A Research Of Maintenance-Effect Failures With Markov's Modelling

Georgi D. Nenov[1] and Borislav D. Boiadjiev[2]

Abstract – **To ensure the properly work without failures of the communication and railway signalling systems is recommended to maintain them. But the preventive maintenance has not only advantages because after it there are some failures caused of work within systems. These failures are called "Maintenance-effect failures".**

To analyze these failures there must be an appropriate model. The maintenance will be optimized with assistance of this model. The research of maintenance-effect failures with Markov's modelling is used in the paper.

Keywords – **Maintenance-effect failures, Markov's models, pre-failures, sudden and parametric failures.**

The communication and railway signalling systems ensure railway traffic against accidents and they must work properly without failures. This is the reason to maintain these systems.

The preventive maintenance of the technical systems is introduced to avoid the parametric (gradual) and to repair the sudden failures, with ensuring of maximum availability of the systems to perform its function algorithm. But it has as well disadvantages because of some failures caused when working within systems. These failures are called "Maintenance-effect failures".

To analyze the mentioned failures there must be an appropriate mathematical model and the preventive maintenance will be optimized with assistance of it. The Markov's modelling for researching of all types of failures and the process of the maintenance of the systems is offered in this paper.

Suggested model is shown on Fig. 1. It has three states: S_0 – a fault-free state; S_1 – pre-fault state and S_2 – a fault state. The states S_0 and S_1 reflect the availability state of the system and S_2 – reflects the unavailability state.

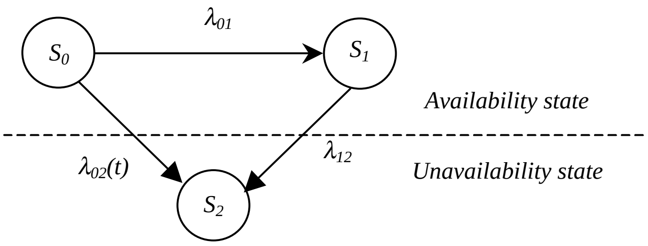

Fig. 1. Three states Markov's model

The model includes an additional fictitious "pre-fault" state (S_1), emulating the "hidden" failures appearing in the system, they are discovered and removed in the preventive maintenance. These hidden failures are identified as "pre-faults". If the system is in the "pre-fault" state, it is still available, since a hidden failure is not yet discovered. The removal of the "pre-fault" state on time prevents arising another one. If the "pre-fault" state is discovered before the preventive maintenance, it arises in the unavailability state. These types of failures are parametric failures and they are emulated by transition intensities $\lambda_{01} = const$ and $\lambda_{12} = const$.

The research of the "maintenance-effect failures" is possible by including the time dependence in the parameter $\lambda_{02}(t)$, which is the transition intensity from the state S_0 to the state S_2. This intensity is

$$\lambda_{02}(t) = \lambda_{02} + \lambda_m \cdot \left(\frac{1}{1 + (t - T_m)^2} \right), \tag{1}$$

where $\lambda_{02} = const$ is intensity of the sudden failures; $\lambda_m = const$ and T_m are respectively the maximal failure intensity and the time after the end of maintenance when the negative effects of it are maximal.

The characteristic of $\lambda_{02}(t)$ is shown on Fig. 2.

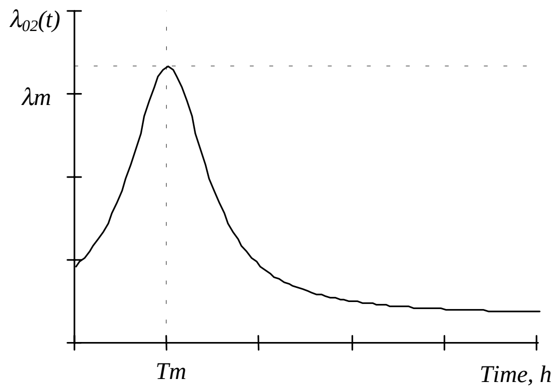

Fig. 2. The $\lambda_{02}(t)$ characteristic

Thus obtained model emulate three type of failures: the sudden failures – with intensity $\lambda_{02}(t)$; the parametric failures – with intensities λ_{01} and λ_{12}; and the "maintenance-effect" failures – through $\lambda_m(t)$ and T_m.

