Performance Analysis of UMTS WCDMA Rake Receiver

Nenad Milošević[1], Bojan Dimitrijević[2], Zorica Nikolić[3], Đorđe Paunović[4]

Abstract – **In this paper we present a theoretical analysis of a number of channel estimation schemes for UMTS WCDMA Rake receiver. In general these schemes are data aided/decision directed and consist of two blocks, channel sample preselection and channel predictor. The first block is introduced to minimize the impact of incorrect data decisions on the channel estimator. This is what makes our schemes different from the solutions known so far. The result of channel predictor (the second block) is used to generate the data estimate for the next sampling interval which is an element used for these purposes by a number of authors.**

Keywords – **UMTS, WCDMA, Rake receiver, channel estimationn**

I. INTRODUCTION

A Rake receiver synchronization in a CDMA system includes, code delay estimation for each path and complex channel coefficient estimation (amplitude and phase) to be used in rake combiner. Code delay estimation is a well elaborated field and a number of references can be found in the open literature covering this issue [2-4]. In this work we will assume that the code delays are already estimated and we focus only on estimating the channel coefficients.

In [5] a ML based algorithm for multiuser (MU) channel estimation is discussed. The second order statistics was used so that the information about the phase could not be extracted. Due to complexity of the likelihood function only single path propagation model has been considered. Further modification of the cost function should be introduced in order to make possible the multipath channel coefficient estimation. Some options, described in [5], remain still to complex to be considered for practical applications.

An algorithm for joint detection and estimation of amplitudes, but with known delays, was developed in [6]. In that paper a tree-search method was used together with least-squares estimation.

A neural-network based algorithm [7] implicitly estimates delays and amplitudes using backpropagation algorithm. This kind of algorithm is rather slow, and can not be used for tracking fast fading parameters.

A recursive signal processing using the Viterbi algorithm for joint tracking of amplitudes and delays was described in [8]. An extended Kalman estimator is used to update the amplitude and delay estimates for each survivor sequence in Viterby Algorithm. The Nyquist samples are used as sufficient statistics. The resulting algorithm requires storage of the survivor sequences and is difficult to simplify.

In the case of advanced CDMA receivers, a rough channel estimates could be found by subtracting the estimated overall MAI from the matched filter outputs, removing the modulation by pilot or estimated symbols and then additionally filtering the result of such preprocessing.

In systems with high processing gain, power control and low crosscorrelations between the users the output of the matched filter of the *k*th user synchronized to the *l*th path, can be considered separately as a digital phase modulated signal in flat fading and Gaussian noise.

For such channels, it was shown even in classical literature, the optimum receiver results in a structure that follows Kailath's separation theorem. In order words, the optimum receiver consist of an estimator that delivers MMSE estimates of the fading distortion and a detector that utilizes these estimates. In [10] a pilot tone signaling to provide channel amplitude and phase information for the detection and the adaptive control of the transmitter, is used.

If the pilot symbols are not available a possible approach is to perform detection using an old channel estimate and then use the detected data to update the channel estimate in a decision-feedback manner.

The main problem of using decision-aided channel estimators comes form the two stochastic processes "data" and "channel" being involved simultaneously.

An alternative is a data aided approach, were a sequence of channel estimates ("snapshots") is obtained via training segments multiplexed into the data stream.

The above solutions are based on using training sequence with or without additional exploitation of the feedback decisions. The existing UMTS standards support such solutions by providing pilot symbols and this approach is the basis for the algorithms used in this paper.

II. SYSTEM MODEL

In W-CDMA (UMTS FDD Mode), the data transmission is organized in frames of 10ms, each divided in slots of 0.625 ms. The slot structure in the uplink is well known and may be seen in [1]. It consists of both data bits (dedicated physical data channel-DPDCH) and control information (dedicated physical control channel-DPCCH): The number of data bits per slot N_d depends on the data rate of the link and varies from 10 to 640. The number of control bits is fixed to 10. It consists of pilot symbols for channel estimation, transmit power control (TPC) bits and transport frame indicator (TFI) bits. The data bits $d(i)$ of the DPDCH are spread with an orthogonal variable spreading factor (OVSF) sequence $s_d(k)$

[1]Autor is with the Faculty of Electronic Engineering, Beogradska 14, 18000 Nis, Serbia and Montenegro, nemilose@elfak.ni.ac.yu

