) is:

$$\sum_{i=0}^{n} a_i z^i=0,\ a_n=1 \tag{4}$$

where a_i are constant parameters. System Eq. (3) is stable if and only if all zeros of the polynomial Eq. (4) is inside unit circle $|z|\le 1$. Stability region of this linear system can be determined by using Schur-Cohn criterion or using the bilinear transformation, $z=\frac{s+1}{s-1}$ and Hurwitz criterion. Let us denote stability region in parametric space by S_n (where n is order of system).

Let define a curve $a(x)$ in parametric space by equations:

$$a(x)\Leftrightarrow \begin{array}{c} a_1=\varphi_1(x)\\ a_2=\varphi_2(x)\\ \vdots\\ a_n=\varphi_n(x). \end{array} \tag{5}$$

This is an oriented curve in the direction increase of $|x|$, starting from $x=0$.

The necessary conditions for the existence of attractors are given [1, 5]. Let us denote with l the set of points of the curve $a(x)$, and with $\vec{L}$ the unit tangent vector of curve $a(x)$ in the point $(a_1,a_2,...,a_n)$.

The necessary conditions for the existence of attractors of system (1) are:

$$\begin{array}{l} S_n\cap l\ne\varnothing\\ \vec{L}\,\mathrm{grad}\,\partial\, \mathrm{S_n}<0 \end{array} \tag{6}$$

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where $\vec{L} = \left(\frac{\partial \varphi_1}{\partial x}, \frac{\partial \varphi_2}{\partial x}, ..., \frac{\partial \varphi_n}{\partial x} \right)$, while $\varnothing$ is the empty set.

The sufficient condition for the absence of attractors of system (1) is:

$$S_n \cap l = \varnothing.$$

$$\text{or } \vec{L} \text{ grad} \partial \text{ S}_n > 0 \tag{7}$$

For example, in the case of second order system we obtain characteristic polynomial:

$$z^2 + a_1 z + a_2 = 0 \tag{8}$$

Using the Schur-Cohn criterion can determined stability region S_2:

$$\begin{aligned} &a_2 < 1 \\ &a_2 - a_1 > -1 \\ &a_2 + a_1 > -1 \end{aligned} \tag{9}$$

The stability region S_2 is given in Fig. 1. The oriented curve $a(x)$ is given in the same Figure also.

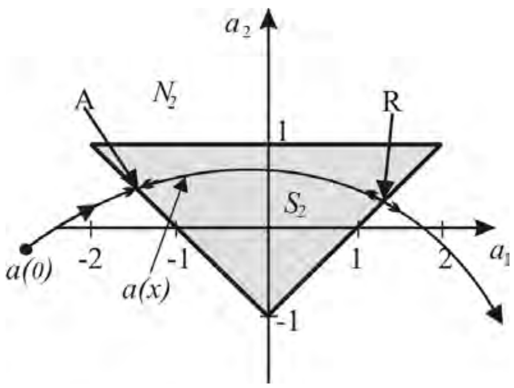

Fig.1 Stability region S_2

where A- attraction point, R- repeller point.

In the case of n–th order system, using bilinear transformation $z = \frac{s+1}{s-1}$ we obtain new characteristic polynomial:

$$b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + b_0 = 0 \tag{10}$$

Using the well known Hurwitz criterion, we obtain:

$$D_n = \begin{vmatrix} b_{n-1} & b_{n-3} & b_{n-5} & ... & 0 \\ b_n & b_{n-2} & b_{n-4} & ... & 0 \\ 0 & b_{n-1} & b_{n-3} & ... & 0 \\ 0 & b_n & b_{n-2} & ... & 0 \\ . & . & . & ... & . \\ 0 & 0 & 0 & ... & b_0 \end{vmatrix} \tag{11}$$

The stability region S_n is given with: $D_1 = b_{n-1} > 0$, $D_2 = \begin{vmatrix} b_{n-1} & b_{n-3} \\ b_n & b_{n-2} \end{vmatrix} = b_{n-1} b_{n-2} - b_n b_{n-3} > 0$,..., $D_i > 0$, where D_i is i-th diagonal minor of D_n.

III. EXAMPLES

Example 1: Consider a discrete nonlinear second order system:

$$\begin{aligned} &x(k+2) + \left(x^2(k)-1\right)\left(x^2(k)-4\right)\left(x^2(k)-9\right)\left(x^2(k)-16\right) \\ &\left(x^2(k)-25\right)\left(x^2(k)-36\right)x(k+1) + x(k) = 0, \end{aligned} \tag{12}$$

Oscillations are given in Fig.2 in phase plain. Note that oscillator can generate six oscillations with different amplitude (denoted by 1, 2, 3, 4, 5 and 6) as shown in the Figure.