To analyse the above mentioned model, is necessary to solve its system of differential equations:

[1] Georgi Dimitrov Nenov, is with the Department of Communication and Railway Signalling Engineering in "Todor Kableshkov" Higher School of Transport. 158 Geo Milev Street, Sofia 1574, Bulgaria.

[2] Borislav Draganov Boiadjiev, is with the Department of Communication and Railway Signalling Engineering in Todor Kableshkov Higher School of Transport. 158 Geo Milev Street, Sofia 1574, Bulgaria. E-mail: bboiadjiev@vtu.bg.

$$\begin{cases} \dfrac{dP_0(t)}{dt} = -\left[\lambda_{01} + \lambda_{02} + \lambda_m \cdot \left(\dfrac{1}{1+(t-T_m)^2}\right)\right] . P_0(t), \\ \dfrac{dP_1(t)}{dt} = \lambda_{01}.P_0(t) - \lambda_{12}.P_1(t), \\ \dfrac{dP_2(t)}{dt} = \left[\lambda_{02} + \lambda_m \left(\dfrac{1}{1+(t-T_m)}\right)\right] . P_0(t) + \lambda_{12}.P_1(t), \\ P_0(t) + P_1(t) + P_2(t) = 1. \end{cases} \quad (2)$$

with the following initial conditions:

$$P_0(0) = 1; \quad P_1(0) = 0; \quad P_2(0) = 0. \quad (3)$$

The first equation of the system given in Eqs. (2), is solved as following:

$$\frac{dP_0(t)}{dt} = -\left[\lambda_{01} + \lambda_{02} + \lambda_m \cdot \left(\frac{1}{1+(t-T_m)^2}\right)\right] . P_0(t), \quad (4)$$

$$\frac{dP_0(t)}{P_0(t)} = -\left[\lambda_{01} + \lambda_{02} + \lambda_m \cdot \left(\frac{1}{1+(t-T_m)^2}\right)\right] . dt, \quad (5)$$

$$\int \frac{dP_0(t)}{P_0(t)} = -\int \left[\lambda_{01} + \lambda_{02} + \lambda_m \cdot \left(\frac{1}{1+(t-T_m)^2}\right)\right] . dt, \quad (6)$$

$$\ln P_0(t) = -[(\lambda_{01} + \lambda_{02}).t + \lambda_m.arctg(t-T_m) + C]. \quad (7)$$

When $t = 0$ it is got *C*, i.e:

$$\ln P_0(0) = -[\lambda_m.arctg(-T_m) + C], \quad (8)$$

$$0 = -\lambda_m.arctg(-T_m) - C, \quad (9)$$

$$C = -\lambda_m.arctg(-T_m) = \lambda_m.arctg(T_m). \quad (10)$$

And for $P_0(t)$ is got:

$$\ln P_0(t) = -(\lambda_{01} + \lambda_{02}).t - \\ -\lambda_m.arctg(t-T_m) - \lambda_m.arctg(T_m), \quad (11)$$

$$P_0(t) = e^{-\lambda_m.arctg(T_m)}.e^{-[(\lambda_{01}+\lambda_{02}).t+\lambda_m.arctg(t-T_m)]}. \quad (12)$$

The second equation of the system given in Eqs. (2), is solved under initial condition $P_1(0) = 0$, since at the initial moment the system is fault-free (it is in S_0 state). This equation is a linear differential equation:

$$\frac{dP_1(t)}{dt} + \lambda_{12}.P_1(t) = \lambda_{01}.P_0(t), \quad (13)$$

It is solved by means of the formula given in [2] and it is got:

$$P_1(t) = e^{-\lambda_{12}.t}.[C_1 + \int \lambda_{01}.P_0(t).e^{\lambda_{12}.t}dt]. \quad (14)$$