[2]Autor is with the Faculty of Electronic Engineering, Beogradska 14, 18000 Nis, Serbia and Montenegro,

[3]Autor is with the Faculty of Electronic Engineering, Beogradska 14, 18000 Nis, Serbia and Montenegro,

[4]Autor is with the Faculty of Electrotechnics, 11000 Belgrade, Serbia and Montenegro,

(channelization sequence for data) with chip rate of 4.096 Mcps. For the fixed chip rate, the spreading factor N is determined by the number of bits per slot and varies from 4 to 256. The control bits $d_c(i)$ of the DPCCH are spread by factor N_c= 256 by using the code $s_c(k)$ (channelization code for control channel, orthogonal to $s_d(k)$).

The chip streams of data and control channel are I/Q multiplexed and scrambled with the complex scrambling sequence $s_{scr}(k)$. Thus the transmit signal can be represented as

$$t(k) = s_{scr}(k)\{s_d(k)d(\lfloor k/N \rfloor) + js_c(k)d_c(\lfloor k/N \rfloor)\} \quad (1)$$

The channel impulse response is defined as

$$h(t,\tau) = \sum_l C^{(l)}(t)\delta(t-\tau_l) \quad (2)$$

In the analysis we use the channel model specified by Rec.ITU-R M.1225. For the channel coefficient correlation function we use the Jack's model where $\rho_c(\tau) = J_0(\omega_D\tau)$.

The general block diagram of Rake receiver may be seen in [1]. A direct sequence spread spectrum signal (DSSS) after propagation through a multipath channel, will be despread in L Rake fingers, and the output of the lth despreading circuit will have the form

$$y_k^{(l)} = C_k^{(l)} \cdot d_k + n_k^{(l)} \quad (3)$$

This operation is performed separately in both, data and control channel. We assume a perfect code synchronization per finger. In (3) k is the sampling index, l = 1, … L is the path index, $C_k^{(l)}$ is the complex channel coefficient, $d_k=d(k)$ for data channel and $d_k=d_c(k)$ for control channel. Parameter $n_k^{(l)}$ is the overall noise in the l-th Rake finger including residual multipath interference (MPI), multiple access interference (MAI), inter-channel interference (ICI) and thermal noise. All together, this component will be approximated as Gaussian noise with zero mean and variance $\sigma^{(l)2}$.

As it was explained above, in the DPCCH channel the pilot symbols are used to facilitate the channel estimate. A sequence of N_p bits is periodically inserted into the control channel stream and used as preamble for channel estimation. In the remaining interval the tentative decisions can be used to remove the modulation from the signal components $y_k^{(l)}$.

III. Performance Analysis

Detailed explanation of channel estimation algorithm, as well as some simulation results, may be found in [1]. In this paper, a theoretical analysis of sustem performance is given.

In general signal $\overline{C}_k^{(l)}$, that will be used for further processing (predictor) can be represented as

$$\overline{C}_k^{(l)} = C_k^{(l)} + \varepsilon_k^{(l)} + n'_k \quad (4)$$

where $\varepsilon_k^{(l)}$ is error due to preprocessing and $n'_k = S(n_k^{(l)})$ is the noise sample after preselection processing. The error and noise will be considered as an equivalent noise of zero mean and variance $\sigma_n^2 = \sigma_\varepsilon^2 + \sigma^2 + 2\rho_{n\varepsilon}\sigma_\varepsilon\sigma$. Parameter $\rho_{n\varepsilon}$ characterizes correlation between the input noise (variance σ^2) and preprocessing error ε (variance σ_ε^2). In this analysis we do not intend to find an exact relation for $\rho_{n\varepsilon}$ but rather use upper and lower bounds on the performance for $\rho_{n\varepsilon}$ = 0 and $\rho_{n\varepsilon}$ = 1 respectively. In the sequel we discuss this parameter for different preselection functions.