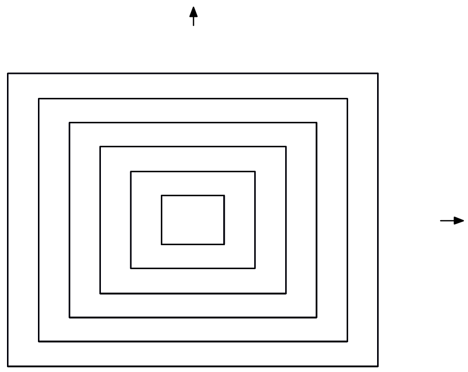

Fig. 2 Oscillations in phase plane

The same oscillations time diagram are given in the Fig.3.

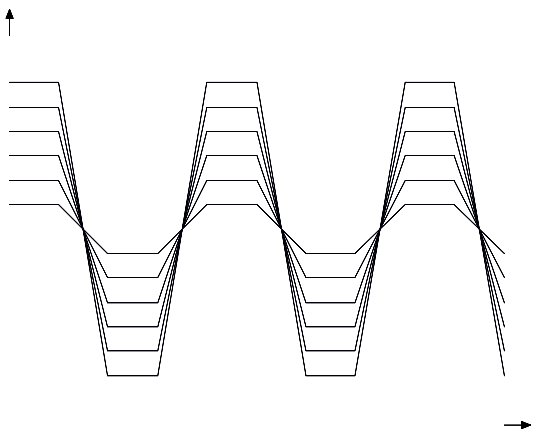

Fig. 3 Oscillations time diagram

Example 2: Let us given discrete time system:

$$\begin{aligned}&x(k+2)+\left(x^2(k)+x^2(k+1)-1\right)\left(x^2(k)+x^2(k+1)-4\right)\\&\left(x^2(k)+x^2(k+1)-9\right)\left(x^2(k)+x^2(k+1)-16\right)\\&\left(x^2(k)+x^2(k+1)-25\right)\left(x^2(k)+x^2(k+1)-36\right)\\&\left(x^2(k)+x^2(k+1)-49\right)x(k+1)+x(k)=0,\end{aligned} \tag{13}$$

Oscillations in the phase plain are shown in Fig.4. The oscillator can generate seven oscillations with different amplitude (denoted by 1, 2, 3, 4, 5, 6 and 7) as shown in the same Figure.

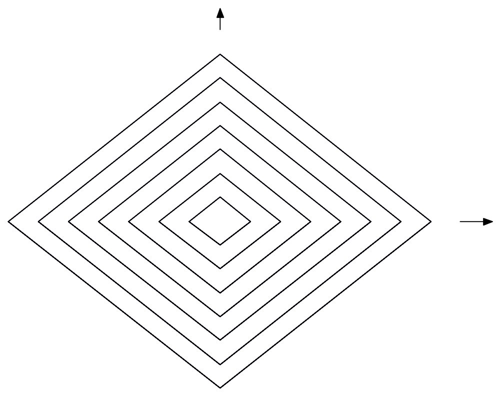

Fig. 4 Oscillations in phase plane

The same oscillations in time diagram are shown in the Fig.5.

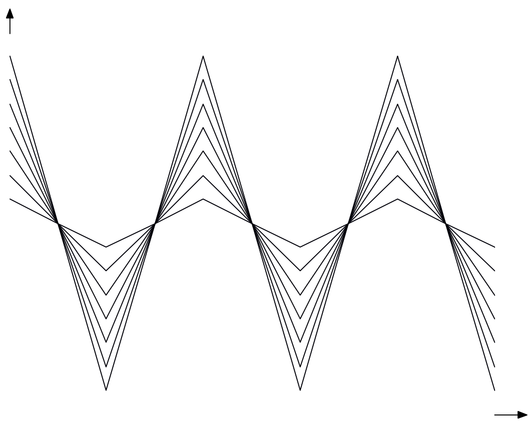

Fig. 5 Oscillations time diagram

IV. CONCLUSION

Discrete nonlinear oscillators which can generate many oscillations with different amplitude and frequencies are considered. Amplitude and frequencies can determine by choosing different initial conditions.

Necessary conditions for existence of steady oscillations and sufficient conditions for absence of the oscillations are given also.

Presented oscillator is able to realized on many different ways. The best solution would be implementation based on microcontroller system, due to its software flexibility the same hardware configuration could be used for various types of oscillators.

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