$P_0(t)$ is replaced from Eq. (12) and it got:

$$P_1(t) = e^{-\lambda_{12}.t}.[C_1 + \lambda_{01}.e^{-\lambda_m.arctg(T_m)} * \\ * \int e^{-(\lambda_{01}+\lambda_{02}+\lambda_{12}).t}.e^{-\lambda_m.arctg(t-T_m)}dt]. \quad (15)$$

The exponent $e^{-\lambda_m.arctg(t-T_m)}$ in Eq. (15) is expanded in a row, using the known formula [2]:

$$e^{-\lambda_m.arctg(T_m)} \approx 1 + (-\lambda_m.arctg(t-T_m)) = \\ = 1 - \lambda_m.arctg(t-T_m). \quad (16)$$

Then, for $P_1(t)$ is got:

$$P_1(t) \approx e^{-\lambda_{12}.t}.\{C_1 + \lambda_{01}.e^{-\lambda_m.arctg(T_m)} * \\ * \int e^{-(\lambda_{01}+\lambda_{02}+\lambda_{12}).t}.[1 - \lambda_m.arctg(t-T_m)]dt\} \quad (17)$$

After solving the integral in Eq. (17), it is got the following:

$$P_1(t) \approx e^{-\lambda_{12}.t}.\left\{C_1 + \frac{\lambda_{01}.e^{\lambda.t-\lambda_m.arctg(T_m)}}{\lambda} * \right. \\ * \{1 - \lambda_m.\{arctg(t-T_m) - \\ \left. - \frac{1}{\lambda.[1+(t-T_m)^2]} - \frac{2.(t-T_m)}{\lambda.[1+(t-T_m)^2]^2}\}\}\right\} \quad (18)$$

where, $\lambda = \lambda_{12} - \lambda_{01} - \lambda_{02}$.

The C_1 constant is got when $t = 0$ and the initial condition for $P_1(t)$:

$$\Rightarrow C_1 = \frac{\lambda_{01}.e^{-\lambda_m.arctg(T_m)}}{\lambda}.\left\{\lambda_m \cdot \left[\frac{2.T_m}{\lambda.(1+T_m^2)^2} - \right.\right. \\ \left.\left. - \frac{1}{\lambda.(1+T_m^2)} - \lambda_m.arctg(T_m)\right] - 1\right\}. \quad (19)$$

Finally for $P_1(t)$ the following expression is got:

$$P_1(t) \approx \frac{\lambda_{01}.e^{-[\lambda_{12}.t+\lambda_m.arctg(T_m)]}}{\lambda}\left\{e^{\lambda.t}.\{1 - \lambda_m.\{arctg(t-T_m) - \right. \\ - \frac{1}{\lambda.[1+(t-T_m)^2]} - \frac{2.(t-T_m)}{\lambda.[1+(t-T_m)^2]^2}\}\} + \\ \left. + \lambda_m \cdot \left[\frac{2.T_m}{\lambda.(1+T_m^2)^2} - \frac{1}{\lambda.(1+T_m^2)} - \lambda_m.arctg(T_m)\right] - 1\right\}. \quad (20)$$

Then the probability that the system is in availability state $P_{av}(t)$ is:

$$P_{av}(t) = P_0(t) + P_1(t). \quad (21)$$

The probability that the system is in the unavailability state $P_2(t)$ is got from fourth equation, given in Eqs. (2).

CONCLUSION

The submitted Markov's model of the process of the failure development, describes both sudden and parametric failures, as well as failures deriving from preventive maintenance – the "maintenance-effect" failures. That can be used for solving a number of special problems, related to reliability, efficiency and safety of communication, railway interlocking and signalling systems, as well as for researching the problem of preventive maintenance optimisation.

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