 # No preselection

$$\begin{aligned} &\overline{C}_k^{(l)} \Rightarrow C_k^{(l)}(1-2P_e) \\ &\sigma_\varepsilon^2 = 2P_e\sigma_c^2 \end{aligned} \quad (5)$$

#1 Hard decision

Let us use notation $1-P_d = P(v_k^{(l)} < th) = P_c$ for preselection correct decision. Then we have

$$\begin{aligned} &\overline{C}_k^{(l)} \Rightarrow \\ &C_k^{(l)}(1-P_e)P_c + 0(1-P_e)P_d - 0P_eP_c - C_k^{(l)}P_eP_d = C_k^{(l)} + \varepsilon \\ &\varepsilon \Rightarrow -C_k^{(l)}(P_e+P_d) \\ &\sigma_\varepsilon^2 = (P_e+P_d)\sigma_c^2 \end{aligned} \quad (6)$$

#2 Interpolation

$$\begin{aligned} &\overline{C}_k^{(l)} \Rightarrow \\ &C_k^{(l)}(1-P_e)P_c + C_{k-1}^{(l)}(1-P_e)P_d + C_{k-1}^{(l)}P_eP_c - P_eP_dC_k^{(l)} = C_k^{(l)} + \varepsilon \end{aligned} \quad (7)$$

By introducing $C_k^{(l)} = C_{k-1}^{(l)} - \Delta C_k^{(l)}$ we have

$$\begin{aligned} \varepsilon &\Rightarrow -(P_e+P_d-P_eP_d)C_k^{(l)} + (1-P_e)P_d(C_k^{(l)} + \Delta C_k^{(l)}) \\ &\quad + P_eP_c(C_k^{(l)} + \Delta C_k^{(l)}) - P_eP_dC_k^{(l)} \\ &= -2P_eP_dC_k^{(l)} + (P_e+P_d-2P_eP_d)\Delta C_k^{(l)} \\ \sigma_\varepsilon^2 &= 2P_eP_d\sigma_c^2 + (P_e+P_d-2P_eP_d)\sigma_{\Delta c}^2 \end{aligned} \quad (8)$$

#3 Substitution

$$\begin{aligned} &\overline{C}_k^{(l)} \Rightarrow \\ &(1-P_e)P_cC_k^{(l)} + (1-P_e)P_d\hat{C}_k^{(l)} + P_eP_c\hat{C}_k^{(l)} - P_eP_dC_k^{(l)} = C_k^{(l)} + \varepsilon \end{aligned} \quad (9)$$

By introducing $\hat{C}_k^{(l)} = C_k^{(l)} + \overline{\varepsilon}$ we have

$$\begin{aligned} \sigma_\varepsilon^2 &= 2P_eP_d\sigma_c^2 + (P_e+P_d-2P_eP_d)\sigma_\varepsilon^2 \\ &= \frac{2P_eP_d}{P_e+P_d-2P_eP_d}\sigma_c^2 \end{aligned} \quad (10)$$

#4 Alternation

$$\begin{aligned} &\overline{C}_k^{(l)} \Rightarrow \\ &(1-P_e)P_cC_k^{(l)} - (1-P_e)P_dC_k^{(l)} + P_eP_cC_k^{(l)} - P_eP_dC_k^{(l)} = C_k^{(l)} + \varepsilon \\ &\varepsilon \Rightarrow -2P_d \\ &\sigma_\varepsilon^2 = 2P_d\sigma_c^2 \end{aligned} \quad (11)$$

The input signal in a Rake finger is

$$y = Cd + n \quad (12)$$

After multiplication by the estimated value of the channel coefficient we have

$$\begin{aligned} f &= (Cd+n)(C^* + \Delta C^*) \\ &= CC^*d + (C\Delta C^*d + nC^* + n\Delta C^*) \end{aligned} \quad (13)$$

The first term is the useful signal. The remaining terms are an equivalent noise.

So, the signal to noise ratio in a finger is

$$SNR_f = \frac{\sigma^2_{signal}}{\sigma^2_e} \tag{14}$$

where

$$\begin{aligned}
\sigma^2_{signal} &= E^2\{(C)(C^*)\} = \sigma^4_c \\
\sigma^2_e &= E\{(C\Delta C^* d + nC^* + n\Delta C^*)(C\Delta C^* d + nC^* + n\Delta C^*)^*\} \\
&= E\{C\Delta C^* C^* \Delta C\} + E\{C\Delta C^* n^* C\} + E\{C\Delta C^* n^* \Delta C\} \\
&+ E\{nC^* C^* \Delta C\} + E\{nC^* n^* C\} \\
&+ E\{nC^* n^* \Delta C\} + E\{n\Delta C^* C^* \Delta C\} \\
&+ E\{n\Delta C^* n^* C\} + E\{n\Delta C^* n^* \Delta C\}
\end{aligned} \tag{15}$$

To evaluate the second term we use the relation

$$\begin{aligned}
E\{x_1x_2x_3x_4\} &= E\{x_1x_2\}E\{x_3x_4\} + E\{x_1x_3\}E\{x_2x_4\} + \\
&+ E\{x_1x_4\}E\{x_2x_3\}
\end{aligned} \tag{16}$$

Losses are defined as

$$LS = \frac{SNR_f(\Delta C = 0)}{SNR_f} \tag{17}$$

Bearing in mind that $\rho_{cn} = 0$ for a real signal we have

$$\begin{aligned}
\sigma^2_e &= 2(\rho_{c\Delta c}\sigma_c\sigma_{\Delta c})^2 + \sigma^2_c\sigma^2_{\Delta c} + \sigma^2_c\rho_{n\Delta c}\sigma\sigma_{\Delta c} + \\
&+ 2\rho_{c\Delta c}\sigma_c\sigma_{\Delta c}\rho_{n\Delta c}\sigma\sigma_{\Delta c} + \rho_{n\Delta c}\sigma\sigma_{\Delta c}\sigma^2_c + \sigma^2\sigma^2_c + \\
&+ \sigma^2\rho_{c\Delta c}\sigma_c\sigma_{\Delta c} + 2\rho_{n\Delta c}\sigma\sigma_{\Delta c}\rho_{c\Delta c}\sigma_c\sigma_{\Delta c} + \\
&+ \sigma^2\rho_{c\Delta c}\sigma_c\sigma_{\Delta c} + 2(\rho_{n\Delta c}\sigma\sigma_{\Delta c})^2 + \sigma^2\sigma^2_{\Delta c}
\end{aligned} \tag{18}$$

Like in the previous case we have no intention to go into a detailed analysis of correlation functions in the previous equation. Instead we will deal again with upper and lower bounds of σ^2_e obtained for ρ_{ab} equal one and zero, respectively.
Simillary, in the case of a complex signal we get

$$\begin{aligned}
\sigma^2_e &= 4\rho_{c\Delta c}\rho_{nc}\sigma\sigma^2_c\sigma_{\Delta c} + 2\rho_{c_1c_2}\rho_{\Delta cn}\sigma_{\Delta c}\sigma\sigma^2_c + \\
&+ 2\rho_{c\Delta c}\sigma^2\sigma_{\Delta c}\sigma_c + 4\rho_{\Delta cn}\rho_{cn}\sigma_{\Delta c}\sigma^2\sigma_c + \\
&+ \sigma^2_c\sigma^2_{\Delta c} + 2\rho^2{}_{c\Delta c}\sigma^2_c\sigma^2_{\Delta c} + \sigma^2\sigma^2_c + 2\rho_{nc}{}^2\sigma^2\sigma^2_c + \\
&+ \sigma^2\sigma^2_{\Delta c} + 2\rho^2_{n\Delta c}\sigma^2\sigma^2_{\Delta c} + 2\rho_{nc}\sigma\sigma^2_{\Delta c}\sigma_c + \\
&+ 4\rho_{\Delta cn}\rho_{c\Delta c}\sigma^2_{\Delta c}\sigma\sigma_c
\end{aligned} \tag{19}$$

For the channel model we accept

$$C_k = \rho C_{k-1} + n_{ck} \tag{20}$$

where n_{ck} is modeling error (zero mean Gaussian variable with variance $\sigma^2_c(1-\rho^2)$.
The channel is estimated using a smoother, adaptive linear predictor with LMS algorithm and with Kalman filter.
The channel smoother is producing

$$\hat{C}_k = \frac{1}{K}\sum_{i=1}^{K}(C_{k-i} + n_{k-i}) \tag{21}$$

The Rake finger output signal is

$$\begin{aligned}
y_k\hat{C}_k^* &= (C_k + n_k)\left\{\frac{1}{K}\sum_{i=1}^{K}(C^*_{k-i} + n^*_{k-i})\right\} = \\
&\frac{C_k}{K}\sum_i C^*_{k-i} + \left\{\frac{C_k}{K}\sum_i n^*_{k-i} + \frac{n_k}{K}\sum_i C^*_{k-i} + \frac{n_k}{K}\sum_i n^*_{k-i}\right\}
\end{aligned} \tag{22}$$

The useful signal power can be represented as

$$\sigma^2_{signal} = E^2\left\{\frac{1}{K}C_k\left(\sum_i C^*_{k-i}\right)\right\} = \frac{\sigma^4_c}{K^2}\left(\sum_{i=1}^{K}\rho_c(i)\right)^2 \tag{23}$$

One can show that the equivalent noise power for a real signal is given as

$$\sigma^2_e = \frac{\sigma^2(\sigma^2 + \sigma^2_c)}{K} + \frac{\sigma^2\sigma^2_c}{K^2}\sum_{i=1}^{K}\sum_{l=1}^{K}\rho_c(i-l) \tag{24}$$

and the signal to noise ratio per finger is given again by (28)
In the case of a complex signal we have

$$\sigma^2_e = \frac{\sigma^2\sigma^2_c}{K} + \frac{\sigma^2_c\sigma^2}{K^2}\sum_i\sum_l(\rho_c(i-l)) + \frac{1}{K}\sigma^4 \tag{25}$$

For a transversal filter with coefficients

$$\mathbf{W}_k = (w_k, w_{k-1}, w_{k-2}, ..., w_{k-L}) \tag{26}$$

and channel sample vector

$$\mathbf{C}_k = (C_k, C_{k-1}, C_{k-2}, ..., C_{k-L}) \tag{27}$$

the steady state tracking error variance (minimum mean square error) is given as

$$\sigma^2_\varepsilon = E\left\{\left|C_k\right|^2\right\} - \mathbf{P}^T\mathbf{W}_o = \sigma^2_c - \mathbf{P}^T\mathbf{W}_o \tag{28}$$

where $\mathbf{W}_o$ is the optimum solution for the prediction coefficients obtained from

$$-2\mathbf{P} + 2\mathbf{R}\mathbf{W}_o = 0 \tag{29}$$

and the vector $\mathbf{P}$ and the matrix $\mathbf{R}$ are defined as

$$\begin{aligned}
\mathbf{P}^T &= E[C_k\mathbf{C}_k] \\
\mathbf{R} &= E[\mathbf{C}_k\mathbf{C}^T_k] = [\rho(k-m)], \quad k, m = 1,..L
\end{aligned} \tag{30}$$

Parameter SNR_f is given by eq(28) with σ^2_ε given by eq(28).
For Kalman filter the estimation error is solution to Ricatti equation [18] which for this case can be expressed as

$$\sigma^2_\varepsilon = \frac{\sigma^2\left[\rho^2\sigma^2_\varepsilon + (1-\rho^2)\sigma^2_c\right]}{\rho^2\sigma^2_\varepsilon + (1-\rho^2)\sigma^2_c + \sigma^2} \tag{31}$$

and the SNR_f is given again by eq(14).

For the BER we use the standard results for the diversity of order L [9].

Expressions for SNR_f per finger should be further modified by modifying the equivalent noise to include interference between different paths and different users. It can be shown that, in case of QPSK modulation, BER may be expressed as

$$BER = \frac{1}{2}\sum_{k=1}^{L}\pi_k\left[1-\sqrt{\frac{\bar{\gamma}_k}{2+\bar{\gamma}_k}}\right] \tag{32}$$

where

$$\pi_k = \prod_{\substack{i=1 \\ i\neq k}}^{L}\frac{\bar{\gamma}_k}{\bar{\gamma}_k-\bar{\gamma}_i} \tag{33}$$

and $\bar{\gamma}_k$ is the average *SNR* for the *k*-th path.

IV. Numerical Results

Fig. 1. shows bit error probability as a function of receiver velocity. Signal to noise ratio per bit is chosen to be SNR = 5 dB, and sample preselection/modification function is *substitution* with th=0.

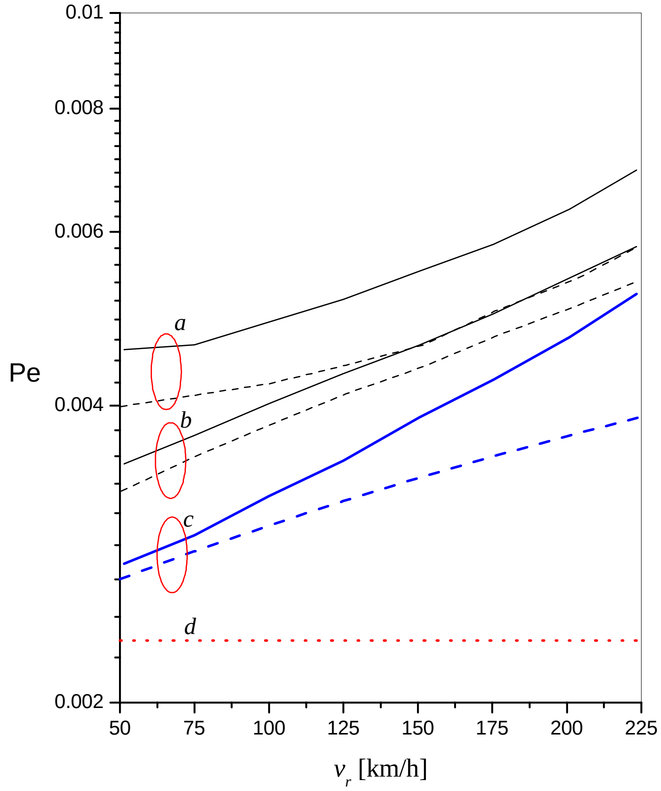

Fig. 1. Bit error probability as a function of receiver velocity
- - - - - theoretical results
——— simulation results

There are results for different channel estimators (*a* – Smoother, *b* – adaptive linear predictor with LMS algorithm, *c* – Kalman filter), as well as for the case if there is a perfect channel estimation (*d*). It can be seen that the theoretical results are very simillar to the simulation ones, especially for low receiver velocity. As expected Kalman filter has the best performances, and the smoother has the worst ones.

V. Conclusions

In this paper we presented a approximate theoretical analysis of a number of channel estimation schemes for UMTS WCDMA Rake receiver. The results show that theoretical performances are in high agreement with the simulation ones. The agreement is higher for low receiver velocities. Since Kalman filter has more information about the channel than the other estimators, its performances are the best. Smoother has the poorest performances, but it is a acceptable solution if we need a simple estimator.

References

[1] B. Dimitrijević, N. Milošević, Z. Nikolić, "Procena feding kanala kod UMTS WCDMA Rake prijemnika", *Proceedings of TELFOR 2003*, Beograd, Novembar 2003.

[2] Polydoros, A., and C. L. Weber, "A Unified Approach to Serial Search Spread-Spectrum Code Acquisition—Part 1: General Theory," *IEEE Transactions on Communications*, Vol. 32, No. 5, May 1984, pp. 542-549.

[3] Polydoros, A., and C. L. Weber, "A Unified Approach to Serial Search Spread-Spectrum Code Acquisition—Part II: A Matched-Filter Receiver," *IEEE Transactions on Communications*, Vol. 32, No. 5, May 1984, pp. SSO-560.

[4] A. Polydoros and S. Glisic - Code Synchronization: A Review of Principles and Techniques - In S.Glisic and P.Leppanen ed. *Code Division Multiple Access*, Kluwer Academic Publishers, 1995

[5] J. S. Bensley and B. Aazhang, " Subspace-Based Channel Estimation for Code Division Multiple Access Communications & Systems", *IEEE Transactions on Communications*, vol. 44, No. 8, pp. 1009-1020, August 1996.

[6] Z. Xie, C. Rushforth, R. Short and T. Moon, "Joint signal detection and parameter estimation in multiuser communications," *IEEE Transactions on Communications*, vol. 41, pp. 1208-1216, Aug. 1993.

[7] B. Aazhang, B. Paris and G. Orsak, "Neural networks for multiuser detection in code-division multiple-access communications," *IEEE Transactions on Communications*, vol. 40, pp. 1212-1222, July 1992.

[8] R. lltis, "A digital receiver for demodulation of CDMA waveforms with a-priori unknown delays and amplitudes," in *Proceedings of the IEEE Military Communications Conference*, (McLean, VA), pp. 113-116, Nov. 1991.

[9] Proakis, *Digital Communiactions*, Mc GrawHill, 1998

[10] T. Kailath, "A general likelihood formula for random signals in Gaussian noise," *IEEE Trans. Inform. Theory*, vol. IT-15, May 1969.